Blast From the Past!

Exam 2

Spring 2018

Problem 3 (30 pts)

Consider the following plane wave that is traveling in air:

$$\underline{E} = \left[(1 - j3)\, \underline{\hat{y}} + (2 + j)\, \underline{\hat{z}} \right] e^{-jk_0 x}$$

(a) Find the polarization (linear, circular, or elliptical) and handedness (left-handed or right-handed) for the wave.

(b) Find the axial ratio of this wave.

(c) Find the magnetic field vector for this plane wave.

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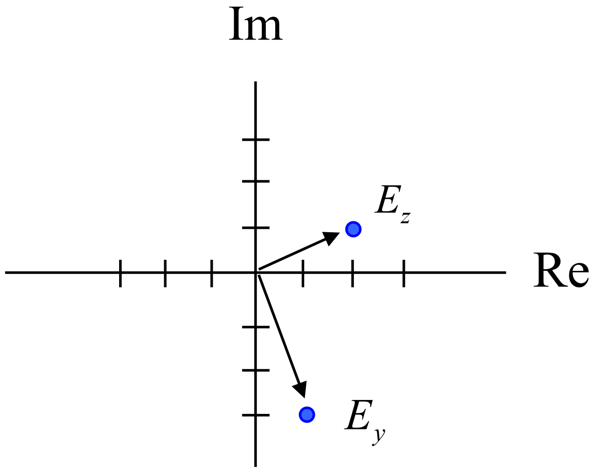
$$\underline{E} = \left[(1 - j3)\,\underline{\hat{y}} + (2 + j)\,\underline{\hat{z}} \right] e^{-jk_0 x}$$

Part (a)

At $x = 0$ we then have

$$E_y = 1 - j3$$

$$E_z = 2 + j$$



Plotting these two points in the complex plane, we see that E_z leads E_y. Therefore the wave rotates from the z axis towards the y axis in time. Since the propagation direction is the positive x axis, the wave is left-handed. The angle between the two phasors is not 90°. Also, the magnitudes are not equal. Thus we have

Polarization = LHEP

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$$\underline{E} = \left[(1 - j3)\,\underline{\hat{y}} + (2 + j)\,\underline{\hat{z}} \right] e^{-jk_0 x}$$

$$\underline{E} = \left[(1 - j3)\,\underline{\hat{x}} + (2 + j)\,\underline{\hat{y}} \right] e^{-jk_0 z}$$

Part (b)

(in rotated coordinates)

$x \rightarrow z$

$y \rightarrow x$

$z \rightarrow y$

Factoring out a $(1\text{-}j3)$ term, we have:

$$\underline{E} = \left[(1)\,\underline{\hat{x}} + \left(\frac{2 + j}{1 - j3} \right) \underline{\hat{y}} \right] e^{-jk_0 z}$$

Hence

$$E_x = 1$$

$$E_y = \frac{2 + j}{1 - j3} = 0.7071 e^{j(1.713)} = 0.7071 \angle 98.13^{\circ}$$

$$a = 1; \quad b = 0.7071; \quad \beta = 1.713 \; [\text{radians}] = 98.13^{\circ}$$

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$$\underline{E} = \left[(1 - j3)\,\underline{\hat{x}} + (2 + j)\,\underline{\hat{y}} \right] e^{-jk_0 z}$$

(in rotated coordinates)

Part (b)

We then have

$$\gamma = 35.264^{\circ}$$

$$\xi = 34.48^{\circ}$$

This gives us

$$\text{AR} = 1.456$$

Since $\xi > 0$, this confirms that the wave is LH. But we already knew this from Part (a).

$$\gamma \equiv \tan^{-1}\left(\frac{b}{a}\right)$$

$$0 \le \gamma \le 90^{\circ}$$

$$\sin 2\xi = \sin 2\gamma \sin \beta$$

$$-45^{\circ} \le \xi \le +45^{\circ}$$

$$\text{AR} = \left|\cot \xi\right|$$

$\xi > 0$: LHEP

$\xi < 0$: RHEP

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$$\underline{E} = \left[(1 - j3)\,\underline{\hat{y}} + (2 + j)\,\underline{\hat{z}} \right] e^{-jk_0 x}$$

(in original coordinates)

Part (c)

Each electric field component of the plane wave has a corresponding magnetic field component, whose amplitude is related by η_0.

Wave traveling in $+x$ direction: $\quad \dfrac{E_y}{H_z} = \eta_0 \qquad \dfrac{E_z}{H_y} = -\eta_0$

Hence, we have:

$$\underline{H} = \frac{1}{\eta_0}\left[(1 - j3)\,\underline{\hat{z}} + (2 + j)\left(-\underline{\hat{y}}\right) \right] e^{-jk_0 x}$$