#### DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

Name:		
PeopleSoft ID:		

# Applied Electromagnetic Waves Final Exam

May 11, 2009

- This exam is closed book and closed notes. No additional material may be used for this exam except for a calculator (no computers), a ruler, and a compass.
- 2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown.
- 3. Perform all your work on the exam in the space allowed.
- 4. Write neatly. You will not be given credit for work that is not **easily** legible.
- 5. Leave answers in terms of the parameters given in the problem.
- 6. Show units in all of your final answers.
- 7. Circle your final answers.
- 8. Double-check your answers. For simpler problems, partial credit may not be given.
- 9. If you have any questions, ask the instructors. You will not be given credit for work that is based on a wrong assumption.
- 10. Make sure you sign the academic honesty statement on the next page.

# **Academic Honesty Statement**

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including getting an F in the class and getting expelled from the University.
Signature

## **FORMULA SHEET**

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \cdot \underline{D} = \rho_{v}$$

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B}$$

$$\nabla \times \underline{\mathbf{H}} = \underline{\mathbf{J}} + j\omega \underline{\mathbf{D}}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \underline{\mathbf{D}} = \rho_{\mathbf{v}}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\varepsilon_0 \doteq 8.8541878 \times 10^{-12} \text{ [F/m]}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ [H/m]}$$

$$\oint_{S} \left( \underline{E} \times \underline{H} \right) \cdot \hat{n} \, dS = -\int_{V} \sigma \left| \underline{E} \right|^{2} \, dV - \int_{V} \frac{\partial}{\partial t} \left( \frac{1}{2} \mu \left| \underline{H} \right|^{2} \right) dV - \int_{V} \frac{\partial}{\partial t} \left( \frac{1}{2} \varepsilon \left| \underline{E} \right|^{2} \right) dV$$

$$\underline{S} = \underline{E} \times \underline{H}$$

$$\underline{\mathbf{S}} \equiv \frac{1}{2} \left( \underline{\mathbf{E}} \times \underline{\mathbf{H}}^* \right)$$

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [F/m]$$

$$L = \frac{\mu_0}{2\pi} \ln \left( \frac{b}{a} \right) \quad [H/m]$$

$$G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)} \quad [S/m]$$

$$R = \frac{1}{\sigma_m \delta} \left( \frac{1}{2\pi a} + \frac{1}{2\pi b} \right) \quad \left[ \Omega / \mathbf{m} \right]$$

$$\frac{\partial V}{\partial z} = -RI - L \frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial z} = -GV - C\frac{\partial V}{\partial t}$$

$$V(z,t) = f(z-c_{i}t) + g(z+c_{i}t)$$

$$I(z,t) = \frac{1}{Z_0} \left[ f(z - c_d t) - g(z + c_d t) \right]$$

$$V(z,t) = V_g(t - z / c_d)$$

$$\Gamma_g = \left(\frac{R_g - Z_0}{R_g + Z_0}\right) \qquad \Gamma_L = \left(\frac{R_L - Z_0}{R_L + Z_0}\right)$$

$$V^{+} = \left(\frac{Z_0}{R_g + Z_0}\right) V_0$$

$$\Gamma_L(t) = 1 - 2e^{-(t-T)/\tau}, \quad t \ge T$$
  $\tau = Z_0C_L$ 

$$\Gamma_L(t) = -1 + 2e^{-(t-T)/\tau}, \ t \ge T$$
  $\tau = L_L / Z_0$ 

$$V(z) = Ae^{-\gamma z} + Be^{-\gamma z}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$LC = \mu \varepsilon = \frac{1}{c_d^2}$$

$$\gamma = \alpha + j\beta$$

$$k_z = -j\gamma = \beta - j\alpha$$

$$v_{p} = \frac{\omega}{\beta}$$

$$\beta = \frac{2\pi}{\lambda}$$

attenuation = 
$$\left(\frac{20}{\ln 10}\right)\alpha = (8.6859)\alpha$$
 [dB/m]

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$V(z) = Ae^{-\gamma z} + Be^{-\gamma z}$$

$$I(z) = \left(\frac{1}{Z_0}\right) \left[Ae^{-\gamma z} - Be^{+\gamma z}\right]$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$V(z) = A(e^{-\gamma z} + \Gamma_L e^{+\gamma z})$$

$$I(z) = \frac{1}{Z_0} A \left( e^{-\gamma z} - \Gamma_L e^{+\gamma z} \right)$$

$$Z_{in}(z) = \frac{V(z)}{I(z)}$$

$$Z_{in}(z) = Z_0 \left( \frac{1 + \Gamma_L e^{+2\gamma z}}{1 - \Gamma_L e^{+2\gamma z}} \right)$$

$$Z_{in} = Z_0 \left( \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} \right)$$

$$Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right)$$

$$Z_{in}(z) = Z_0 \left( \frac{1 + \Gamma(z)}{1 - \Gamma(z)} \right)$$

$$Z_{in} = jZ_0 \tan(\beta l)$$

$$Z_{in} = -jZ_0 \cot(\beta l)$$

$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

$$|V(z)| = |A| |1 + |\Gamma_{L}| e^{+j(\phi + 2\beta z)}|$$

$$\left|\frac{\mathbf{V}(z)}{V^{+}}\right| = \left|1 + \left|\Gamma_{L}\right|e^{+j(\phi+2\beta z)}\right| = \left|1 + \Gamma(z)\right|$$

$$\Gamma(z) = \Gamma_L e^{j2\beta z} = \Gamma_L e^{-j2\beta l}$$

$$VSWR \equiv \frac{V_{\text{max}}}{V_{\text{min}}}$$

$$VSWR = \frac{1 + \left| \Gamma_{L} \right|}{1 - \left| \Gamma_{L} \right|}$$

$$SWR = \max\left(\frac{R_L}{Z_0}, \frac{Z_0}{R_L}\right)$$

$$Z_{in}^{N}(z) = \frac{1 + \Gamma_{L} e^{+2j\beta z}}{1 - \Gamma_{L} e^{+2j\beta z}}$$

$$Z_{in} = \frac{Z_{0T}^2}{Z_I}$$

$$Z_{0T} = \sqrt{Z_0 R_L}$$

$$k = \omega \sqrt{\mu \varepsilon}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \doteq 376.730313 \, [\Omega]$$

$$\underline{\mathbf{S}} = \hat{\underline{z}} \frac{\left| E_0 \right|^2}{2\eta}$$

$$v_p = \frac{\omega}{k}$$

$$\lambda = \frac{2\pi}{k}$$

$$\lambda = \frac{c_d}{f}$$

$$E_x = E_0 e^{-jkz}$$

$$H_{y} = \frac{1}{\eta} E_{0} e^{-jkz}$$

$$\varepsilon_c = \varepsilon - j \left( \frac{\sigma}{\omega} \right)$$

$$k = k' - jk''$$

$$\lambda = \frac{2\pi}{k'}$$

$$d_p = 1/k$$
"

$$\tan \delta = \frac{\varepsilon_c''}{\varepsilon_c'} = \frac{\sigma}{\omega \varepsilon}$$

$$\delta = d_p = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$k' \approx k'' \approx \frac{1}{\delta}$$

$$Z_s \equiv \frac{\mathrm{E}_{x0}}{\mathrm{J}_{sx}}$$

$$R_s = \frac{1}{\sigma \delta} = \sqrt{\frac{\omega \mu}{2\sigma}}$$

$$X_s = R_s$$

$$R = X = R_s \left(\frac{l}{2\pi a}\right)$$

$$R = R_s \left( \frac{1}{2\pi a} + \frac{1}{2\pi b} \right)$$

$$\underline{\mathbf{E}}(z) = (\hat{\underline{x}} \mathbf{E}_x + \hat{y} \mathbf{E}_y) e^{-jkz}$$

(a) 
$$0 < \beta < \pi$$
 LHEP

(b) 
$$-\pi < \beta < 0$$
 RHEP

$$n_1 \sin \theta_i = n_2 \sin \theta_i$$

$$\Gamma_{TM} = \frac{Z_2^{TM} - Z_1^{TM}}{Z_2^{TM} + Z_1^{TM}}$$

$$\Gamma_{TE} = \frac{Z_2^{TE} - Z_1^{TE}}{Z_2^{TE} + Z_1^{TE}}$$

$$Z_1^{TE} = \left(\frac{\omega \mu_1}{k_{zi}}\right) \qquad Z_2^{TE} = \left(\frac{\omega \mu_2}{k_{zt}}\right)$$

$$Z_1^{TM} = \left(\frac{k_{zi}}{\omega \varepsilon_1}\right) \qquad Z_2^{TM} = \left(\frac{k_{zt}}{\omega \varepsilon_2}\right)$$

$$\theta_i = \theta_b = \tan^{-1} \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$k_z = k\sqrt{1 - \left(f_c / f\right)^2}$$

$$f_c = \frac{c_d}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\beta = k\sqrt{1-(f_c/f)^2}, \qquad f > f_c$$

$$\alpha = k_c \sqrt{1 - (f/f_c)^2}, \quad f < f_c$$

$$H_z(x, y, z) = A_{10} \cos\left(\frac{\pi x}{a}\right) e^{-jk_z z}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(f_c / f\right)^2}}$$

$$v_p = \frac{c_d}{\sqrt{1 - \left(f_c / f\right)^2}}$$

$$v_{g} = \frac{d\omega}{d\beta} = \frac{1}{\frac{d\beta}{d\omega}}$$

$$v_g = c_d \sqrt{1 - \left(f_c / f\right)^2}$$

$$\underline{\mathbf{S}} = \hat{\underline{r}} \left( \frac{\left| \underline{\mathbf{E}} \right|^2}{2\eta_0} \right)$$

$$\underline{\mathbf{E}}(r,\theta,\phi) = \left(\frac{e^{-jk_0r}}{r}\right)\underline{\mathbf{E}}^F(\theta,\phi)$$

$$P_{rad} = \frac{1}{2\eta_0} \int_{0}^{2\pi} \int_{0}^{\pi} \left| \underline{\mathbf{E}}^F (\theta, \phi) \right|^2 \sin \theta \, d\theta d\phi$$

$$D(\theta,\phi) = \frac{S_r(\theta,\phi)}{P_{rad}/(4\pi r^2)} \qquad r \to \infty$$

$$D(\theta,\phi) = \frac{4\pi \left| \underline{\mathbf{E}}^{F}(\theta,\phi) \right|^{2}}{\int_{0}^{2\pi} \int_{0}^{\pi} \left| \underline{\mathbf{E}}^{F}(\theta,\phi) \right|^{2} \sin \theta \, d\theta d\phi}$$

$$G(\theta,\phi) \equiv e_r D(\theta,\phi)$$

$$E_{\theta}^{F} = \frac{\mathrm{I}l}{4\pi} (j\omega\mu_{0}) \sin\theta$$

$$H_{\phi}^{F} = \frac{\mathrm{I}l}{4\pi}(jk_{0})\sin\theta$$

$$D(\theta,\phi) = \frac{3}{2}\sin^2\theta$$

$$AF(\theta) \equiv \int_{-h}^{h} I(z') e^{+jk_0 z' \cos \theta} dz'$$

$$E_{\theta} \approx \frac{1}{4\pi} (j\omega\mu_0) \sin\theta \left(\frac{e^{-jk_0r}}{r}\right) AF(\theta)$$

$$AF(\theta) = 2\left(\frac{I_0}{\sin(k_0 h)}\right)\left[\frac{\cos(k_0 h \cos \theta) - \cos(k_0 h)}{k_0 \sin^2 \theta}\right]$$

$$D(\pi/2,\phi) = 1.643$$

$$R_{rad} \approx 73 \ \left[\Omega\right]$$

$$\mathbf{V}_{Th} = l_{eff} \left( \hat{\underline{l}} \cdot \underline{\mathbf{E}}^{inc} \right)$$

$$Z_{Th} = Z_{in}$$

$$V^{monopole} = \frac{1}{2} V^{dipole}$$

$$l_{\it eff} = \begin{cases} l/2, & l << \lambda_0 \text{ (electrically small)} \\ l\bigg(\frac{2}{\pi}\bigg) \cos\bigg(\frac{\pi}{2}\cos\theta\bigg), & l = \lambda_0/2 \text{ (resonant)} \end{cases}$$

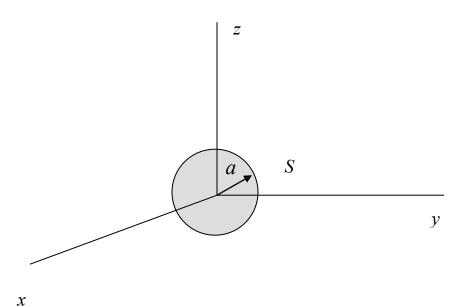
Assume that we have a plane wave in free space at 100 [MHz] given as

$$\underline{\mathbf{E}} = \underline{\hat{z}} e^{+jkx}$$

$$\underline{\mathbf{H}} = (0.002654) \underline{\hat{y}} e^{+jkx},$$

where k is a real number. The surface S shown below is a circle of radius a = 1 [m], centered at the origin and located in the plane x = 0.

- a) Find the value of k.
- b) Find the complex Poynting vector.
- c) Find the instantaneous Poynting vector.
- d) Find the power flowing through the surface S shown below at the time  $t = 10^{-9}$  [s] (crossing the surface from the positive x region to the negative region).
- e) Find the time-average power flowing through the surface S shown below (crossing the surface from the positive x region to the negative region).
- f) Find the amount of reactive power (in VARs) flowing through the surface S shown below (crossing the surface from the positive x region to the negative region).

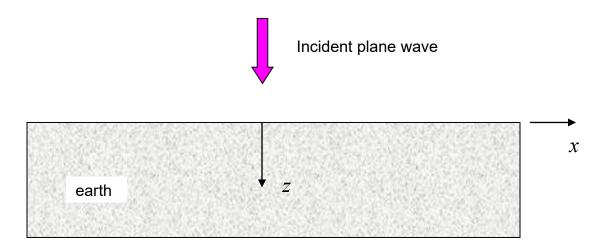


A 75 [ $\Omega$ ] coaxial line used for TV has an outer radius of b = 0.25 [cm] and an inner radius of a = 0.039 [cm]. The coax is filled with Teflon (nonmagnetic, with  $\varepsilon_r = 2.2$ ) that has a loss tangent of 0.001. The conductors are made of copper, which is nonmagnetic. The conductivity of copper is  $3.0 \times 10^7$  [S/m]. Assume that the transmission line is operating at a UHF frequency of 500 [MHz].

- a) Calculate the parameters R, L, G, C for the line.
- b) Calculate the attenuation on the transmission line, in both nepers/m and dB/m.

A plane wave is incident at an angle of zero degrees from air onto the earth, as shown below (For an incident angle of zero degrees, the plane wave may be considered to be either  $TM_z$  or  $TE_z$ .) The earth has  $\varepsilon_r = 10$  and  $\sigma = 0.1$  [S/m]. The frequency is 200 [MHz].

- a) Find the percent power that is reflected.
- b) Find the depth below the surface of the earth at which the magnitude of the field is down by 20 dB from the value at the surface.
- c) If the power density of the incident wave is 1.0 [W/m²], find the power density at a depth of 1.0 [m]. (Your calculation should account for both reflection and attenuation.)



Sunlight contains equal power densities in both the parallel and perpendicular polarizations. Assume that sunlight is incident at a  $60^{\circ}$  angle (measured from the vertical) on a puddle of water. Because the water is relatively pure fresh water, it is nonconductive and has a dielectric constant (relative permittivity) of  $\varepsilon_r = 1.78$ . The water is nonmagnetic.

What is the percentage of power that is reflected?

A standard X-band rectangular waveguide has dimensions  $0.9 \times 0.4$  [inches]. (One inch equals 2.54 cm.) Assume that the waveguide is air filled and it operates in the dominant TE<sub>10</sub> mode.

- a) Find the cutoff frequency of the TE<sub>10</sub> mode.
- b) Find the cutoff frequency of the next highest mode, and identify which mode this is.
- c) Find the normalized phase constant  $\beta / k_0$  for the TE<sub>10</sub> mode at 10.0 GHz.
- d) Find the normalized attenuation constant  $\alpha / k_0$  for the TE<sub>10</sub> mode at 5.0 GHz.
- e) How far do you have to go in the waveguide at 10.0 GHz before the field of the TE<sub>10</sub> mode repeats itself?
- f) How far do you have to go in the waveguide at 5.0 GHz before the field of the TE<sub>10</sub> mode has attenuated by 40 dB?
- g) If the waveguide is made from aluminum, how thick does the wall of the waveguide have to be if we wish it to be ten skin depths thick at 5.0 GHz? The conductivity of aluminum is  $2.0 \times 10^7$  [S/m]. Aluminum is nonmagnetic ( $\mu = \mu_0$ ).

## EXTRA CREDIT PROBLEM

Suppose we wish to communicate between two resonant half-wavelength dipole wire antennas at 10.0 [GHz]. The antennas are located 10.0 [km] apart. Suppose the receive antenna is connected to a transmission line that has a characteristic impedance of 50 [ $\Omega$ ]. At the end of the transmission line is a load that is also 50 [ $\Omega$ ]. Suppose we transmit a power of 10 [W] from the transmit antenna. What will the amplitude of the voltage be that appears across the load that is connected to the receive antenna?

Assume that both antenna are vertical and the receive antenna is in the horizontal plane ( $\theta = 90^{\circ}$ ). Also assume that both antennas are 100% efficient, and that they radiate in free space.