Blast From the Past!

Exam 2

Spring 2013

Problem 3 (30 pts)

An air-filled rectangular waveguide has $b < a$. For a certain application, we want to make sure that all of the modes are attenuated sufficiently fast inside the waveguide, so that the entire field is attenuated very fast. Note that all modes will attenuate at least as fast as the dominant mode, if the dominant mode attenuates.

What is the largest value that the dimension a can be, so that the dominant mode is attenuated at a rate of at least A [dB/m] at a given frequency f?

Your answer should be a formula that has the result for the dimension a in terms of A and f.

(assume air-filled waveguide)

Blast From the Past!

$$\alpha = \sqrt{k_c^2 - k^2}, \quad f < f_c$$

$$\alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2}, \quad f < f_c$$

Start with:

$$A = \alpha\left(8.686\right)$$

$$\alpha = \sqrt{k_c^2 - k^2} = k_c\sqrt{1-\left(\frac{k}{k_c}\right)^2} = k_c\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

TE$_{10}$ mode:

$$\left(\frac{\pi}{a}\right)\sqrt{1-\left(\frac{f}{\left(\frac{c}{2a}\right)}\right)^2}\left(8.686\right) = A$$

$$\left(\frac{\pi}{a}\right)^2\left(1-\left(\frac{(2a)f}{c}\right)^2\right)\left(8.686\right)^2 = A^2$$

or

$$\pi^2\left(1-\left(\frac{(2a)f}{c}\right)^2\right)\left(8.686\right)^2 = A^2 a^2$$

Blast From the Past!

Collecting terms:

$$a^2\left(A^2+\frac{4\pi^2 f^2}{c^2}(8.686)^2\right)=\pi^2(8.686)^2$$

so

$$a=\left(\frac{8.686\pi}{\sqrt{A^2+\frac{4\pi^2 f^2}{c^2}(8.686)^2}}\right)$$

We must keep a smaller than this value.