

DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

Name: _____

PeopleSoft ID: _____

ECE 3317
Applied Electromagnetic Waves
FINAL EXAM
May 7, 2013

1. This exam is open book and open notes. However, you are not allowed to use a computer or any electronic device other than a calculator. Any devices that may be used to communicate are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily legible**.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Make sure you sign the academic honesty statement on the next page.

Academic Honesty Statement

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including getting an F in the class and getting expelled from the University.

Signature

$$\nabla \times \underline{\mathcal{E}} = -\frac{\partial \underline{\mathcal{B}}}{\partial t}$$

$$\nabla \times \underline{\mathcal{H}} = \underline{\mathcal{J}} + \frac{\partial \underline{\mathcal{Q}}}{\partial t}$$

$$\nabla \cdot \underline{\mathcal{B}} = 0$$

$$\nabla \cdot \underline{\mathcal{Q}} = \rho_v$$

$$\nabla \times \underline{E} = -j\omega \underline{B}$$

$$\nabla \times \underline{H} = \underline{J} + j\omega \underline{D}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \cdot \underline{D} = \rho_v$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\varepsilon_0 \doteq 8.8541878 \times 10^{-12} \text{ [F/m]}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ [H/m]}$$

$$\oint_s (\underline{\mathcal{E}} \times \underline{\mathcal{H}}) \cdot \hat{n} dS = - \int_V \sigma |\underline{\mathcal{E}}|^2 dV - \int_V \frac{\partial}{\partial t} \left(\frac{1}{2} \mu |\underline{\mathcal{H}}|^2 \right) dV - \int_V \frac{\partial}{\partial t} \left(\frac{1}{2} \varepsilon |\underline{\mathcal{E}}|^2 \right) dV$$

$$\underline{\mathcal{J}} = \underline{\mathcal{E}} \times \underline{\mathcal{H}}$$

$$\underline{S} \equiv \frac{1}{2} (\underline{E} \times \underline{H}^*)$$

$$C = \frac{2\pi\epsilon_0\epsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\text{F/m}]$$

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \quad [\text{H/m}]$$

$$G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)} \quad [\text{S/m}]$$

$$R = \frac{1}{\sigma_m\delta} \left(\frac{1}{2\pi a} + \frac{1}{2\pi b} \right) \quad [\Omega/\text{m}]$$

$$\frac{\partial v}{\partial z} = -Ri - L \frac{\partial i}{\partial t}$$

$$\frac{\partial I}{\partial z} i = -Gv - C \frac{\partial v}{\partial t}$$

$$v(z,t) = f(z - c_d t) + g(z + c_d t)$$

$$i(z,t) = \frac{1}{Z_0} [f(z - c_d t) - g(z + c_d t)]$$

$$v(z,t) = v_g(t - z / c_d)$$

$$\Gamma_g = \left(\frac{R_g - Z_0}{R_g + Z_0} \right) \qquad \Gamma_L = \left(\frac{R_L - Z_0}{R_L + Z_0} \right)$$

$$V^+ = \left(\frac{Z_0}{R_g + Z_0} \right) V_0$$

$$\Gamma_L(t)=1-2e^{-(t-T)/\tau}, \quad t \geq T \qquad \tau = Z_0 C_L$$

$$\Gamma_L(t)=-1+2e^{-(t-T)/\tau}, \quad t \geq T \qquad \tau = L_L / Z_0$$

$$V(z)=Ae^{-\gamma z}+Be^{+\gamma z}$$

$$\gamma=\sqrt{(R+j\omega L)(G+j\omega C)}$$

$$LC=\mu\varepsilon=\frac{1}{c_d^2}$$

$$\gamma=\alpha+j\beta$$

$$k_z=-j\gamma=\beta-j\alpha$$

$$v_p=\frac{\omega}{\beta}$$

$$\beta=\frac{2\pi}{\lambda}$$

$$\text{attenuation}=\left(\frac{20}{\ln 10}\right)\alpha=(8.6859)\alpha \quad [\text{dB/m}]$$

$$Z_0=\sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$i(z) = \left(\frac{1}{Z_0} \right) \left[A e^{-\gamma z} - B e^{+\gamma z} \right]$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$V(z) = A \left(e^{-\gamma z} + \Gamma_L e^{+\gamma z} \right)$$

$$I(z) = \frac{1}{Z_0} A \left(e^{-\gamma z} - \Gamma_L e^{+\gamma z} \right)$$

$$Z_{in}(z) = \frac{V(z)}{I(z)}$$

$$Z_{in}(z) = Z_0 \left(\frac{1 + \Gamma_L e^{+2\gamma z}}{1 - \Gamma_L e^{+2\gamma z}} \right)$$

$$Z_{in} = Z_0 \left(\frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} \right)$$

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right)$$

$$Z_{in}(z) = Z_0 \left(\frac{1 + \Gamma(z)}{1 - \Gamma(z)} \right)$$

$$Z_{in} = jZ_0 \tan(\beta l)$$

$$Z_{in} = -jZ_0 \cot(\beta l)$$

$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

$$|V(z)| = |A| \left| 1 + |\Gamma_L| e^{+j(\phi+2\beta z)} \right|$$

$$\left| \frac{V(z)}{V^+} \right| = \left| 1 + |\Gamma_L| e^{+j(\phi+2\beta z)} \right| = \left| 1 + \Gamma(z) \right|$$

$$\Gamma(z) = \Gamma_L e^{j2\beta z} = \Gamma_L e^{-j2\beta l}$$

$$\text{VSWR} \equiv \frac{V_{\max}}{V_{\min}}$$

$$\text{VSWR} \equiv \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$$\text{SWR} = \max\left(\frac{R_L}{Z_0}, \frac{Z_0}{R_L}\right)$$

$$Z_{in}^N(z) = \frac{1 + \Gamma_L e^{+2j\beta z}}{1 - \Gamma_L e^{+2j\beta z}}$$

$$Z_{in} = \frac{Z_{0T}^2}{Z_L}$$

$$Z_{0T} = \sqrt{Z_0 R_L}$$

$$k=\omega\sqrt{\mu\varepsilon}$$

$$\eta=\sqrt{\frac{\mu}{\varepsilon}}$$

$$\eta_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}\doteq 376.730313\text{ }[\Omega]$$

$$\underline{S}=\underline{\hat{z}}\frac{\left|E_0\right|^2}{2\eta}$$

$$v_p=\frac{\omega}{k}$$

$$\lambda=\frac{2\pi}{k}$$

$$\lambda=\frac{c_d}{f}$$

$$E_x=E_0\,e^{-jkz}$$

$$H_y=\frac{1}{\eta}E_0\,e^{-jkz}$$

$$\varepsilon_c=\varepsilon-j\left(\frac{\sigma}{\omega}\right)$$

$$k=k'-jk''$$

$$\lambda = \frac{2\pi}{k'}$$

$$d_p = 1/k''$$

$$\tan \delta = \frac{\varepsilon_c''}{\varepsilon_c'} = \frac{\sigma}{\omega \varepsilon}$$

$$\delta = d_p = \sqrt{\frac{2}{\omega \mu \sigma}}$$

$$k' \approx k'' \approx \frac{1}{\delta}$$

$$Z_s \equiv \frac{E_{x0}}{J_{sx}}$$

$$R_s = \frac{1}{\sigma \delta} = \sqrt{\frac{\omega \mu}{2 \sigma}}$$

$$X_s = R_s$$

$$R=X=R_s\left(\frac{l}{2\pi a}\right)$$

$$R=R_s\left(\frac{1}{2\pi a}+\frac{1}{2\pi b}\right)$$

$$\underline{E}(z) = (\underline{\hat{x}}\,E_x + \underline{\hat{y}}\,E_y)\,e^{-jkz}$$

$$(a) \quad 0 < \beta < \pi \quad \text{LHEP}$$

$$(b) \quad -\pi < \beta < 0 \quad \text{RHEP}$$

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$\Gamma_{TM} = \frac{Z_2^{TM} - Z_1^{TM}}{Z_2^{TM} + Z_1^{TM}}$$

$$\Gamma_{TE} = \frac{Z_2^{TE} - Z_1^{TE}}{Z_2^{TE} + Z_1^{TE}}$$

$$Z_1^{TE} = \left(\frac{\omega \mu_1}{k_{zi}} \right) \quad Z_2^{TE} = \left(\frac{\omega \mu_2}{k_{zt}} \right)$$

$$Z_1^{TM} = \left(\frac{k_{zi}}{\omega \varepsilon_1} \right) \quad Z_2^{TM} = \left(\frac{k_{zt}}{\omega \varepsilon_2} \right)$$

$$\theta_i = \theta_b = \tan^{-1} \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$k_z = k \sqrt{1 - (f_c / f)^2}$$

$$f_c = \frac{c_d}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\beta = k \sqrt{1 - (f_c / f)^2}, \quad f > f_c$$

$$\alpha = k_c \sqrt{1 - (f / f_c)^2}, \quad f < f_c$$

$$H_z(x, y, z) = A_{10} \cos\left(\frac{\pi x}{a}\right) e^{-jk_z z}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1 - (f_c / f)^2}}$$

$$v_p = \frac{c_d}{\sqrt{1 - (f_c / f)^2}}$$

$$v_g = \frac{d\omega}{d\beta} = \frac{1}{\frac{d\beta}{d\omega}}$$

$$v_g = c_d \sqrt{1 - (f_c / f)^2}$$

$$\underline{S} = \underline{\hat{r}} \left(\frac{|\underline{E}|^2}{2\eta_0} \right)$$

$$\underline{E}(r,\theta,\phi) = \left(\frac{e^{-jk_0 r}}{r} \right) \underline{E}^F(\theta,\phi)$$

$$P_{rad} = \frac{1}{2\eta_0} \int_0^{2\pi} \int_0^\pi \left| \underline{E}^F(\theta,\phi) \right|^2 \sin\theta d\theta d\phi$$

$$D(\theta,\phi) \equiv \frac{S_r(\theta,\phi)}{P_{rad} / (4\pi r^2)} \quad r \rightarrow \infty$$

$$D(\theta, \phi) = \frac{4\pi \left| \underline{E}^F(\theta, \phi) \right|^2}{\int_0^{2\pi} \int_0^\pi \left| \underline{E}^F(\theta, \phi) \right|^2 \sin \theta d\theta d\phi}$$

$$G(\theta, \phi) \equiv e_r D(\theta, \phi)$$

$$E_\theta^F = \frac{Il}{4\pi} (j\omega\mu_0) \sin \theta$$

$$H_\phi^F = \frac{Il}{4\pi} (jk_0) \sin \theta$$

$$D(\theta, \phi) = \frac{3}{2} \sin^2 \theta$$

$$AF(\theta) \equiv \int_{-h}^h I(z') e^{+jk_0 z' \cos \theta} dz'$$

$$E_\theta \approx \frac{1}{4\pi} (j\omega\mu_0) \sin \theta \left(\frac{e^{-jk_0 r}}{r} \right) AF(\theta)$$

$$AF(\theta) = 2 \left(\frac{I_0}{\sin(k_0 h)} \right) \left[\frac{\cos(k_0 h \cos \theta) - \cos(k_0 h)}{k_0 \sin^2 \theta} \right]$$

$$D(\pi/2, \phi) = 1.643$$

$$R_{rad} \approx 73 \text{ } [\Omega]$$

$$V_{Th} = l_{eff} \left(\hat{\underline{l}} \cdot \underline{E}^{inc} \right)$$

$$Z_{Th} = Z_{in}$$

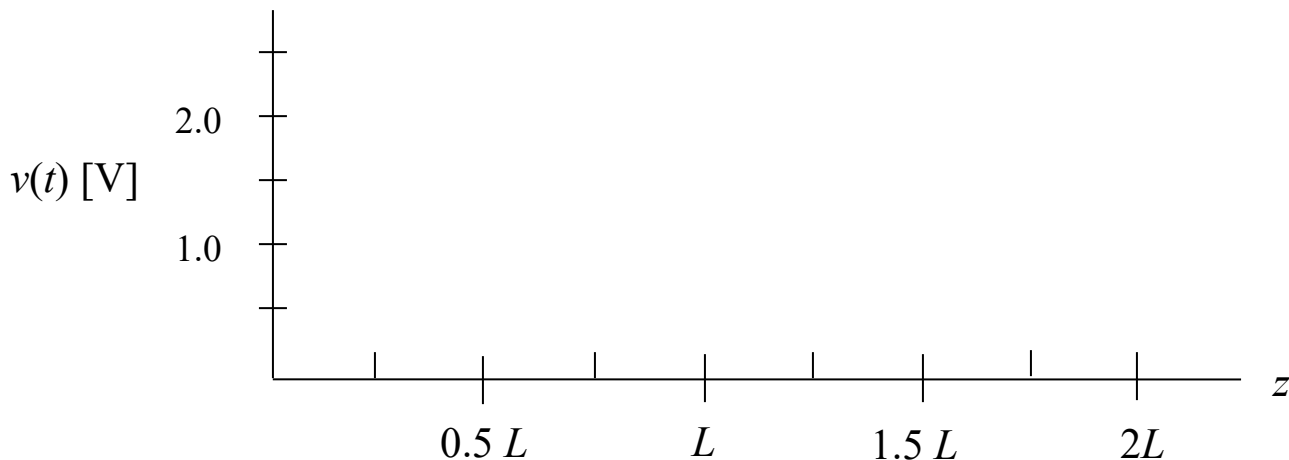
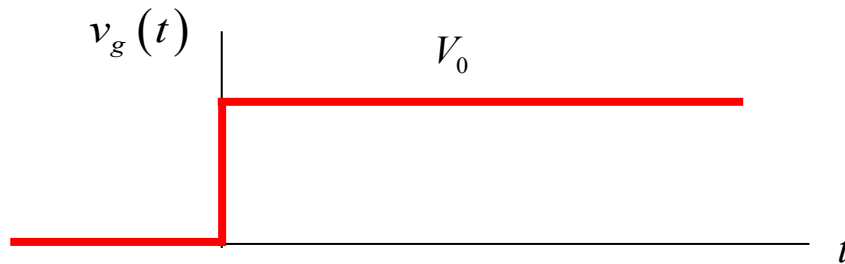
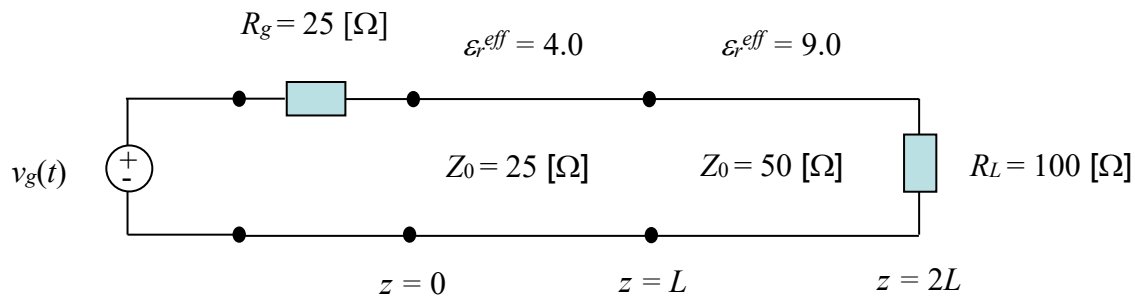
$$V^{monopole} = \frac{1}{2} V^{dipole}$$

$$l_{eff} = \begin{cases} l/2, & l \ll \lambda_0 \text{ (electrically small)} \\ l \left(\frac{2}{\pi} \right) \cos \left(\frac{\pi}{2} \cos \theta \right) / \sin^2 \theta, & l = \lambda_0 / 2 \text{ (resonant)} \end{cases}$$

Problem 1 (40 pts.)

A step function of amplitude $V_0 = 2.0$ [V] is applied at the input to the transmission line circuit shown below. Each of the two transmission lines has a length L that is 3.0 [m]. Each line has a different effective relative permittivity, however.

- Construct a bounce diagram that extends to a time of 50 [ns], labeling with 10 [ns] divisions on your time scale. (Make your bounce diagram on the next page.)
- Make an accurate “snapshot” trace of the voltage on the line at $t = 35$ [ns]. Make your snapshot trace on the graph shown below, plotting out to $2L$. Label all voltage values on your plot as well as all z values at which the voltage on your plot changes. Also, indicate in which direction (to the left or right) each “wavefront” (point of voltage discontinuity) is moving.



Room for Work

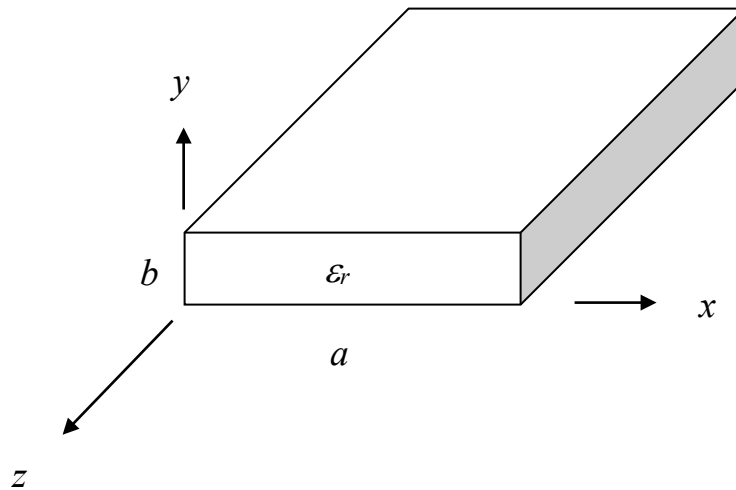
Problem 2 (40 pts.)

A rectangular waveguide has a dominant TE₁₀ mode that has an electric field that is given by

$$\underline{E} = \underline{\hat{y}} \sin\left(\frac{\pi x}{a}\right) e^{-jk_z z}.$$

The waveguide is filled with a lossless material having a relative permittivity ϵ_r .

- Find the complex power flow P_f that is flowing in the z direction down the waveguide at any frequency f . (Evaluate all integrals so that your result is a closed-form expression.)
- Find the time-average power flowing down the waveguide when $f > f_c$. Your result should be in closed form. Do not leave any “Re” or “Im” operators in your answer.
- Find the VARS flowing down the waveguide when $f < f_c$. Your result should be in closed form. Do not leave any “Re” or “Im” operators in your answer.

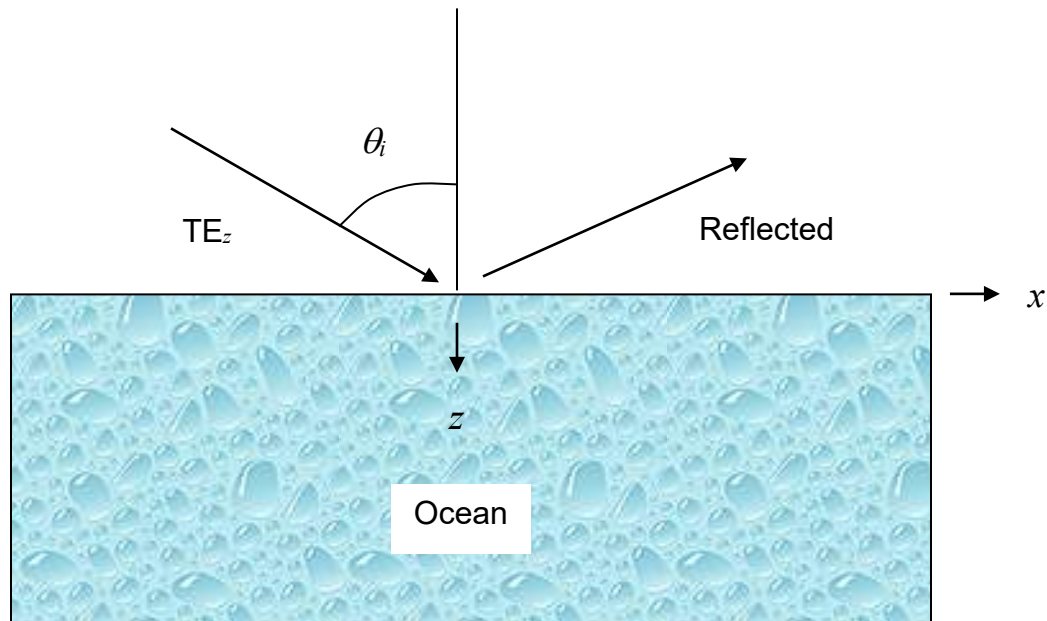


Room for Work

Problem 3 (40 pts.)

A TE_z plane wave at a frequency of 1.0 GHz is incident on the ocean (which is nonmagnetic). The ocean has a relative permittivity of $\epsilon_r = 81$ and a conductivity of $\sigma = 4.0$ [S/m]. The angle of incidence is $\theta_i = 60^\circ$.

- Calculate the total power density in the reflected plane wave relative to that of the incident plane wave.
- Find the distance z below the surface of the ocean for which the field E_y has attenuated by 10 dB from what it is at the surface ($z = 0$).



Room for Work

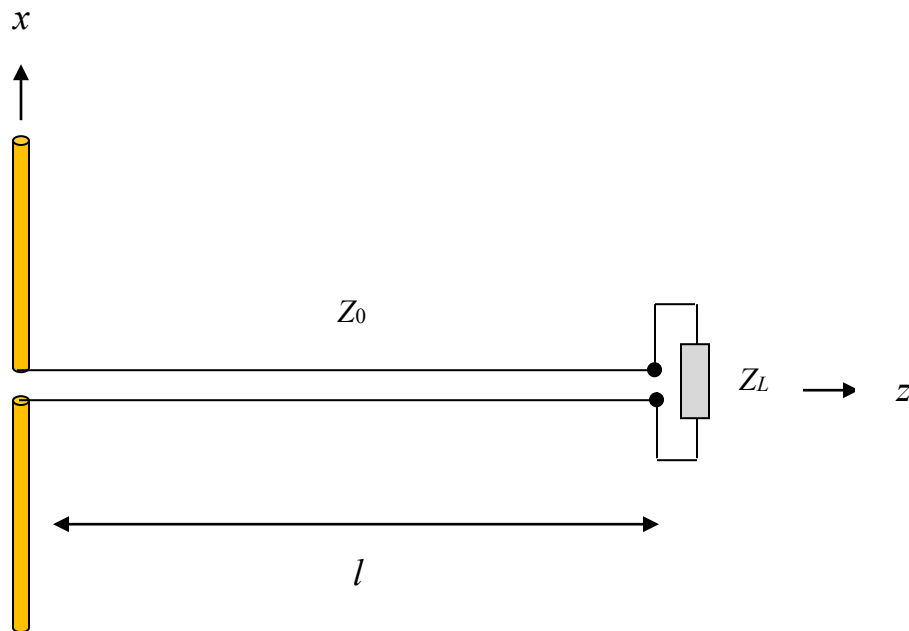
Problem 4 (60 pts.)

A plane wave from a distant transmitter is incident on a receive antenna as shown below. The incident wave is a RHCP wave and it has the mathematical form

$$\underline{E} = (\underline{\hat{x}}(1) + \underline{\hat{y}}(-j))e^{-jk_0z}.$$

The frequency is 1.0 [GHz]. The receive antenna is a resonant half-wavelength dipole wire antenna that is aligned in the x direction. The receive antenna is connected to a lossless transmission line of length l , which is then connected to a load. The load is an impedance $Z_L = 100 - j(50)$ [Ω] while the transmission line has a characteristic impedance of $Z_0 = 50$ [Ω]. The length of the transmission line is $l = \lambda / 8$, where λ is the wavelength on the transmission line.

- Find the Thévenin equivalent circuit for the receive antenna.
- Find the input impedance seen by the receive antenna, using the Smith chart (attached at the end of the exam).
- Find the power being delivered to the load.



Room for Work

Problem 5 (50 pts.)

An antenna radiates an electric field in the far field that is given by

$$\underline{E}^F(\theta, \phi) = \begin{cases} E_0 (\hat{\theta} \cos \theta \cos \phi + \hat{\phi} \sin \phi), & \theta \leq \pi/2 \\ 0, & \theta \geq \pi/2 \end{cases}$$

where E_0 is a constant.

- Calculate the directivity $D(\theta, \phi)$. Make sure that you evaluate all integrals to get a closed-form result.
- Find the direction $(\theta_{max}, \phi_{max})$ that maximizes the directivity, and find the value of the maximum directivity; i.e., find $D_{max} = D(\theta_{max}, \phi_{max})$.
- If the antenna radiates 1 [W] of power, find the magnitude of the electric field vector in the direction $\theta = 0$, at a distance of 1.0 [km] from the antenna.

Room for Work

