DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

Name: ___________SOLUTION_____________________

PeopleSoft ID: ___________________________________

ECE 3317

Applied Electromagnetic Waves

Exam 1
Oct. 24, 2018

1. This exam is open book and open notes. However, you are not allowed to use a computer or any electronic device other than a calculator. Any devices that may be used to communicate are not allowed.

2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.

3. Perform all your work on the exam in the space allowed.

4. Write neatly. You will not be given credit for work that is not **easily legible**.

5. Leave answers in terms of the parameters given in the problem.

6. Show units in all of your final answers.

7. Circle your final answers.

8. Double-check your answers. For simpler problems, partial credit may not be given.

9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.

10. Make sure you sign the academic honesty statement on the next page.

Academic Honesty Statement

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including getting an F in the class and getting expelled from the University.

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Signature

FORMULA SHEET

$$\nabla \times \underline{\mathscr{E}} = -\frac{\partial \underline{\mathscr{B}}}{\partial t}$$

$$\nabla \times \underline{\mathscr{H}} = \underline{\mathscr{J}} + \frac{\partial \underline{\mathscr{D}}}{\partial t}$$

$$\nabla \cdot \underline{\mathscr{B}} = 0$$

$$\nabla \cdot \underline{\mathscr{D}} = \rho_v$$

$$\nabla \times \underline{E} = -j\omega \underline{B}$$

$$\nabla \times \underline{H} = \underline{J} + j\omega \underline{D}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \cdot \underline{D} = \rho_v$$

$$\nabla \times \underline{V} = \underline{\hat{x}}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) + \underline{\hat{y}}\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right) + \underline{\hat{z}}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

$$\nabla \times \underline{V} = \underline{\hat{\rho}}\left(\frac{1}{\rho}\frac{\partial V_z}{\partial \phi} - \frac{\partial V_\phi}{\partial z}\right) + \underline{\hat{\phi}}\left(\frac{\partial V_\rho}{\partial z} - \frac{\partial V_z}{\partial \rho}\right) + \underline{\hat{z}}\,\frac{1}{\rho}\left(\frac{\partial\left(\rho V_\phi\right)}{\partial \rho} - \frac{\partial V_\rho}{\partial \phi}\right)$$

$$\nabla \times \underline{V} = \underline{\hat{r}}\,\frac{1}{r\sin\theta}\left[\frac{\partial\left(V_\phi \sin\theta\right)}{\partial \theta} - \frac{\partial V_\theta}{\partial \phi}\right] + \underline{\hat{\theta}}\,\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial V_r}{\partial \phi} - \frac{\partial\left(rV_\phi\right)}{\partial r}\right] + \underline{\hat{\phi}}\,\frac{1}{r}\left[\frac{\partial\left(rV_\theta\right)}{\partial r} - \frac{\partial V_r}{\partial \theta}\right]$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\varepsilon_0 \doteq 8.8541878 \times 10^{-12} \text{ [F/m]}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ [H/m]}$$

$$\oint_S \left(\underline{\mathscr{E}} \times \underline{\mathscr{H}}\right) \cdot \hat{n}\, dS = -\int_V \sigma \left|\underline{\mathscr{E}}\right|^2 dV - \int_V \frac{\partial}{\partial t}\left(\frac{1}{2}\mu \left|\underline{\mathscr{H}}\right|^2\right) dV - \int_V \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon \left|\underline{\mathscr{E}}\right|^2\right) dV$$

$$\underline{\mathscr{S}} = \underline{\mathscr{E}} \times \underline{\mathscr{H}}$$

$$\underline{S} \equiv \frac{1}{2}\left(\underline{E} \times \underline{H}^*\right)$$

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\text{F/m}]$$

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) \quad [\text{H/m}]$$

$$G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)} \quad [\text{S/m}]$$

$$R = \frac{1}{\sigma_m \delta}\left(\frac{1}{2\pi a} + \frac{1}{2\pi b}\right) \quad [\Omega/\text{m}]$$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma_m}}$$

$$\frac{\partial v}{\partial z} = -Ri - L\frac{\partial i}{\partial t}$$

$$\frac{\partial i}{\partial z} = -Gv - C\frac{\partial v}{\partial t}$$

$$v(z,t) = f\left(z - c_d t\right) + g\left(z + c_d t\right)$$

$$i(z,t) = \frac{1}{Z_0}\left[f(z - c_d t) - g(z + c_d t) \right]$$

$$v(z,t) = v_g(t - z / c_d)$$

$$\Gamma_g = \left(\frac{R_g - Z_0}{R_g + Z_0} \right) \qquad \Gamma_L = \left(\frac{R_L - Z_0}{R_L + Z_0} \right)$$

$$V^+ = \left(\frac{Z_0}{R_g + Z_0} \right) V_0$$

$$\Gamma_L(t) = 1 - 2e^{-(t-T)/\tau}, \quad t \geq T \qquad \tau = Z_0 C_L$$

$$\Gamma_L(t) = -1 + 2e^{-(t-T)/\tau}, \quad t \geq T \qquad \tau = L_L / Z_0$$

$$V(z) = Ae^{-\gamma z} + Be^{+\gamma z}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$LC = \mu\varepsilon = \frac{1}{c_d^2}$$

$$\gamma = \alpha + j\beta$$

$$k_z = -j\gamma = \beta - j\alpha$$

$$v_p = \frac{\omega}{\beta}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\text{attenuation} = \left(\frac{20}{\ln 10}\right)\alpha = (8.6859)\alpha \quad \text{[dB/m]}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$V(z) = Ae^{-\gamma z} + B\,e^{+\gamma z}$$

$$I(z) = \left(\frac{1}{Z_0}\right)\left[Ae^{-\gamma z} - B\,e^{+\gamma z}\right]$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$V(z) = A\left(e^{-\gamma z} + \Gamma_L\, e^{+\gamma z}\right)$$

$$I(z) = \frac{1}{Z_0} A\left(e^{-\gamma z} - \Gamma_L\, e^{+\gamma z}\right)$$

$$Z_{in}(z) = \frac{V(z)}{I(z)}$$

$$Z_{in}(z) = Z_0\left(\frac{1 + \Gamma_L\, e^{+2\gamma z}}{1 - \Gamma_L\, e^{+2\gamma z}}\right)$$

$$Z_{in} = Z_0\left(\frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}\right)$$

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right)$$

$$Z_{in}(z) = Z_0 \left(\frac{1+\Gamma(z)}{1-\Gamma(z)} \right)$$

$$Z_{in} = jZ_0 \tan(\beta l)$$

$$Z_{in} = -jZ_0 \cot(\beta l)$$

$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

$$|V(z)| = |A| \left| 1 + |\Gamma_L| e^{+j(\phi+2\beta z)} \right|$$

$$\left| \frac{V(z)}{V^+} \right| = \left| 1 + |\Gamma_L| e^{+j(\phi+2\beta z)} \right| = \left| 1 + \Gamma(z) \right|$$

$$\Gamma(z) = \Gamma_L e^{j2\beta z} = \Gamma_L e^{-j2\beta l}$$

$$\text{VSWR} \equiv \frac{V_{\max}}{V_{\min}}$$

$$\text{VSWR} \equiv \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$\text{SWR} = \max\left(\frac{R_L}{Z_0}, \frac{Z_0}{R_L} \right)$$

$$\mathrm{Z}_{in}^{N}(z) = \frac{1+\Gamma_L\, e^{+2j\beta z}}{1-\Gamma_L\, e^{+2j\beta z}}$$

$$Z_{in} = \frac{Z_{0\mathrm{T}}^{2}}{Z_L}$$

$$Z_{0\mathrm{T}} = \sqrt{Z_0 R_L}$$

Problem 1 (30 pts)

An electric field in free space is described by

$$\underline{\mathscr{E}}(x, y, z, t) = \underline{\hat{z}} \cos(\omega t + \pi / 4 + k_0 y) \quad \text{[V/m]},$$

where $k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$.

a) Find the electric field vector in the phasor domain.

b) Find the magnetic field vector in the phasor domain.

c) Find the complex power going through a surface S in the upward sense. The surface S is a square that is 2 meters × 2 meters, and the face of the square is perpendicular to the vector $\underline{V} = \left(\underline{\hat{x}}(1) + \underline{\hat{y}}(2) + \underline{\hat{z}}(3)\right)$.

Note: Please evaluate all constants that appear in your answers.

Solution

Part (a)

$$\underline{E}(x, y, z) = \underline{\hat{z}}\, e^{j\pi/4} e^{jk_0 y} \quad \text{[V/m]}$$

Part (b)

From Faraday's law (taking the curl of $\underline{E}$), we have

$$\underline{H}(x, y, z) = \underline{\hat{x}}\left(-\frac{1}{\eta_0}\right) e^{j\pi/4} e^{jk_0 y} \quad \text{[A/m]}$$

Part (c)

The complex Poynting vector is

$$\underline{S} = \frac{1}{2}\underline{E} \times \underline{H}^* = -\underline{\hat{y}}\frac{1}{2\eta_0}.$$

The unit normal to the surface is

$$\underline{\hat{n}} = \frac{1}{\sqrt{14}}\left(\underline{\hat{x}}(1) + \underline{\hat{y}}(2) + \underline{\hat{z}}(3)\right).$$

The power flowing through the surface is

$$P = \int_S \underline{S} \cdot \underline{\hat{n}}\, dS \, .$$

Because the Poynting vector is constant on the surface, we have

$$P = 4\left(\underline{S} \cdot \underline{\hat{n}}\right).$$

This gives us

$$P = 4\left(-\underline{\hat{y}}\frac{1}{2\eta_0}\right) \cdot \left(\frac{1}{\sqrt{14}}\left(\underline{\hat{x}}(1) + \underline{\hat{y}}(2) + \underline{\hat{z}}(3)\right)\right).$$

We then have

$$P = 4\left(-\frac{1}{2\eta_0}\frac{2}{\sqrt{14}}\right)$$

or

$$P = -\frac{4}{\sqrt{14}}\frac{1}{\eta_0} \;\; [\mathrm{VA}].$$

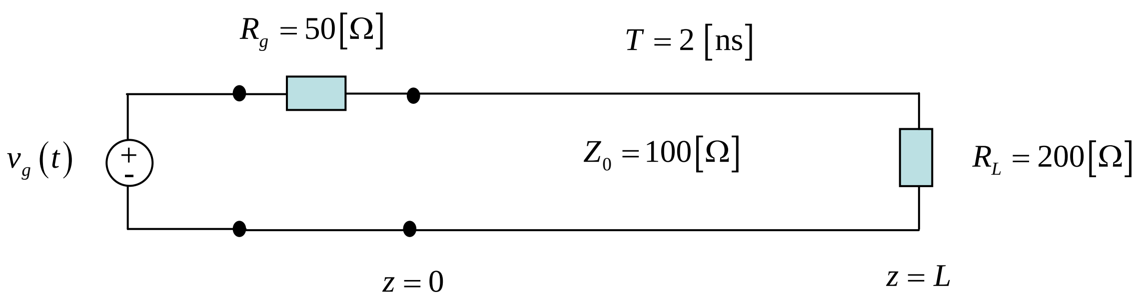
Note: Because the complex power is purely real, we can also say that this is the time-average power in Watts flowing through the surface.

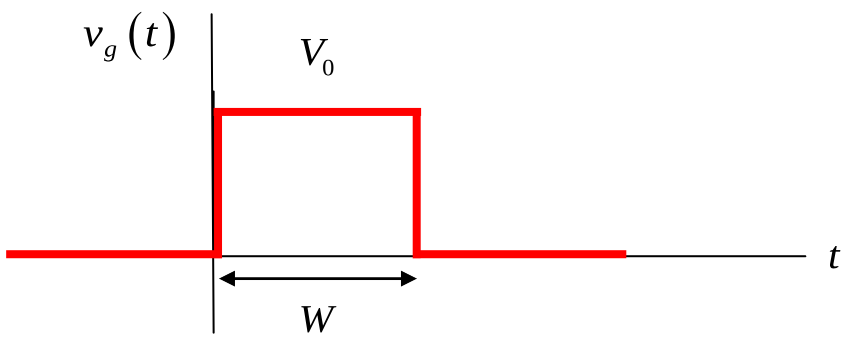
Problem 2 (35 pts)

A digital pulse of amplitude V_0 = 3.0 [V] and duration W = 1.5 [ns] is applied at the input to the transmission line circuit shown below.

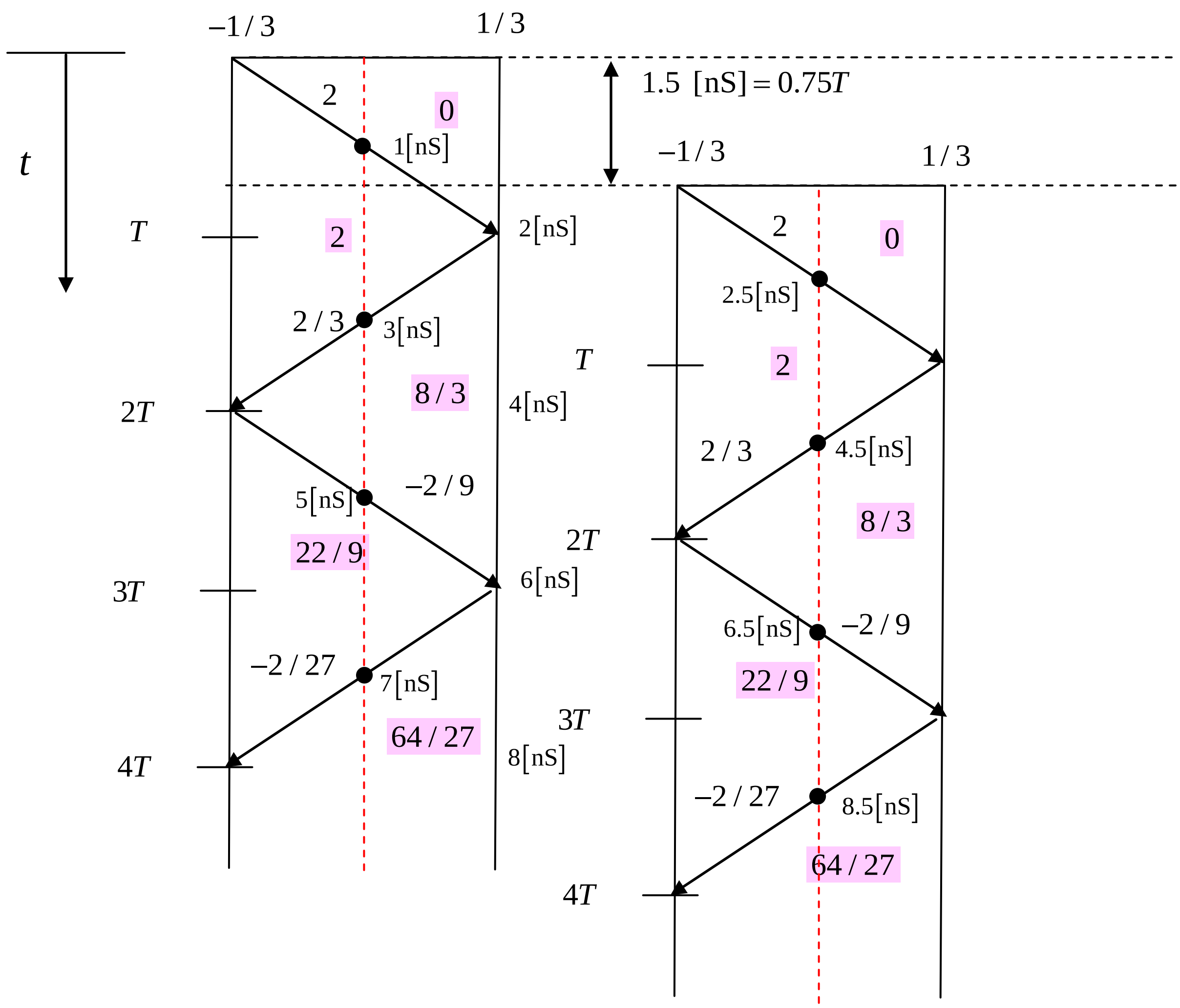
a) Construct a bounce diagram for this problem that extends to a time of 4T. (Make your bounce diagram on the next page.)

b) Make an accurate "oscilloscope trace" plot of the voltage $v(t)$ on the line at $z = L/2$. Make your plot on the graph that is given below. Label all voltage values on your plot, and label all times where the voltage changes. Plot to a time of 8 [ns].

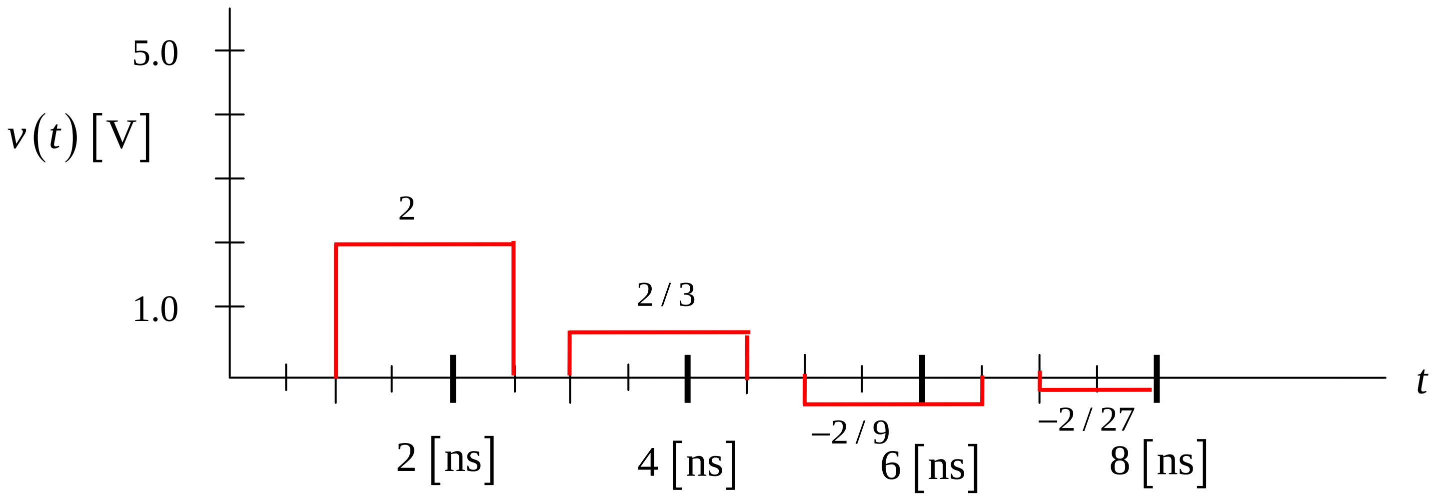




Make your bounce diagram here:



Make your plot here:

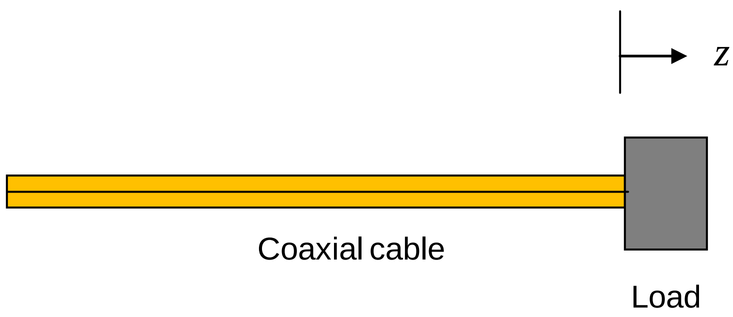


Problem 3 (35 pts.)

A coaxial cable transmission line has a characteristic impedance of 50 [Ω]. The relative permittivity of the (nonmagnetic) Teflon filling the line is 2.1. It is found that a voltage minimum of 2.0 [V] occurs on the line at z = -5 [cm] and a voltage maximum of 4.0 [V] occurs on the line at z = -15 [cm]. (This voltage maximum is the one that is the closest to the voltage minimum.)

What is the unknown load impedance at z = 0?

Note: This is not supposed to be a Smith chart problem, so please do not use a Smith chart.



Solution

We have

$\text{SWR} = 2$.

Next, use

$$|\Gamma_L| = \frac{\text{SWR-1}}{\text{SWR+1}}$$

to obtain

$$|\Gamma_L| = \frac{1}{3}.$$

For the phase of the load reflection coefficient, we have

$$\phi + 2\beta z = -\pi$$

so

$$\phi - 2\left(\frac{2\pi}{\lambda}\right)d = -\pi\ .$$

From the given data we have

$$d = -5[\text{cm}],$$

$$\lambda = \lambda_d = 4(-5-(-15))\ [\text{cm}] = 40[\text{cm}].$$

Hence, we have

$$\phi = -\pi / 2\ [\text{radians}].$$

Hence, we have

$$\Gamma_L = \frac{1}{3}e^{-j\pi/2} = -j/3.$$

The load impedance is then

$$Z_L = Z_0\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right).$$

This gives us

$$Z_L = 50\left(\frac{1+\left(-j/3\right)}{1-\left(-j/3\right)}\right)$$

or

$$Z_L = 50\left(\frac{1-j/3}{1+j/3}\right).$$

This gives us

$$Z_L = 40 - j30\ [\Omega].$$