ECE 3317
Applied Electromagnetic Waves

Exam 1

Oct. 20, 2022

Name ___________SOLUTION____

General Information:

The exam is open-book and open-notes. You are not allowed to use any device that has communication functionality (laptop, cell phone, ipad, etc.).

Remember, you are bound by the UH Academic Honesty Policy during the exam!

Instructions:

- Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
- Write neatly. You will not be given credit for work that is not easily legible.
- Leave answers in terms of the parameters given in the problem.
- Show units in all of your final answers.
- Circle your final answers.
- Double-check your answers. For simpler problems, partial credit may not be given.
- If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
- Make sure you sign the academic honesty statement below.

Academic Honesty Statement

By taking this exam, you agree to abide by the UH Academic Honesty Policy during this exam. You understand and agree that the punishment for violating this policy will be most severe, including getting an F in the class and getting expelled from the University.

Signature: ____

Problem 1 (25 pts)

A lossy coaxial cable is shown below. The coax is filled with a lossy dielectric. The fields inside the coax are given (in cylindrical coordinates) by

$$\underline{E}(\rho, z) = \underline{\hat{\rho}}\left(\frac{1}{\rho}\right)e^{-j\beta z}e^{-\alpha z} \quad \text{[V/m]},$$

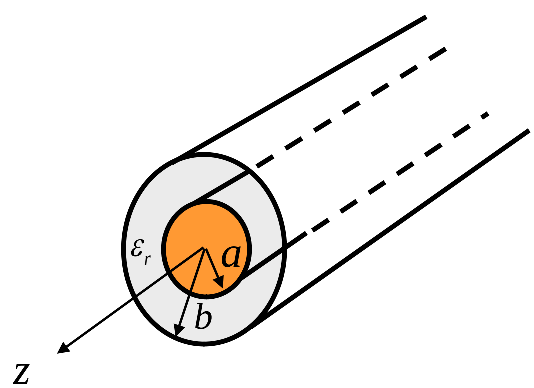
$$\underline{H}(\rho, z) = \underline{\hat{\phi}}\frac{1}{\eta}\left(\frac{1}{\rho}\right)e^{-j\beta z}e^{-\alpha z} \quad \text{[A/m]},$$

where

$$\eta = |\eta| e^{j\phi}$$

is a complex number written in polar form (this is the complex intrinsic impedance of the dielectric material).

Find the complex power that is flowing in the positive z direction at any point z on the coax.



ROOM FOR WORK

Solution

$$\underline{S} = \frac{1}{2}\underline{E} \times \underline{H}^*$$

$$\begin{aligned}
\underline{S} &= \frac{1}{2}\left(\underline{\hat{\rho}}\left(\frac{1}{\rho}\right)e^{-j\beta z}e^{-\alpha z}\right) \times \left(\underline{\hat{\phi}}\frac{1}{\eta}\left(\frac{1}{\rho}\right)e^{-j\beta z}e^{-\alpha z}\right)^* \\
&= \frac{1}{2}\left(\underline{\hat{\rho}} \times \underline{\hat{\phi}}\right)\left(\frac{1}{\rho^2}\right)\left(e^{-j\beta z}e^{-\alpha z}\right)\left(e^{-j\beta z}e^{-\alpha z}\right)^* \frac{1}{\eta^*} \\
&= \frac{1}{2}\underline{\hat{z}}\left(\frac{1}{\rho^2}\right)\left(e^{-j\beta z}e^{-\alpha z}\right)\left(e^{+j\beta z}e^{-\alpha z}\right)\frac{1}{\eta^*} \\
&= \frac{1}{2}\underline{\hat{z}}\left(\frac{1}{\rho^2}\right)\left(e^{-2\alpha z}\right)\frac{1}{\eta^*} \\
&= \frac{1}{2}\underline{\hat{z}}\left(\frac{1}{\rho^2}\right)\left(e^{-2\alpha z}\right)\frac{1}{|\eta|e^{-j\phi}} \\
&= \frac{1}{2}\underline{\hat{z}}\left(\frac{1}{\rho^2}\right)\left(e^{-2\alpha z}\right)\frac{1}{|\eta|}e^{+j\phi}.
\end{aligned}$$

Hence,

$$\underline{S} = \frac{1}{2}\underline{\hat{z}}\left(\frac{1}{\rho^2}\right)\left(e^{-2\alpha z}\right)\frac{1}{|\eta|}e^{+j\phi} \ \left[\text{VA/m}^2\right].$$

We then have

$$\begin{aligned}
P_c &= \int_S \underline{S} \cdot \underline{\hat{z}}\, dS \\
&= \int_S S_z dS \\
&= \int_0^{2\pi}\int_a^b S_z\, \rho d\rho d\phi \\
&= \int_0^{2\pi}\int_a^b \frac{1}{2}\left(\frac{1}{\rho^2}\right)\left(e^{-2\alpha z}\right)\frac{1}{|\eta|}e^{+j\phi}\, \rho d\rho d\phi \\
&= \frac{1}{2}\left(e^{-2\alpha z}\right)\frac{1}{|\eta|}e^{+j\phi}\int_0^{2\pi}\int_a^b \frac{1}{\rho} d\rho d\phi.
\end{aligned}$$

Hence,

$$P_c = \frac{1}{2}\left(e^{-2\alpha z}\right)\frac{1}{|\eta|}e^{+j\phi}\int_a^b \frac{1}{\rho}d\rho\int_0^{2\pi} d\phi$$
$$= \frac{1}{2}\left(e^{-2\alpha z}\right)\frac{1}{|\eta|}e^{+j\phi}\left(\ln\left(\frac{b}{a}\right)\right)(2\pi).$$

We then have

$$P_c = \pi\left(e^{-2\alpha z}\right)\frac{1}{|\eta|}\ln\left(\frac{b}{a}\right)(\cos\phi + j\sin\phi).$$

We then also have

$$\operatorname{Re} P_c = \pi\left(e^{-2\alpha z}\right)\frac{1}{|\eta|}\ln\left(\frac{b}{a}\right)\cos\phi\ [\mathrm{W}].$$

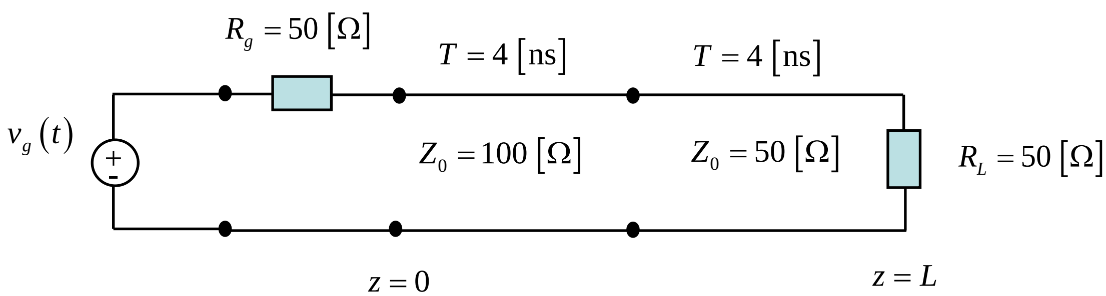
$$\operatorname{Im} P_c = \pi\left(e^{-2\alpha z}\right)\frac{1}{|\eta|}\ln\left(\frac{b}{a}\right)\sin\phi\ [\mathrm{Vars}].$$

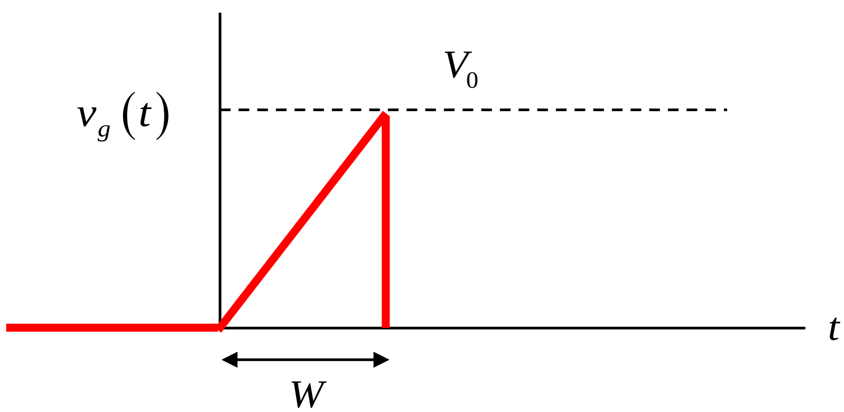
Problem 2 (25 pts)

A voltage source is applied at the left end of a transmission line as shown below. The transmission line meets a second transmission line, which is then terminated by a load.

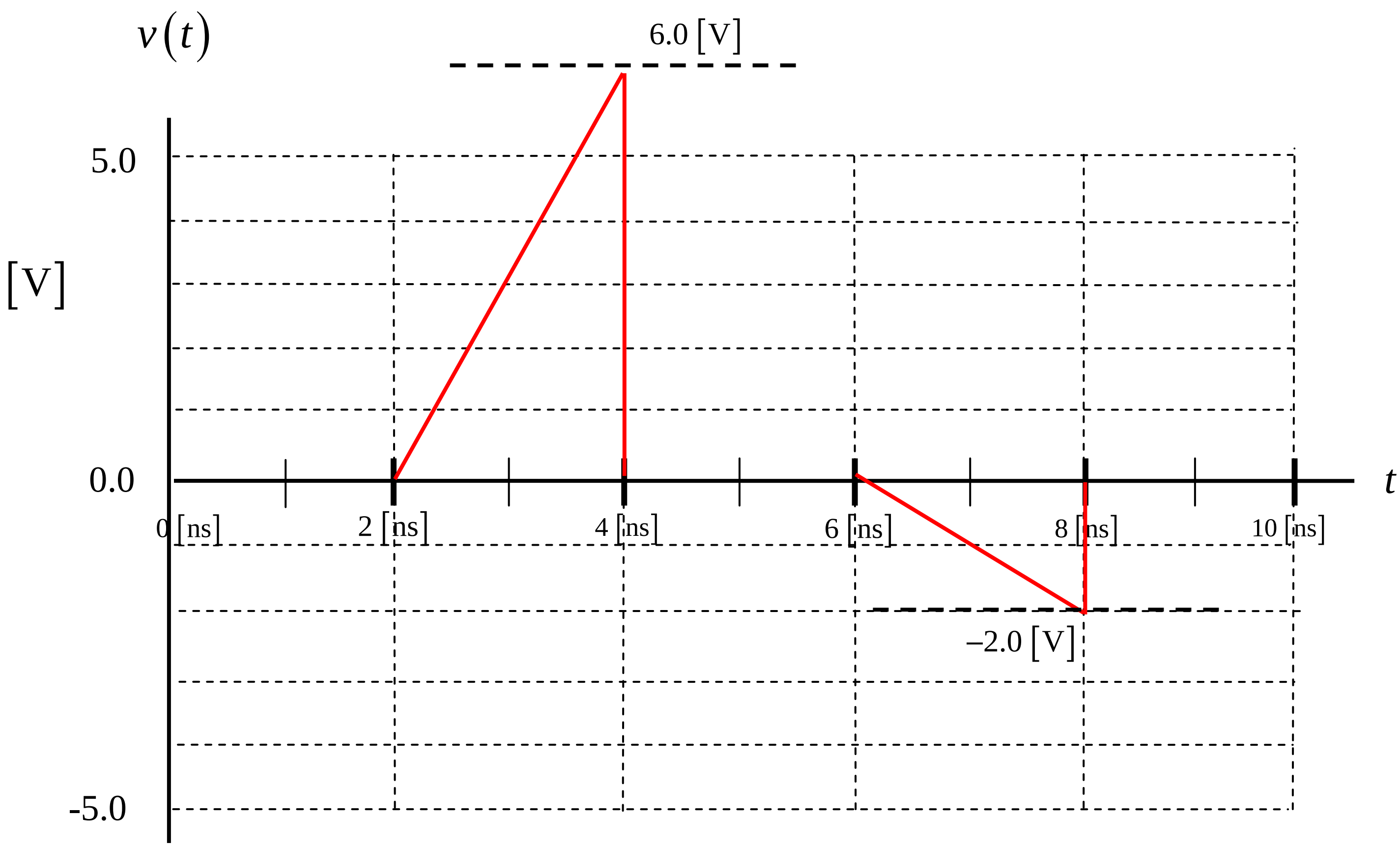
A plot of the generator voltage $v_g(t)$ is shown below. The pulse peak is $V_0 = 9\ [\text{V}]$ and the width of the pulse is $W = 2\ [\text{ns}]$.

Plot the voltage $v(t)$ measured by an oscilloscope that is connected to the first (left) line at a point halfway down the first line (halfway between the generator and the junction). Plot to a time of 10 [ns]. Use the graph on the next page to make your plot. Label all voltage values on your plot. (Please show your work on the page that is after the page with the plot.)





Make your plot here:



Solution

$$A=\frac{2}{3};\quad \Gamma_L=-\frac{1}{3};\quad \Gamma_g=-\frac{1}{3}$$

$$v(t)=Av_g\left(t-2[\text{nS}]\right)+A\Gamma_L v_g\left(t-6[\text{nS}]\right)+A\Gamma_L\Gamma_g v_g\left(t-10[\text{nS}]\right)+\ldots[\text{V}]$$

Keeping the first two terms, we have

$$v(t)=\left(\frac{2}{3}\right)v_g\left(t-2[\text{nS}]\right)+\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)v_g\left(t-6[\text{nS}]\right)+\ldots[\text{V}]$$

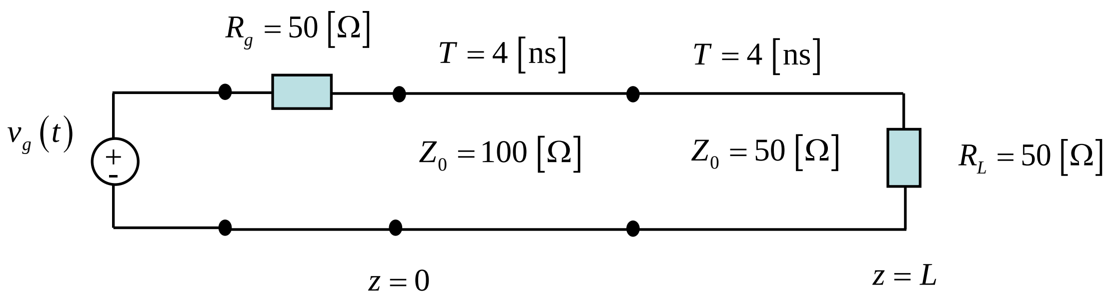
Problem 3 (25 pts)

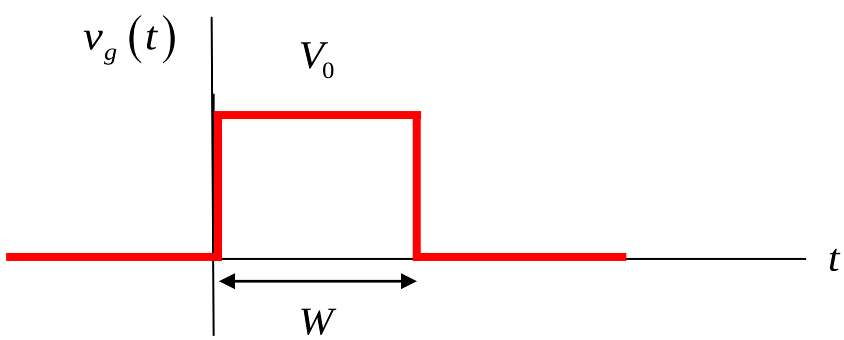
A voltage source is applied at the left end of a transmission line as shown below. The transmission line meets a second transmission line, which is then terminated by a load.

A plot of the generator voltage $v_g(t)$ is shown below. The pulse peak is $V_0 = 9\,[\text{V}]$ and the width of the pulse is $W = 8\,[\text{ns}]$.

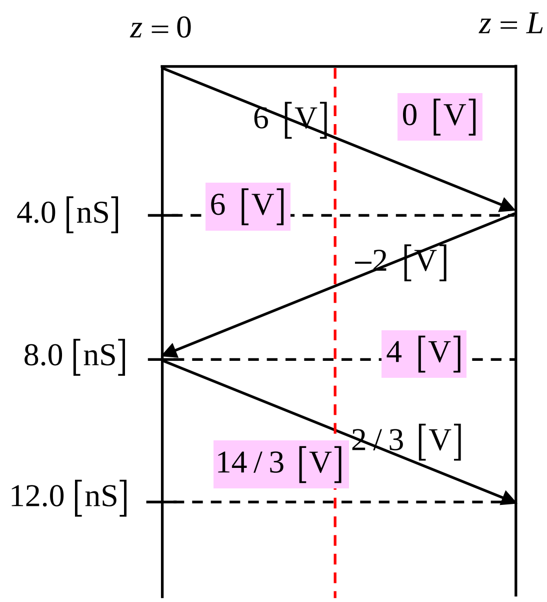
(a) Make a bounce diagram for this problem, for the left line. Plot to a time of 12 [ns]. Put your bounce diagram on the next page.

(b) Plot the voltage $v(t)$ measured by an oscilloscope that is connected to the first (left) line at a point halfway down the first line (halfway between the generator and the junction). Plot to a time of 10 [ns]. Use the graph on the next page to make your plot. Label all voltage values on your plot. Please show your work on the page that is after the page with your plot.

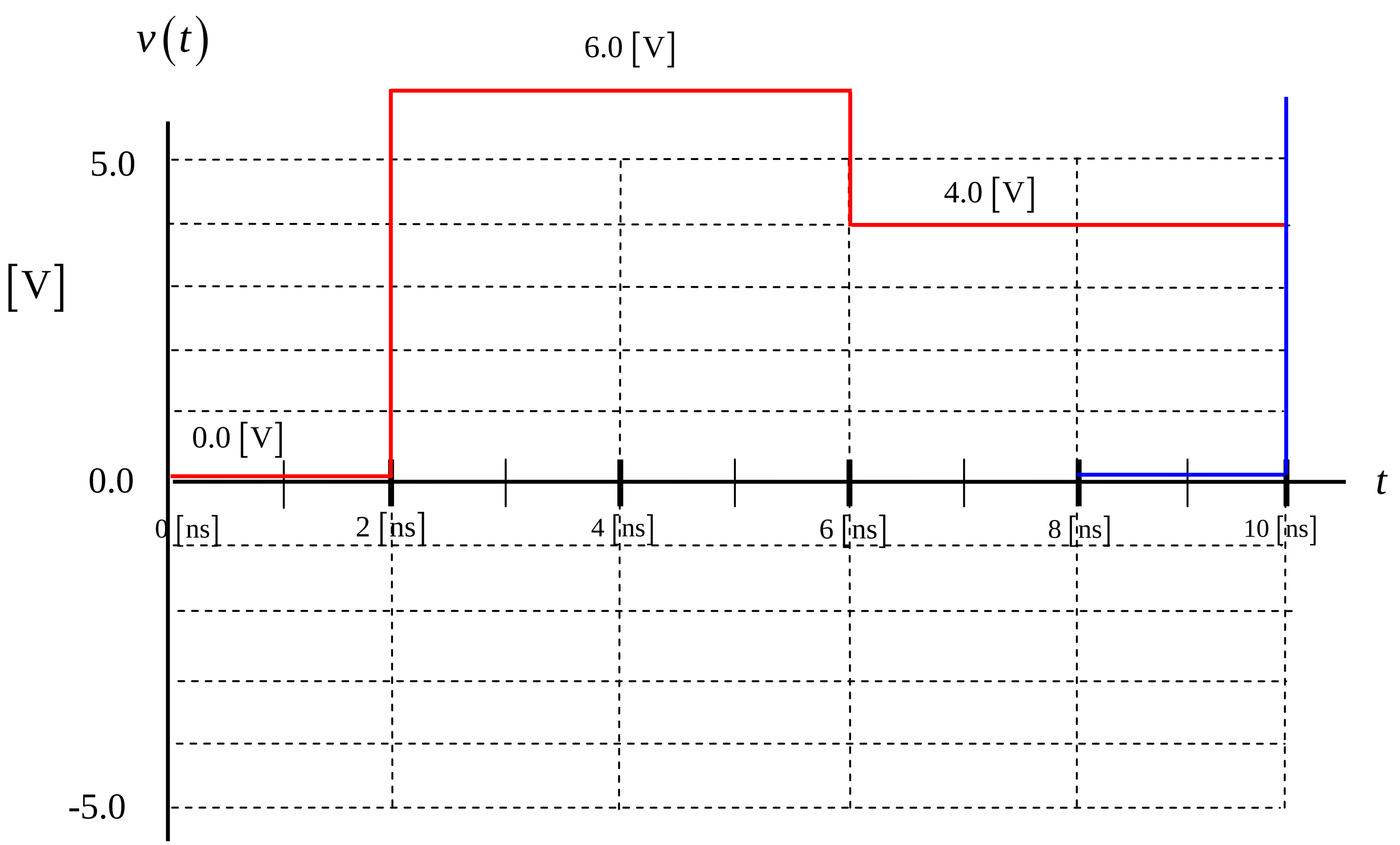




Make your bounce diagram here:



Make your plot here:



Solution

The oscilloscope trace from the step function of amplitude 6 [V] is shown in red on the plot. The oscilloscope trace due to the delayed step function of amplitude 6 [V] is shown in blue on the plot. As can be seen, when we subtract the blue curve from the red curve, the blue curve does not have any effect when we plot up to 10 [nS]. Hence, the red plot is also the final oscilloscope trace.

Problem 4 (25 pts)

At a frequency of 1.0 GHz, a coaxial cable has the following parameters:

$R = 4.5$ [Ω/m]

$L = 4.0\times10^{-7}$ [H/m]

$G = 5.0\times10^{-4}$ [S/m]

$C = 7.0\times10^{-11}$ [F/m]

Find the maximum length of the coaxial cable that we can have, if we wish to keep the loss less than 10 dB at 1.0 GHz.

Solution

We have

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}\,.$$

At 1.0 GHz this gives us

$$\gamma = 0.048663 + j33.247493\ [1/\mathrm{m}]\,.$$

Hence,

$$\alpha = 0.048663\ [\mathrm{np/m}]\,.$$

Therefore,

$$\mathrm{Att}_{\mathrm{dB/m}} = 0.048663(8.686)\ [\mathrm{dB/m}]\,.$$

We then have for the length L of the coaxial cable

$$(\mathrm{Att}_{\mathrm{dB/m}})L = 10\ [\mathrm{dB}]\,.$$

We thus have

$$L = \frac{10}{\mathrm{Att}_{\mathrm{dB/m}}}\,.$$

This gives us

$$L = \frac{10}{0.048663(8.686)}$$

so

$$L = 23.6582\ [\mathrm{m}]\,.$$