ECE 3317
Applied Electromagnetic Waves

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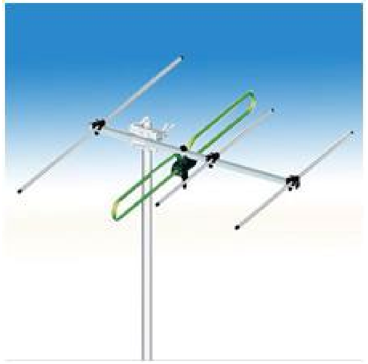
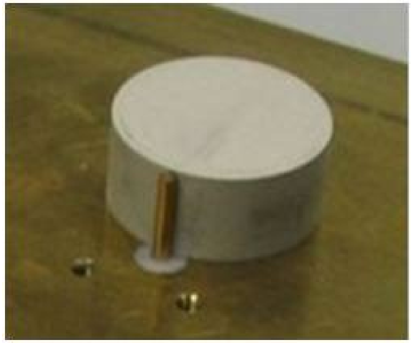
Fall 2024

Notes 21

Antenna Properties





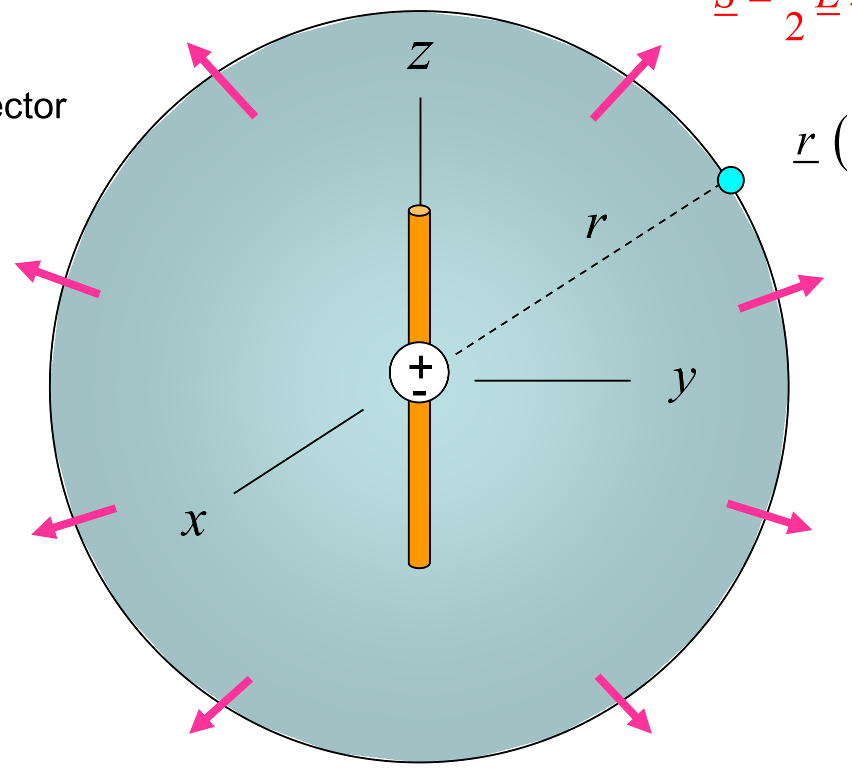
Antenna Radiation

We consider here the radiation from an antenna.

$\underline{S} = \frac{1}{2}\underline{E} \times \underline{H}^*$

$\underline{S}$ = Poynting vector

$\underline{r}\,(r, \theta, \phi)$



"far field"

$r \to \infty$

$$\underline{E}(r,\theta,\phi) = \left(\frac{e^{-jk_0 r}}{r}\right)\underline{E}^{\mathrm{FF}}(\theta,\phi)$$

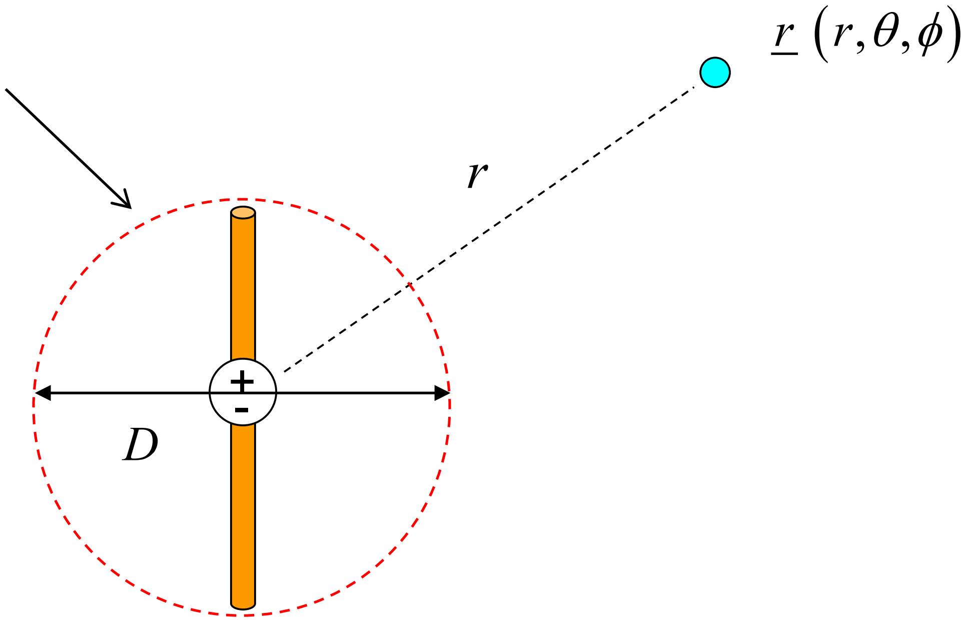
(phasor electric field)

- The far-field radiation acts like a plane wave going in the radial direction.
- The shape of the pattern in the far field is only a function of (θ, ϕ).

Antenna Radiation (cont.)

How far do we have to go to be in the far field?

Sphere of minimum diameter
D that encloses the antenna.



$$r > \frac{2D^2}{\lambda_0}$$

Antenna Radiation (cont.)

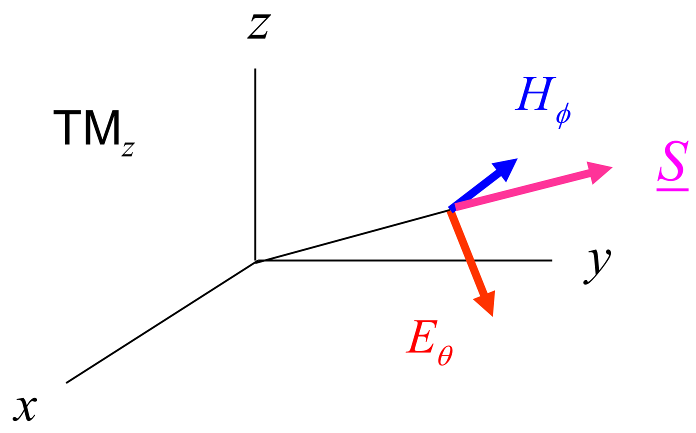
The far-field has the following form:

$$\underline{E} = \underline{\hat{\theta}}\, E_\theta + \underline{\hat{\phi}}\, E_\phi \quad \text{(phasor electric field vector)}$$

$$\underline{H} = \underline{\hat{\theta}}\, H_\theta + \underline{\hat{\phi}}\, H_\phi \quad \text{(phasor magnetic field vector)}$$

Example: vertical dipole

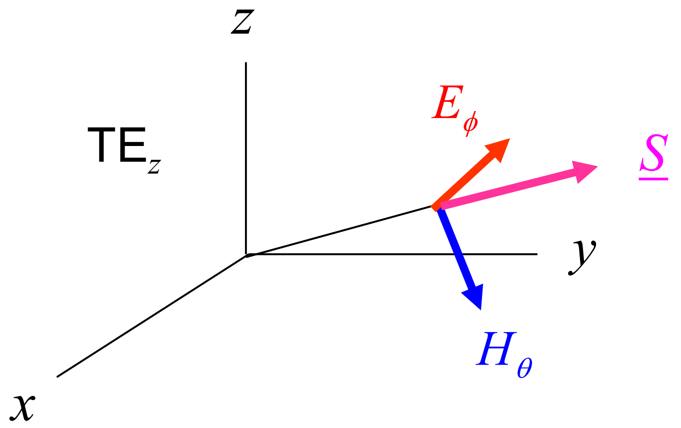
TM_z



$$\frac{E_\theta}{H_\phi} = \eta_0$$

Example: horizontal loop antenna

TE_z



$$\frac{E_\phi}{H_\theta} = -\eta_0$$

Depending on the type of antenna, either or both polarizations may be radiated (e.g., a vertical wire antenna radiates only E_θ polarization).

Antenna Radiation (cont.)

The power density in the far field is:

$$\underline{S}(r,\theta,\phi) = \underline{\hat{r}}\left(\left|E_\theta\right|^2 + \left|E_\phi\right|^2\right)\left(\frac{1}{2\eta_0}\right)$$

or

$$\underline{S}(r,\theta,\phi) = \underline{\hat{r}}\left(\frac{\left|\underline{E}\right|^2}{2\eta_0}\right)$$

Antenna Radiation (cont.)

The far field always has the following form:

$$\underline{E}(r,\theta,\phi) = \left(\frac{e^{-jk_0 r}}{r}\right)\underline{E}^{\mathrm{FF}}(\theta,\phi)$$

$$\underline{E}^{\mathrm{FF}}(\theta,\phi) \equiv \text{Normalized far-field electric field vector}$$

In decibels (dB):

$$\mathrm{dB}(\theta,\phi) = 20\log_{10}\left(\frac{\left|\underline{E}^{\mathrm{FF}}(\theta,\phi)\right|}{\left|\underline{E}^{\mathrm{FF}}(\theta_m,\phi_m)\right|}\right)$$

$$(\theta_m,\phi_m) = \text{direction of maximum radiation}$$

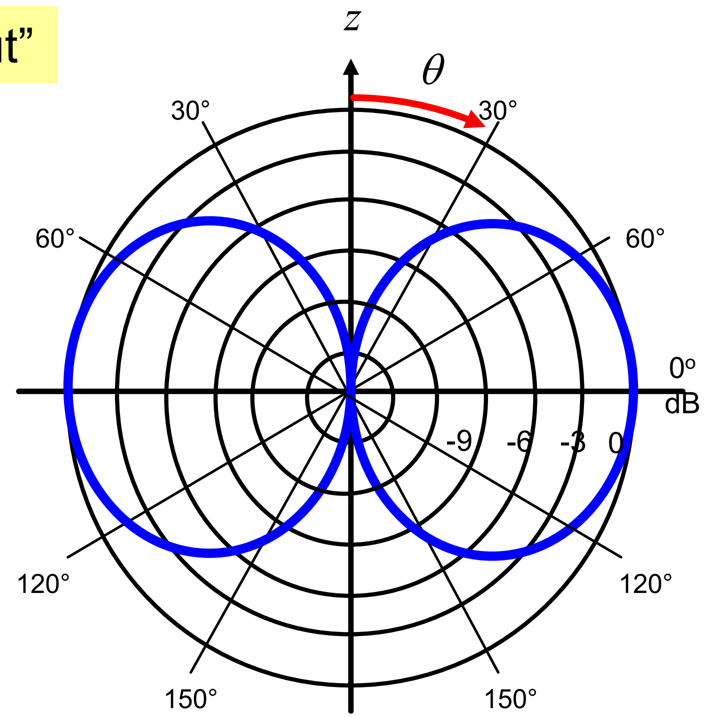
Antenna Radiation (cont.)

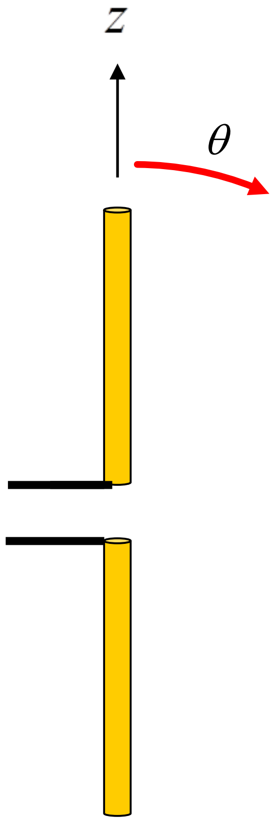
A normalized far-field pattern is usually shown vs. the angle θ (for a fixed angle $\phi = \phi_0$) in polar coordinates.

$$\mathrm{dB}(\theta,\phi_0) = 20\log_{10}\left(\frac{\left|\underline{E}^{\mathrm{FF}}(\theta,\phi_0)\right|}{\left|\underline{E}^{\mathrm{FF}}(\theta_m,\phi_0)\right|}\right)$$

The subscript "m" denotes the beam maximum.

An "elevation cut"

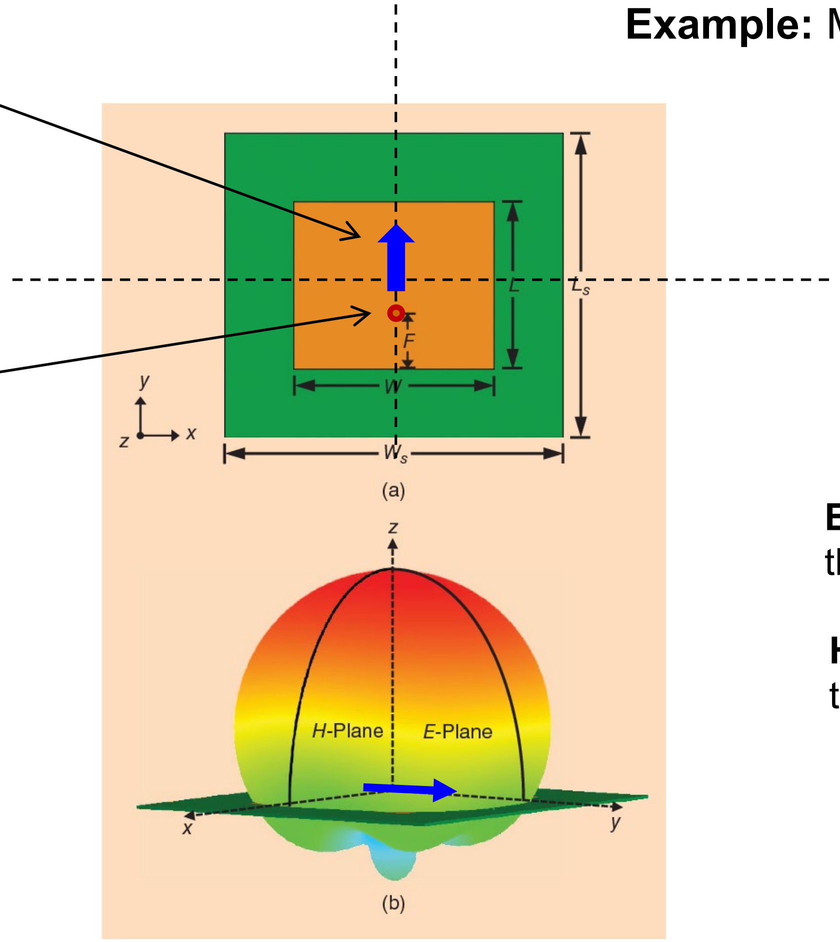




Antenna Radiation (cont.)

E-plane and H-plane

Example: Microstrip antenna



E-plane:
the plane containing the $\underline{E}$ vector. $\underline{E} = \underline{\hat{\theta}} E_\theta$

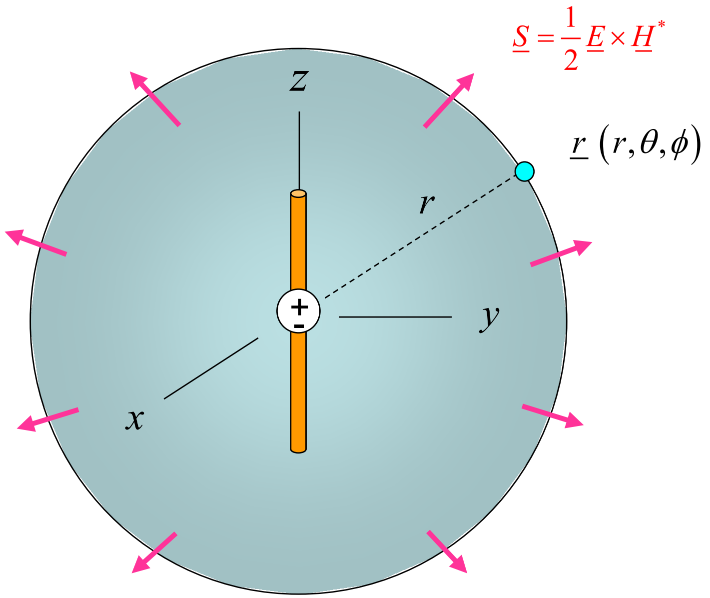
H-plane:
the plane containing the $\underline{H}$ vector. $\underline{H} = \underline{\hat{\theta}} H_\theta$

Note:
For the vertical dipole antenna, every elevation cut is an E plane.

Radiated Power

The Poynting vector in the far field is

$$\underline{S}(r,\theta,\phi) = \underline{\hat{r}}\left(\frac{\left|\underline{E}^{\mathrm{FF}}(\theta,\phi)\right|^2}{2\eta_0}\right)\left(\frac{1}{r^2}\right)\ \left[\mathrm{W/m^2}\right]$$



The total power radiated (in Watts) is then given by

$$P_{\mathrm{rad}} = \int_0^{2\pi}\int_0^{\pi}\left(\underline{S}\cdot\underline{\hat{r}}\right)r^2\sin\theta\,d\theta d\phi = \int_0^{2\pi}\int_0^{\pi}\left(\frac{\left|\underline{E}^{\mathrm{FF}}(\theta,\phi)\right|^2}{2\eta_0}\right)\sin\theta\,d\theta d\phi$$

Hence, we have

$$P_{\mathrm{rad}} = \frac{1}{2\eta_0}\int_0^{2\pi}\int_0^{\pi}\left|\underline{E}^{\mathrm{FF}}(\theta,\phi)\right|^2\sin\theta\,d\theta d\phi$$

Directivity

The directivity of the antenna in the directions (θ, ϕ) is defined as

$$D(\theta,\phi) \equiv \frac{S_r(\theta,\phi)}{P_{\text{rad}} / \left(4\pi r^2\right)} \qquad r \to \infty$$

The directivity (in a particular direction) is the ratio of the actual power density radiated in that direction to the power density that would be radiated in that direction if the antenna were an isotropic radiator (radiates equally in all directions).

In dB,

$$D_{\text{dB}}(\theta,\phi) = 10\log_{10} D(\theta,\phi)$$

Note: The directivity is sometimes referred to as the "directivity with respect to an isotropic radiator."

Directivity (cont.)

The directivity is then directly expressed in terms of the far field pattern:

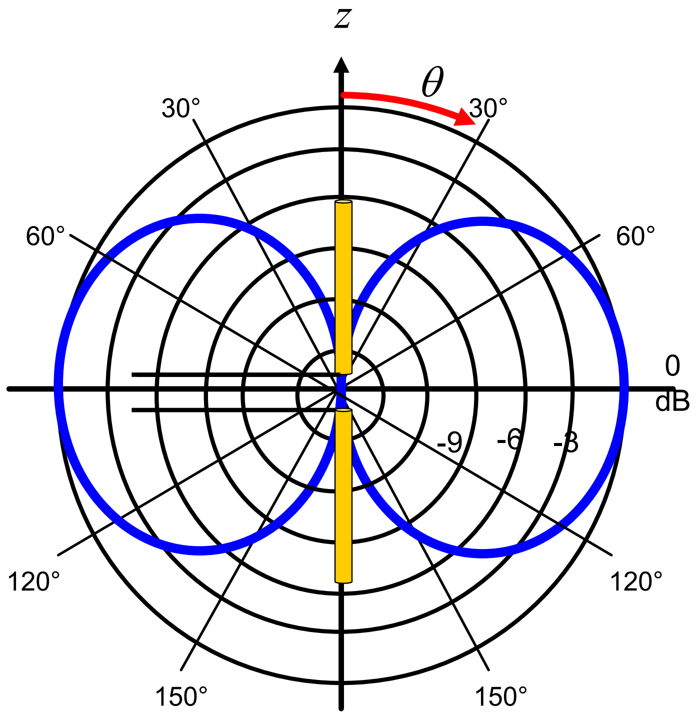
$$D(\theta,\phi) = \frac{4\pi \left| \underline{E}^{\mathrm{FF}}(\theta,\phi) \right|^2}{\int_0^{2\pi}\int_0^{\pi} \left| \underline{E}^{\mathrm{FF}}(\theta,\phi) \right|^2 \sin\theta \, d\theta d\phi}$$

$$\left\{ \begin{aligned} & D(\theta,\phi) \equiv \frac{S_r(\theta,\phi)}{P_{\mathrm{rad}} / (4\pi r^2)} \qquad r \to \infty \\ & \underline{S}(r,\theta,\phi) = \underline{\hat{r}} \left(\frac{\left| \underline{E}^{\mathrm{FF}}(\theta,\phi) \right|^2}{2\eta_0} \right) \left(\frac{1}{r^2} \right) \ \left[\mathrm{W/m^2} \right] \\ & P_{\mathrm{rad}} = \frac{1}{2\eta_0} \int_0^{2\pi}\int_0^{\pi} \left| \underline{E}^{\mathrm{FF}}(\theta,\phi) \right|^2 \sin\theta \, d\theta d\phi \end{aligned} \right.$$

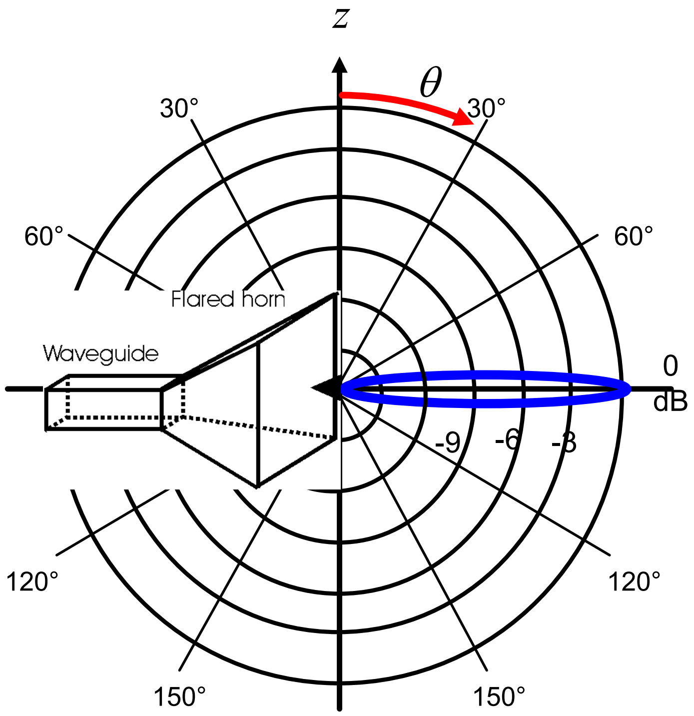
Usually, the directions are chosen to corresponds to the main beam direction:

$$D = D(\theta_m, \phi_m)$$

Directivity (cont.)



Antenna with moderate directivity
(e.g., dipole)



Antenna with high directivity
(e.g., horn or dish)

Directivity (cont.)

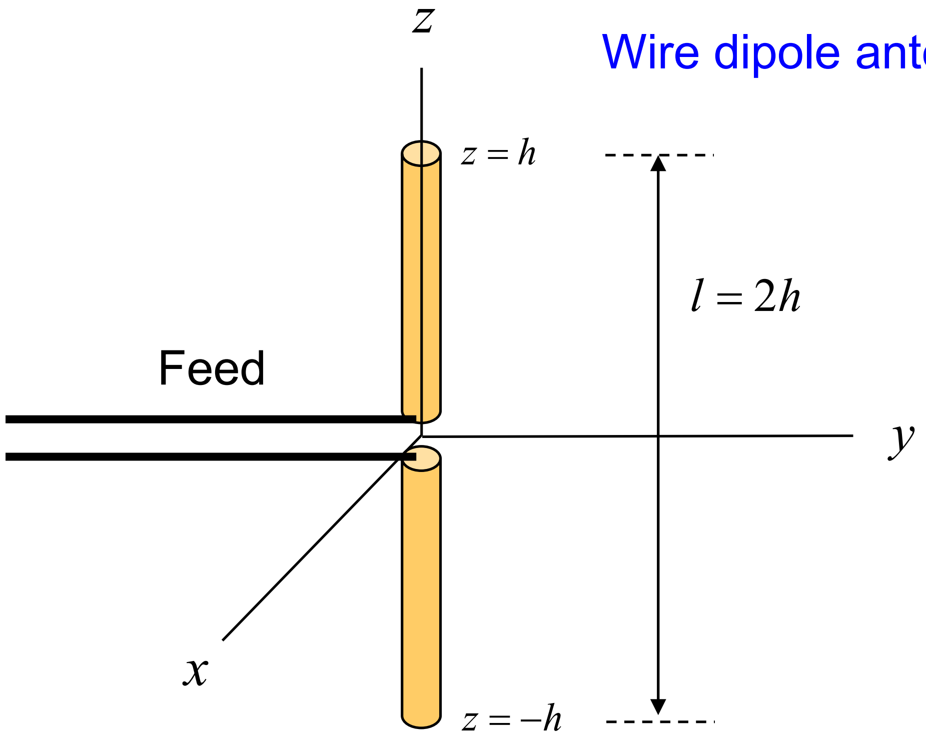
Two Common Cases: Dipole Antennas

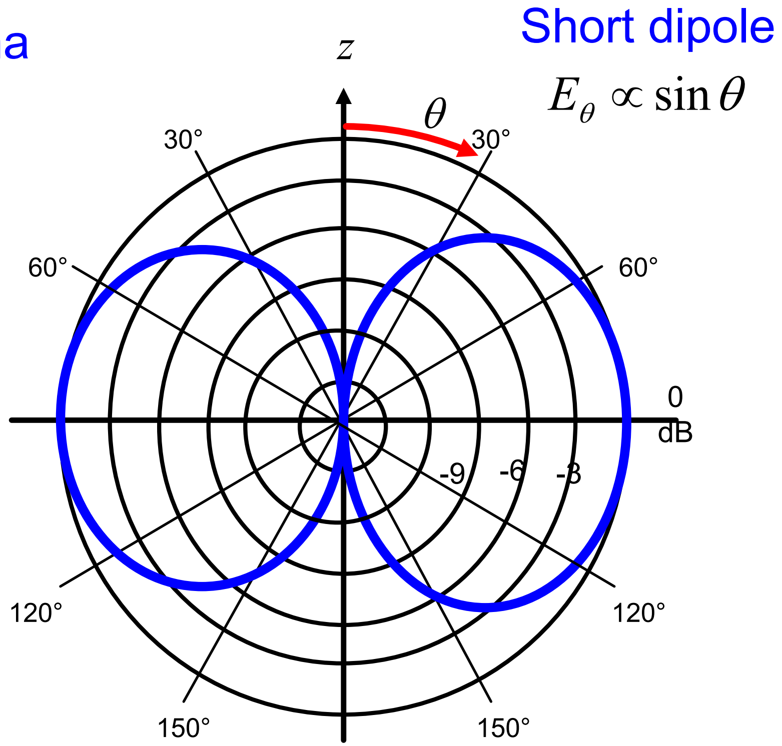
$\theta_m = \pi / 2$

$$D = D_{\max} = D(\pi / 2, \phi)$$

Short dipole wire antenna ($l << \lambda_0$): $\boldsymbol{D = 1.5}$

Resonant half-wavelength dipole wire antenna ($l = \lambda_0 / 2$): $\boldsymbol{D = 1.643}$

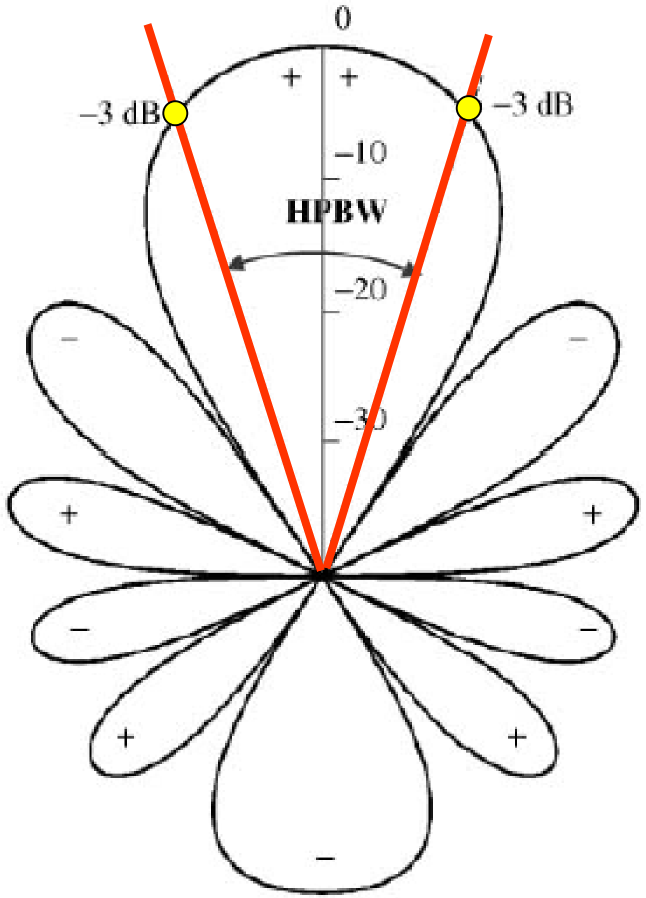




Beamwidth

The beamwidth measures how narrow the beam is
(the narrower the beamwidth, the higher the directivity).

HPBW = half-power beamwidth



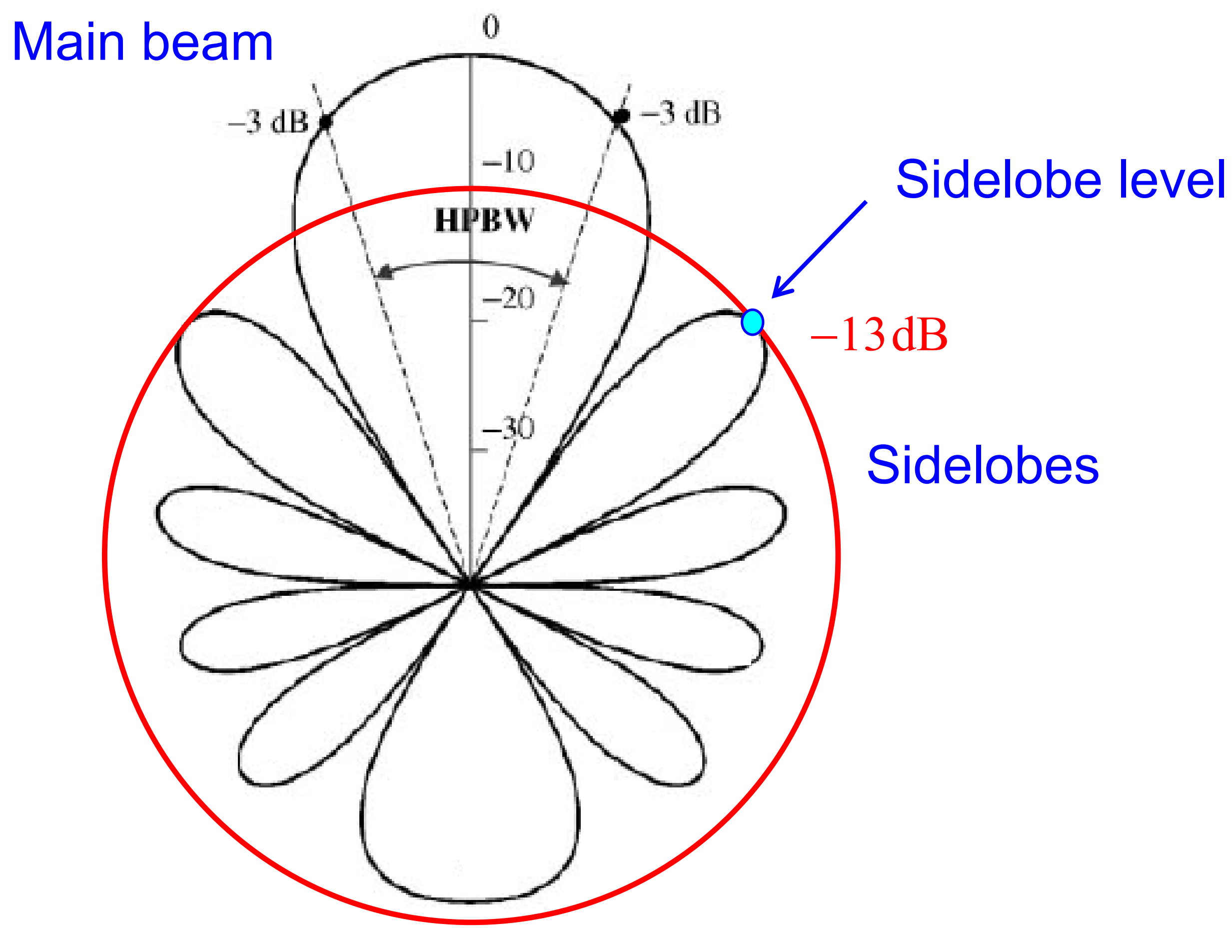
At the "half-power" points:

The power density is down by a factor of 1/2.
The field is down by a factor of $1/\sqrt{2} = 0.707$.
In dB, we are down by 3 dB.

Sidelobe Level

The sidelobe level measures how large the sidelobes are.

In this example the sidelobe level is about -13 dB.



Gain and Efficiency

The radiation efficiency of an antenna is defined as

$$e_r \equiv \frac{P_{\text{rad}}}{P_{\text{in}}}$$

$P_{\text{rad}} =$ power radiated by the antenna

$P_{\text{in}} =$ power input to the antenna

The gain of an antenna in the directions (θ, ϕ) is defined as

$$G(\theta,\phi) \equiv e_r\, D(\theta,\phi)$$

In dB, we have

$$G_{\text{dB}}(\theta,\phi) = 10\log_{10} G(\theta,\phi)$$

Gain and Efficiency (cont.)

The gain tells us how strong the radiated power density is in a certain direction, for a given amount of input power.

Recall that

$$D(\theta,\phi) \equiv \frac{S_r(\theta,\phi)}{P_{\text{rad}}/(4\pi r^2)} \qquad r \to \infty$$

Therefore, in the far field:

$$S_r(\theta,\phi) = \left[P_{\text{rad}}/(4\pi r^2)\right] D(\theta,\phi)$$

$$\Downarrow$$

Recall: $e_r \equiv \dfrac{P_{\text{rad}}}{P_{\text{in}}}$

$$S_r(\theta,\phi) = \left[e_r P_{\text{in}}/(4\pi r^2)\right] D(\theta,\phi)$$

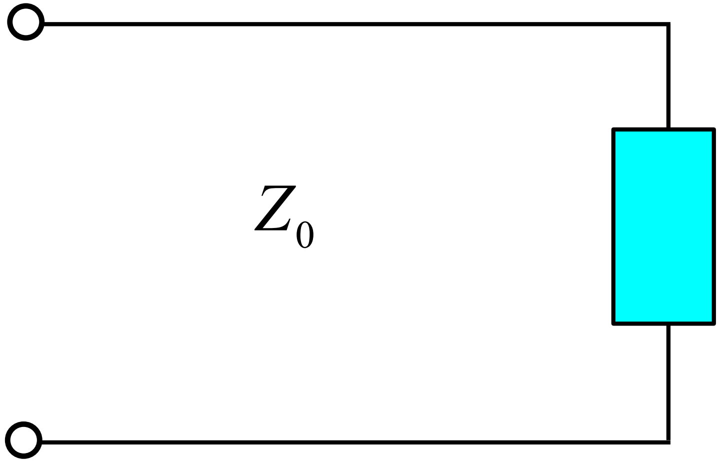
$$\Downarrow$$

Recall: $G(\theta,\phi) \equiv e_r\, D(\theta,\phi)$

$$S_r(\theta,\phi) = \left[P_{\text{in}}/(4\pi r^2)\right] G(\theta,\phi)$$

Input Impedance

The antenna acts like a load impedance during transmit.



$$Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}}$$

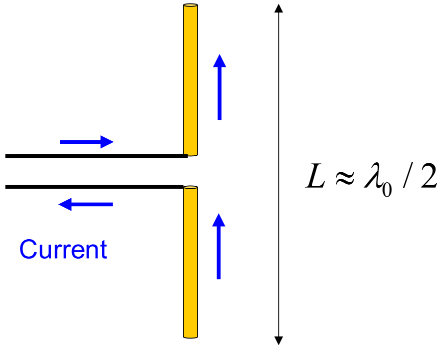
At resonance, the input reactance X_{in} is zero (the desired situation).

Note: We usually want a match between the input impedance and the characteristic impedance Z_0 of the feeding transmission line, to avoid reflection.

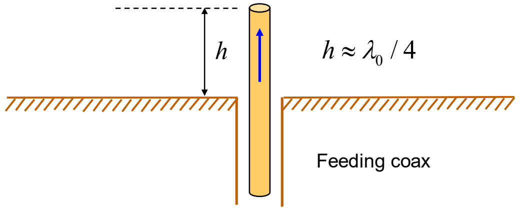
Input Impedance (cont.)

At resonance:

Dipole: $Z_{\text{in}} \approx 73\ [\Omega]$

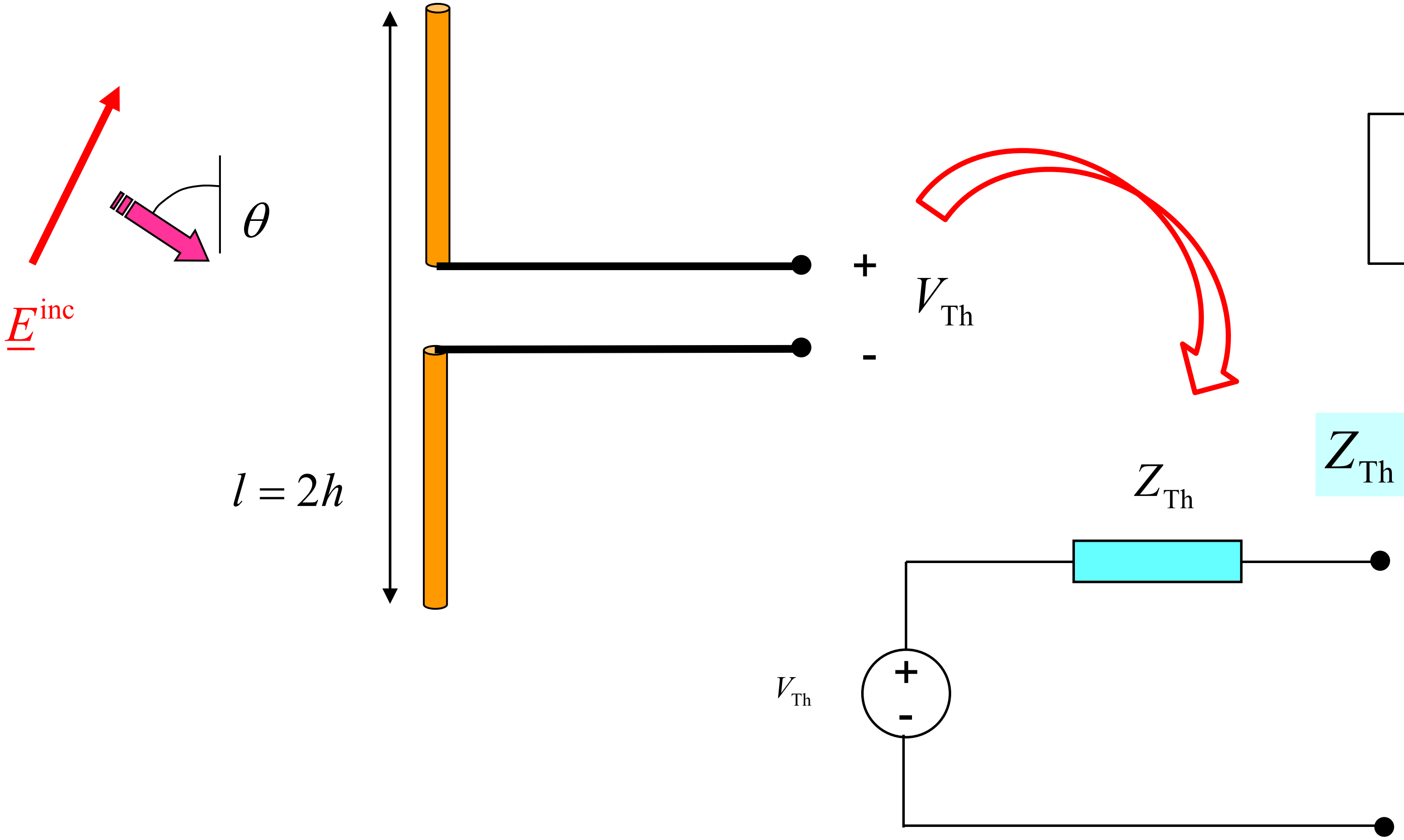


Monopole: $Z_{\text{in}} \approx 36.5\ [\Omega]$



Receive Antenna

The Thévenin equivalent circuit of an antenna being used as a receive antenna is shown below.



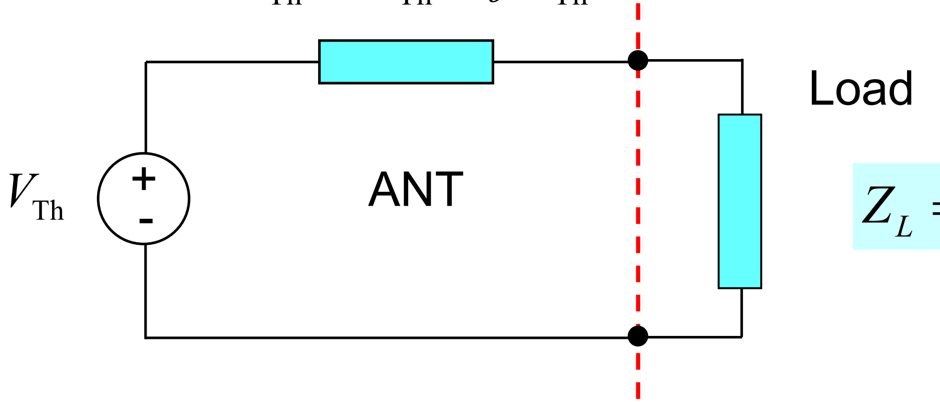
The Thevenin impedance is the same as the input impedance of the antenna.

$$Z_{\text{Th}} = Z_{\text{in}}$$

Receive Antenna (cont.)

The power received by an optimum conjugate-matched load:

$$Z_{\text{Th}} = R_{\text{Th}} + jX_{\text{Th}}$$



$$Z_L = Z_{\text{Th}}^* = R_{\text{Th}} - jX_{\text{Th}}$$

$P_{\text{rec}} = P_L =$ power absorbed by load

For a <u>resonant</u> dipole wire antenna:

$$X_{\text{Th}} = X_{\text{in}} = 0$$

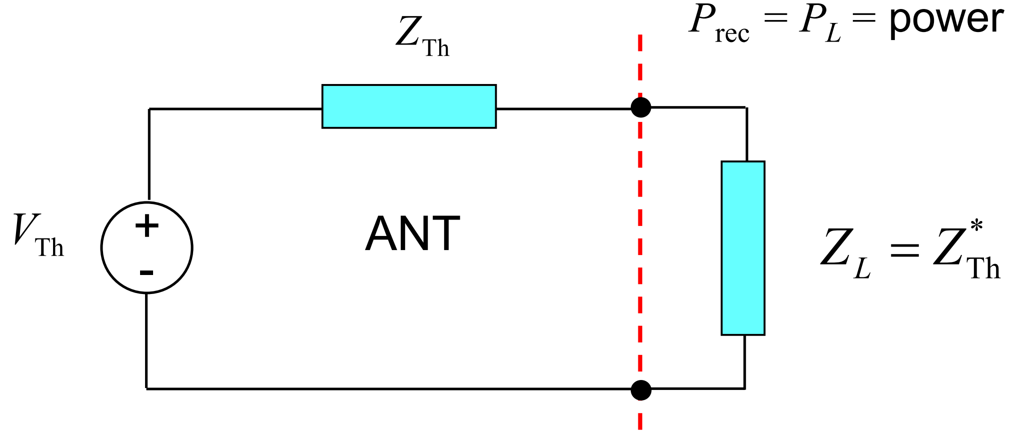
$$R_{\text{Th}} = R_{\text{in}} = 73\ [\Omega] \;\Rightarrow\; R_{\text{Th}} = 73\ [\Omega] \;\Rightarrow\; R_L^{\text{opt}} = 73\ [\Omega]$$

Receive Antenna (cont.)

We can find the power received using an effective area.

Receive circuit: Assume an optimum conjugate-matched load:

$P_{\mathrm{rec}} = P_L =$ power absorbed by load



$$P_{\mathrm{rec}} = A_{\mathrm{eff}} P_d^{\mathrm{inc}}$$

$A_{\mathrm{eff}} =$ effective area of antenna

$P_d^{\mathrm{inc}} =$ power density of incident wave $\left[\mathrm{W/m^2}\right]$

Receive Antenna (cont.)

We have the following general formula*:

$$A_{\text{eff}} = G\left(\frac{\lambda_0^2}{4\pi}\right)$$

$G = G(\theta,\phi) =$ gain of antenna in direction (θ,ϕ)

(Usually, we assume that (θ,ϕ) is in the main beam direction.)

*A derivation is given in the following book: C. A. Balanis, *Antenna Engineering,* 3rd Ed., 2016, Wiley.

Receive Antenna (cont.)

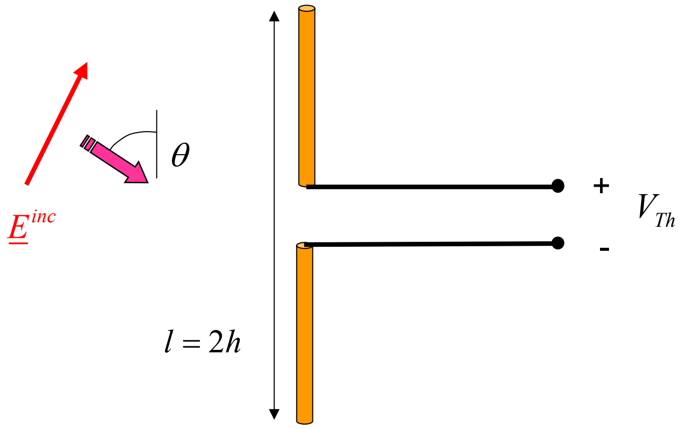
Effective area of a lossless resonant half-wave dipole antenna:

Assuming normal incidence ($\theta = 90^o$):

$$A_{\text{eff}} = G\left(\frac{\lambda_0^2}{4\pi}\right)$$

$$= 1.643\left(\frac{\lambda_0^2}{4\pi}\right) \qquad \left(D = D_{\max} = 1.643\right)$$

$$= 1.643\left(\frac{(2l)^2}{4\pi}\right) \qquad \left(l = \lambda_0 / 2\right)$$



Assume lossless antenna:
$e_r = 100\%$
$(G = D)$

Hence:

$$A_{\text{eff}} = 0.523\ l^2$$

Note:
The dipole will receive more power at a lower frequency (larger l), assuming the same incident power.

Example with Wire Antennas

Example

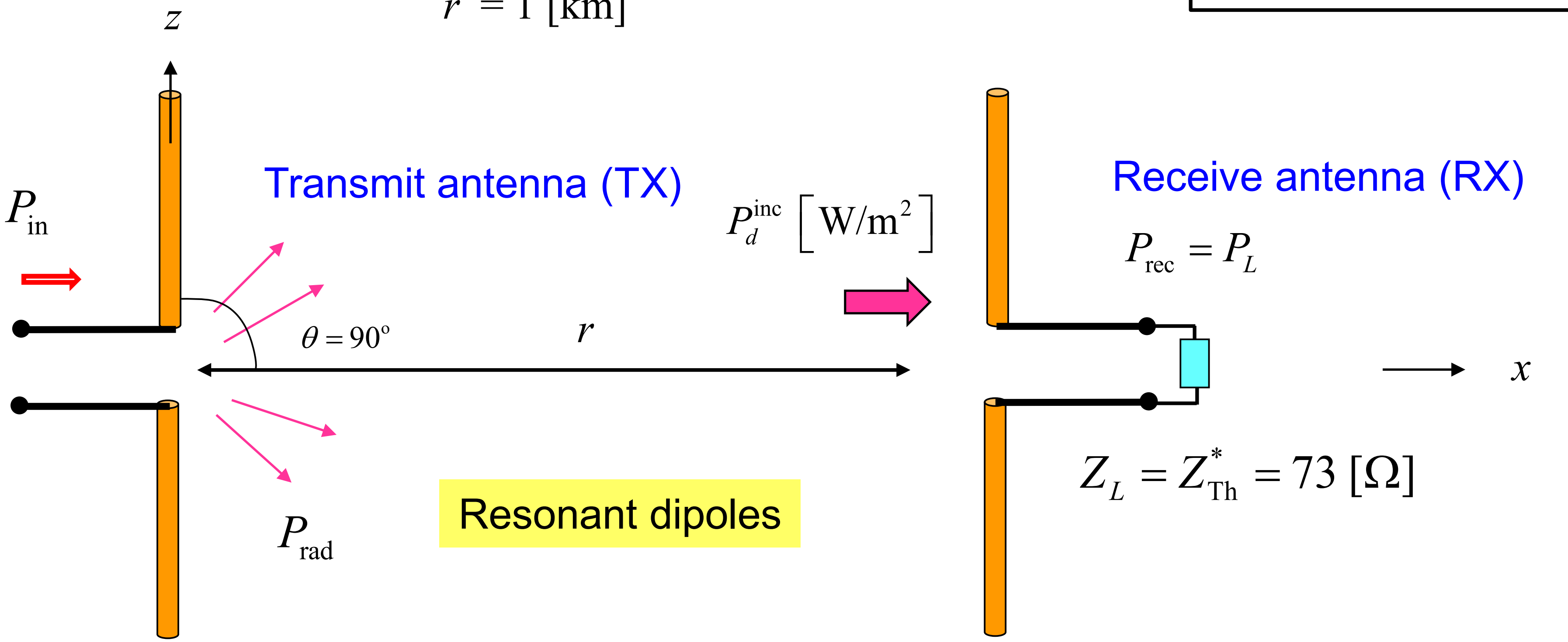
Find the received power P_{rec} in the example below, assuming that the receiver is connected to an optimum conjugate-matched load.

$$f = 1 \text{ [GHz]} \quad (\lambda_0 = 0.29979 \text{ [m]})$$

$$P_{\text{in}} = 10 \text{ [W]}$$

$$r = 1 \text{ [km]}$$

Assume lossless antennas:
$e_r = 100\%$
$(G = D)$



Example with Wire Antennas (cont.)

$$P_{\text{rec}} = A_{\text{eff}} P_d^{\text{inc}}$$

Gain of RX

$$A_{\text{eff}} = 1.643\left(\frac{\lambda_0^2}{4\pi}\right)$$

Gain of TX

$$P_d^{\text{inc}} = \frac{P_{\text{in}}}{4\pi r^2}(1.643)$$

Hence:

$$P_{\text{rec}} = \left[1.643\left(\frac{\lambda_0^2}{4\pi}\right)\right]\left[\frac{P_{\text{in}}}{4\pi r^2}(1.643)\right]$$

The result is:

$$P_{\text{rec}} = 1.54 \times 10^{-8} \text{ [W]}$$

Receive Antenna (cont.)

Effective area of dish (reflector) antenna

In the maximum gain (main beam) direction:

$$A_{\text{eff}} = A_{\text{phy}} e_{\text{ap}}$$

Now it is the effective area that we know, and from this we can calculate the gain.

A_{phy} = physical area of dish

e_{ap} = "aperture efficiency"

$$A_{\text{eff}} = G\left(\frac{\lambda_0^2}{4\pi}\right)$$

The aperture efficiency is usually less than 1 (less than 100%).

Gain of Dish Antenna

Dish antenna: Obtaining a higher gain means having a larger dish.

$$G = A_{\text{eff}}\left(\frac{4\pi}{\lambda_0^2}\right)$$

$$A_{\text{eff}} = A_{\text{phy}}\, e_{\text{ap}}$$

$$G = 4\pi\left(\frac{A_{\text{phy}}}{\lambda_0^2}\right)e_{\text{ap}}$$

Example with Dish Antenna

Example

A microstrip antenna on a CubeSat with a gain of 8 (9.03 dB) transmits with an input power of 1 [W] at 10.0 GHz from a distance of 50,000,000 [km] (near Mars).

How much power will be received by the NASA Deep Space Network dish at Goldstone, CA, which has a diameter of 70 [m]? Assume an aperture efficiency of 0.75 (75%).

Express answer in Watts and in dBm (dB relative to a milliwatt).

Note : $P_{\text{rec}}^{\text{dBm}} \equiv 10\log_{10}\left(\dfrac{P_{\text{rec}}}{0.001\ [\text{W}]}\right)$

Example with Dish Antenna (cont.)

Example (cont.)

$$P_{\text{rec}} = P_d^{\text{inc}} A_{\text{eff}} = P_d^{\text{inc}} A_{\text{phy}} e_{\text{ap}}$$

$$\uparrow$$

$$P_d^{\text{inc}} = \left(\frac{P_{\text{in}}}{4\pi r^2} \right) G_{\text{trans}}$$

$$P_{\text{rec}} = 1.014^{-19} \left[\text{W} \right]$$

$$P_{\text{rec}}^{\text{dBm}} = -151.3$$

Parameters:

$$r = 5.0 \times 10^{10} \left[\text{m} \right]$$

$$A_{\text{phy}} = \pi \left(70/2 \right)^2 \left[\text{m}^2 \right]$$

$$e_{\text{ap}} = 0.75$$

$$P_{\text{in}} = 1 \left[\text{W} \right]$$

$$G_{\text{trans}} = 8$$