Blast From the Past!

Exam 1

Fall 2018

Problem 1 (30 pts)

An electric field in free space is described by

$$\underline{\mathscr{E}}(x, y, z, t) = \underline{\hat{z}}\cos(\omega t + \pi/4 + k_0 y) \quad [\text{V/m}],$$

where $k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$.

a) Find the electric field vector in the phasor domain.

b) Find the magnetic field vector in the phasor domain.

c) Find the complex power going through a surface S in the upward sense. The surface S is a square that is 2 meters $\times$ 2 meters, and the face of the square is perpendicular to the vector $\underline{V} = \left(\underline{\hat{x}}(1) + \underline{\hat{y}}(2) + \underline{\hat{z}}(3)\right)$.

Note: Please evaluate all constants that appear in your answers.

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$$\underline{\mathscr{E}}(x,y,z,t) = \underline{\hat{z}}\cos(\omega t + \pi/4 + k_0 y) \quad [\text{V/m}]$$

$$\underline{E}(x,y,z) = \underline{\hat{z}}\, e^{j\pi/4} e^{jk_0 y} \quad [\text{V/m}]$$

$$\nabla \times \underline{E} = -j\omega\mu_0 \underline{H} \;\Rightarrow\; \underline{H} = \frac{1}{-j\omega\mu_0}\nabla \times \underline{E} \;\Rightarrow\; \underline{H} = \frac{1}{-j\omega\mu_0}\underline{\hat{x}}\left(\frac{\partial E_z}{\partial y}\right) \;\Rightarrow\; \underline{H} = \frac{1}{-j\omega\mu_0}\underline{\hat{x}}\left(jk_0\, e^{j\pi/4} e^{jk_0 y}\right)$$

$$\nabla \times \underline{E} = \underline{\hat{x}}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \underline{\hat{y}}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \underline{\hat{z}}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

Note:

$$\frac{k_0}{\omega\mu_0} = \frac{\omega\sqrt{\mu_0\varepsilon_0}}{\omega\mu_0} = \sqrt{\frac{\varepsilon_0}{\mu_0}} = \frac{1}{\eta_0} \qquad \left(\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7603\ [\Omega]\right)$$

So

$$\underline{H}(x,y,z) = \underline{\hat{x}}\left(-\frac{1}{\eta_0}\right) e^{j\pi/4} e^{jk_0 y} \quad [\text{A/m}]$$

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$$\underline{E}(x,y,z) = \underline{\hat{z}}\, e^{j\pi/4} e^{jk_0 y} \quad [\text{V/m}]$$

$$\underline{H}(x,y,z) = \underline{\hat{x}}\left(-\frac{1}{\eta_0}\right) e^{j\pi/4} e^{jk_0 y} \quad [\text{A/m}]$$

We then have:

$$\underline{S} = \frac{1}{2}\underline{E} \times \underline{H}^* = -\underline{\hat{y}}\frac{1}{2\eta_0} \qquad \left(\underline{\hat{z}} \times \underline{\hat{x}} = +\underline{\hat{y}}\right)$$

The complex power flowing through S is then:

$$P = \int_S \underline{S} \cdot \underline{\hat{n}}\, dS$$

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$$\underline{S} = -\hat{\underline{y}}\left(\frac{1}{2\eta_0}\right)$$

The unit normal to the surface is:

$$\hat{\underline{n}} = \frac{1}{\sqrt{14}}\left(\hat{\underline{x}}(1) + \hat{\underline{y}}(2) + \hat{\underline{z}}(3)\right)$$

Hence, we have:

$$P = \int_S \left(-\frac{1}{2\eta_0}\frac{2}{\sqrt{14}}\right) dS = \left(-\frac{1}{2\eta_0}\frac{2}{\sqrt{14}}\right)\int_S dS = \left(-\frac{1}{2\eta_0}\frac{2}{\sqrt{14}}\right) A = \left(-\frac{1}{2\eta_0}\frac{2}{\sqrt{14}}\right) 4$$

$$\left(A = 4\left[\mathrm{m}^2\right]\right)$$

$$P = -\frac{4}{\eta_0\sqrt{14}} \ [\mathrm{VA}]$$

Watts $= \langle \mathscr{P}(t) \rangle = \mathrm{Re}\, P = -\frac{4}{\eta_0\sqrt{14}} \ [\mathrm{W}]$

Reactive power $= \mathrm{Im}\, P = 0 \ [\mathrm{VARS}]$