ECE 3317
Applied Electromagnetic Waves

Exam 1
Oct. 22, 2024

Name ________SOLUTION____

General Information:

The exam is open-book and open-notes. You are not allowed to use any device that has communication functionality (laptop, cell phone, ipad, etc.).

Remember, you are bound by the UH Academic Honesty Policy during the exam!

Instructions:

- Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
- Write neatly. You will not be given credit for work that is not easily legible.
- Leave answers in terms of the parameters given in the problem.
- Show units in all of your final answers.
- Circle your final answers.
- Double-check your answers. For simpler problems, partial credit may not be given.
- If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
- Make sure you sign the academic honesty statement below.

Academic Honesty Statement

By taking this exam, you agree to abide by the UH Academic Honesty Policy during this exam. You understand and agree that the punishment for violating this policy will be most severe, including getting an F in the class and getting expelled from the University.

Signature: ____

Problem 1 (35 pts)

A hollow rectangular pipe (called a rectangular waveguide) is shown below. It has the following electric field inside of it:

$$\underline{E}(x,z) = \underline{\hat{y}}\, E_0 \sin\left(\frac{\pi x}{a}\right) e^{-jk_z z} \quad [\mathrm{V/m}],$$

where k_z is a real number and E_0 is an amplitude coefficient that we can assume is real.

a) Find the electric field vector in the time domain.

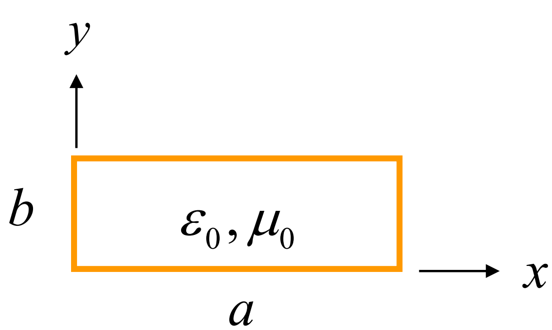
b) Find the magnetic field vector in the phasor domain.

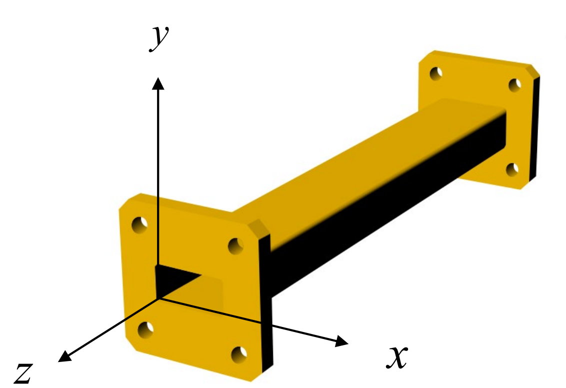
c) Find the complex Poynting vector.

d) Find the complex power flowing (in the positive z direction) through the cross section of the pipe.

e) Find the time-average power and the vars flowing (in the positive z direction) through the cross section of the pipe.

Helpful integral: $\int_0^a \sin^2\left(\frac{\pi x}{a}\right) dx = \frac{a}{2}$





SOLUTION

Part (a)

In the time domain we have

$$\underline{\mathcal{E}} = \underline{\hat{y}} E_0 \sin\left(\frac{\pi x}{a}\right) \cos\left(\omega t - kz\right) \;[\text{V/m}].$$

Part (b)

From Faraday's law we have

$$\underline{H} = -\frac{1}{j\omega\mu_0} \nabla \times \underline{E}\,.$$

This gives us

$$\underline{H} = -\frac{E_0}{\omega\mu_0}\left(\underline{\hat{x}} k_z \sin\left(\frac{\pi x}{a}\right) + \underline{\hat{z}} \frac{\pi}{a} \cos\left(\frac{\pi x}{a}\right)\right) e^{-jk_z z} \;[\text{A/m}]$$

Part (c)

The complex Poynting vector is

$$\underline{S} = \frac{1}{2} \underline{E} \times \underline{H}^*\,.$$

This gives us

$$\underline{S} = \frac{1}{2} \underline{\hat{y}} E_0 \sin\left(\frac{\pi x}{a}\right) e^{-jk_z z} \times \left[-\frac{E_0}{\omega\mu_0}\left(\underline{\hat{x}} k_z \sin\left(\frac{\pi x}{a}\right) + \underline{\hat{z}} \frac{\pi}{a} \cos\left(\frac{\pi x}{a}\right)\right) e^{-jk_z z}\right]^*.$$

The result is

$$\underline{S} = \frac{E_0^2}{2\omega\mu_0}\left(\underline{\hat{z}} k_z \sin^2\left(\frac{\pi x}{a}\right) - \underline{\hat{x}} \frac{\pi}{a} \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi x}{a}\right)\right) \left[\text{VA/m}^2\right]$$

Part (d)

The complex power is

$$p_c = \int_0^b \int_0^a \underline{S} \cdot \underline{\hat{z}}\, dxdy = \int_0^b \int_0^a S_z\, dxdy$$

where

$$S_z = \frac{E_0^2}{2\omega\mu_0} k_z \sin^2\left(\frac{\pi x}{a}\right)\ \left[\text{VA/m}^2\right].$$

This gives us

$$P_c = \frac{E_0^2}{2\omega\mu_0} k_z \left(\frac{ab}{2}\right)\ \left[\text{VA}\right].$$

Part (e)

The time-average power is

$$P = \left\langle \mathscr{P}(t) \right\rangle = \text{Re}\left(P_c\right) = \frac{E_0^2}{2\omega\mu_0} k_z \left(\frac{ab}{2}\right)\ \left[\text{W}\right].$$

The reactive power is

$$Q = \text{Im}\left(P_c\right) = 0\ \left[\text{vars}\right].$$

ROOM FOR WORK

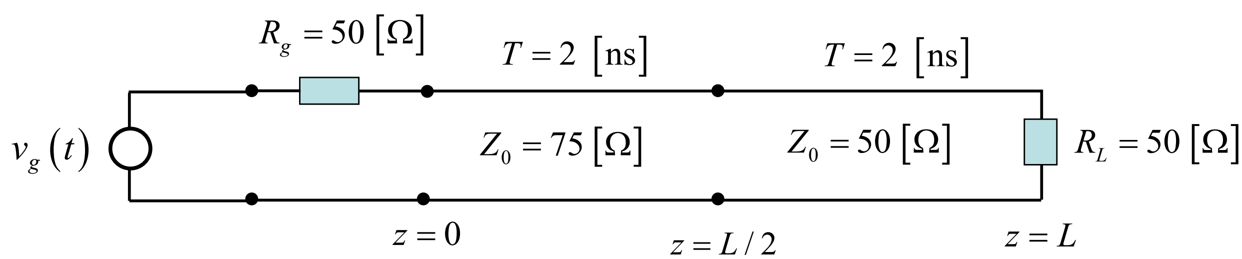
Problem 2 (30 pts)

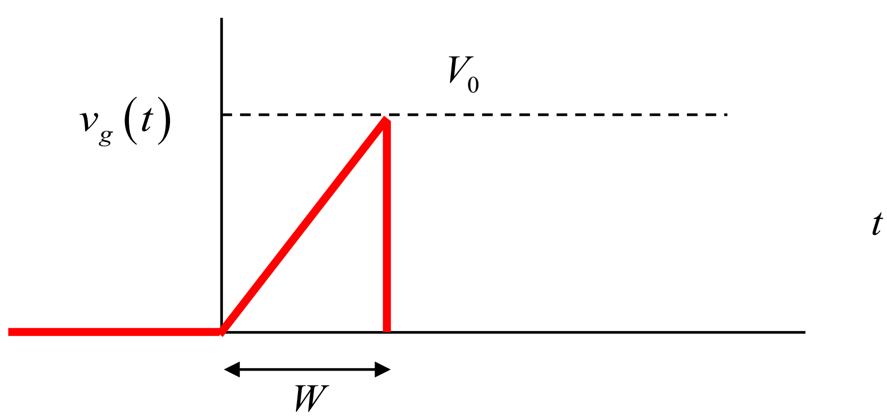
A voltage source is applied at the left end of a two-section transmission line as shown below. A plot of the generator voltage $v_g(t)$ is shown below. The pulse peak is $V_0 = 5.0\ [\mathrm{V}]$ and the width of the pulse is $W = 0.5\ [\mathrm{ns}]$.

a) Plot the voltage $v(t)$ measured by an oscilloscope that is connected to the left line at $z = L/4$ [m]. Plot to a time of 5 [ns].

b) Plot a snapshot of the voltage on the left line at 1.0 [nS].

Use the graphs on the next page to make your plots. Label all important values of voltage, time, and distance on your plot, so that the amplitude and the start and end times (or locations) of the waveforms can be clearly seen.





SOLUTION

Part (a)

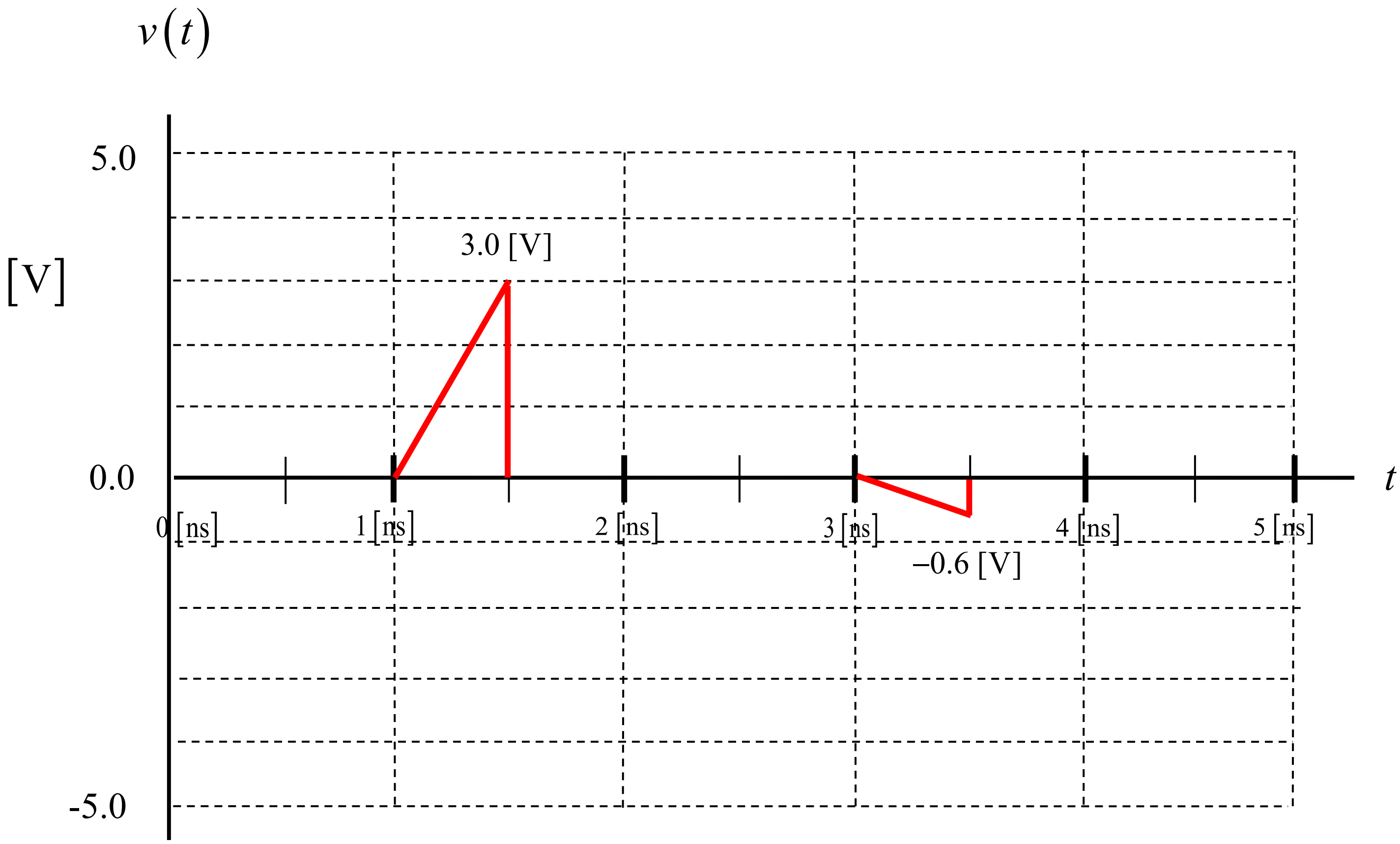
The initial pulse sweeping past the oscilloscope pulse is delayed by 1 [nS], and also the amplitude is decreased by a factor of 3/5 (from the voltage divider equation), giving a peak amplitude of 3.0 [V]. After reflecting from the junction, the pulse will be delayed by a total of 3 [ns] and will be decreased in amplitude by another factor of Γ_J^+, which is -1/5. The reflected pulse thus has an amplitude of -0.6 [V]. The plot is shown below.

Part (b)

At 1.0 [ns], the leading edge of the pulse (the tip of the sawtooth wave) is at the middle of the first line, at $z = L/4$. The width of the pulse on the line is $\Delta z = L/2$ (0.5 [ns] / [2.0 [ns]) = $L/8$ = 0.125 L. The plot is shown below.

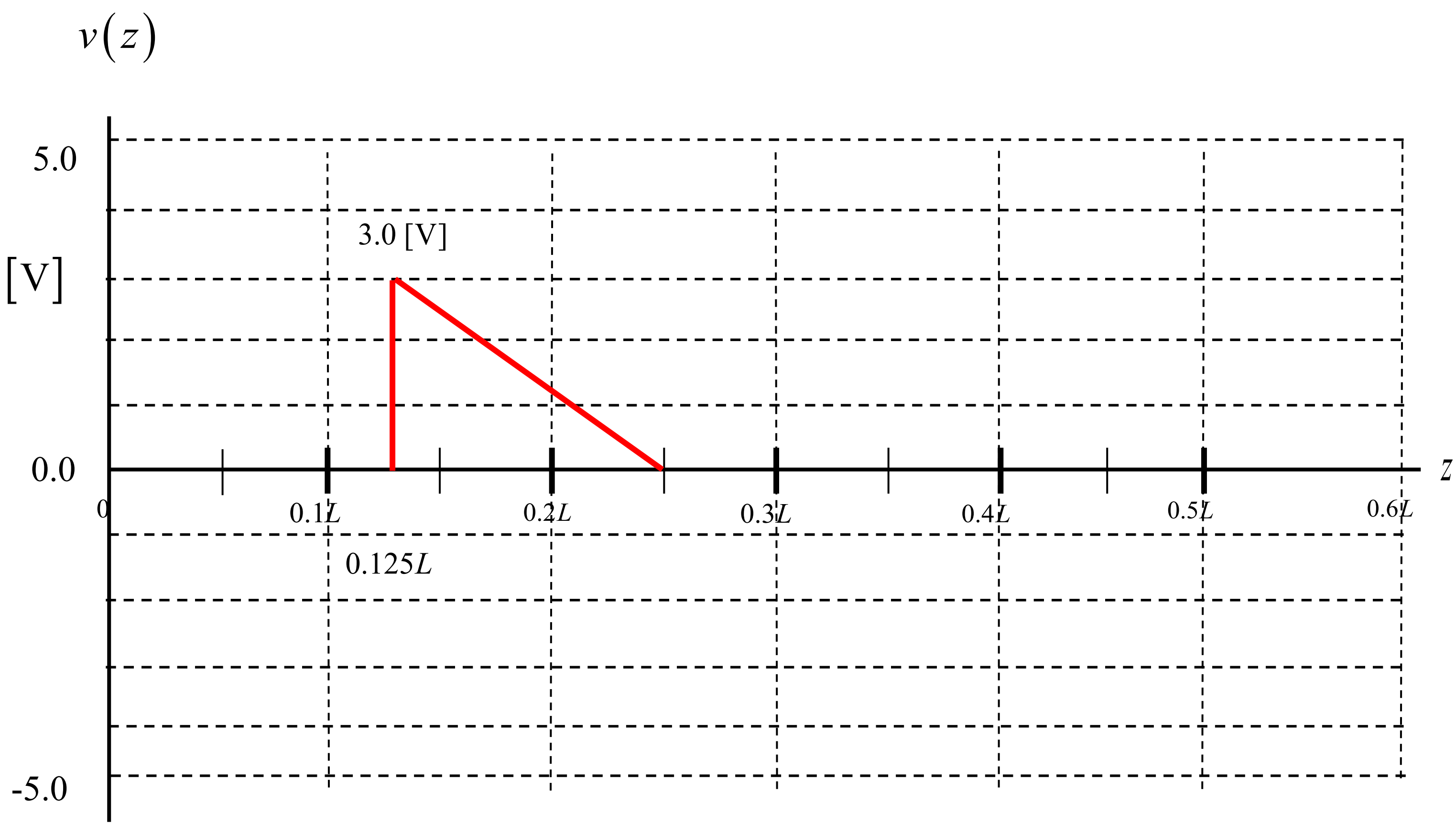
Make your plots here:

Part (a)



Make your plots here:

Part (b)



Problem 3 (35 pts)

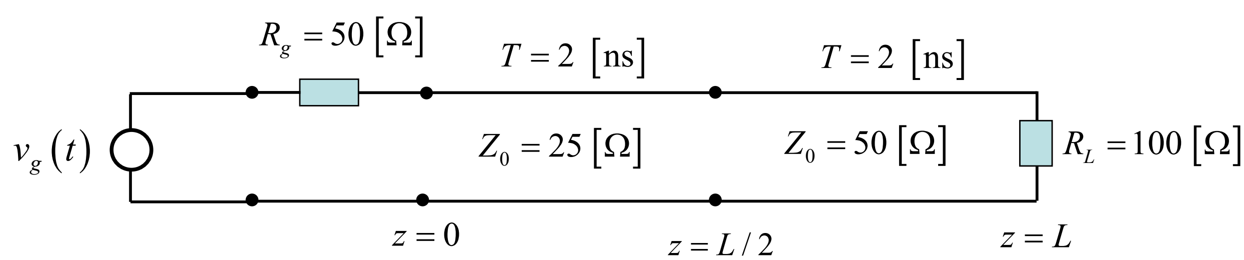
A voltage source is applied at the left end of a two-section transmission line as shown below. (Note that this is not the same two-section line as in Prob. 2.) A plot of the generator voltage $v_g(t)$ is shown below. The pulse peak is $V_0 = 6.0\,[\text{V}]$ and the width of the pulse is $W = 1.0\,[\text{ns}]$.

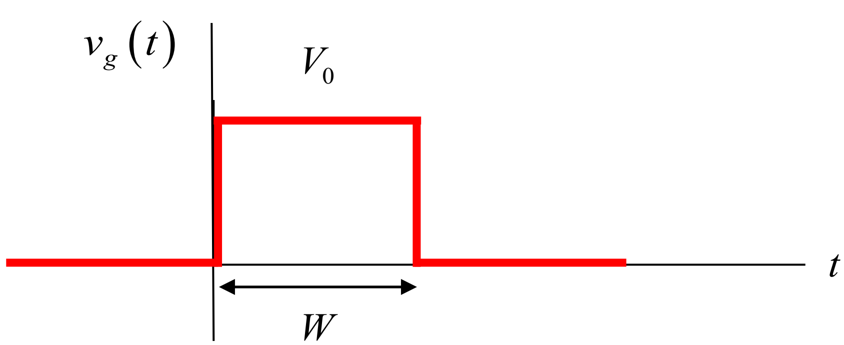
a) Make a bounce diagram for this problem. Plot up to 6 [ns].

b) Plot the voltage $v(t)$ measured by an oscilloscope that is connected to the right line at z = 3L/4. Plot to a time of 5 [ns].

c) Plot a snapshot of the voltage on the right line at 3.0 [nS].

Use the graphs on the next page to make your plots. Label all important values of voltage, time, and distance on your plot, so that the pulse amplitude and the start and end times (or locations) of the waveform can be clearly seen.





SOLUTION

Part (a)

The bounce diagram is shown below.

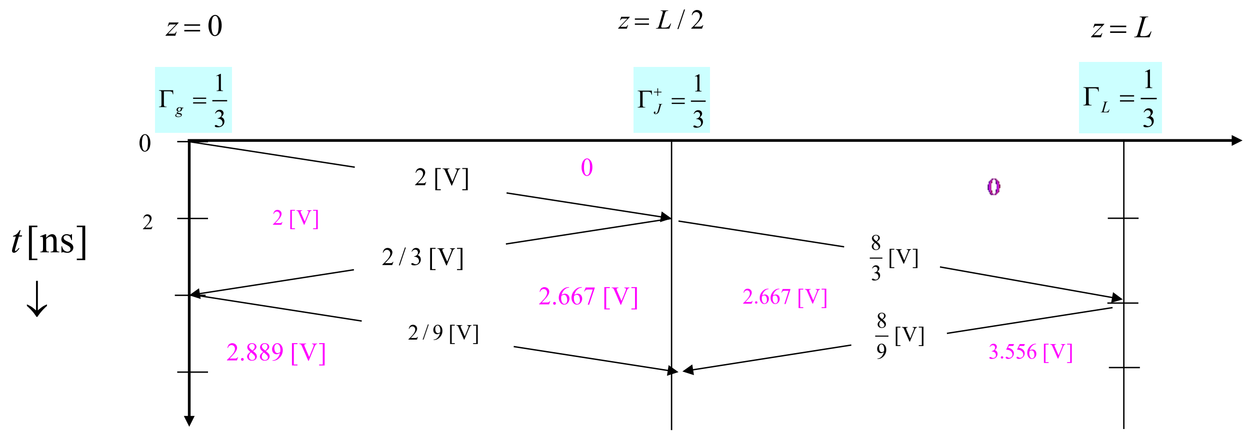
Part (b)

The plot below shows the plot for the step function, and the shifted step function (shifted by W = 1.0 [ns]). It also shows the difference between the two plots (original (black) minus the shifted (blue)) to get the final plot (red) for the rectangular pulse function.

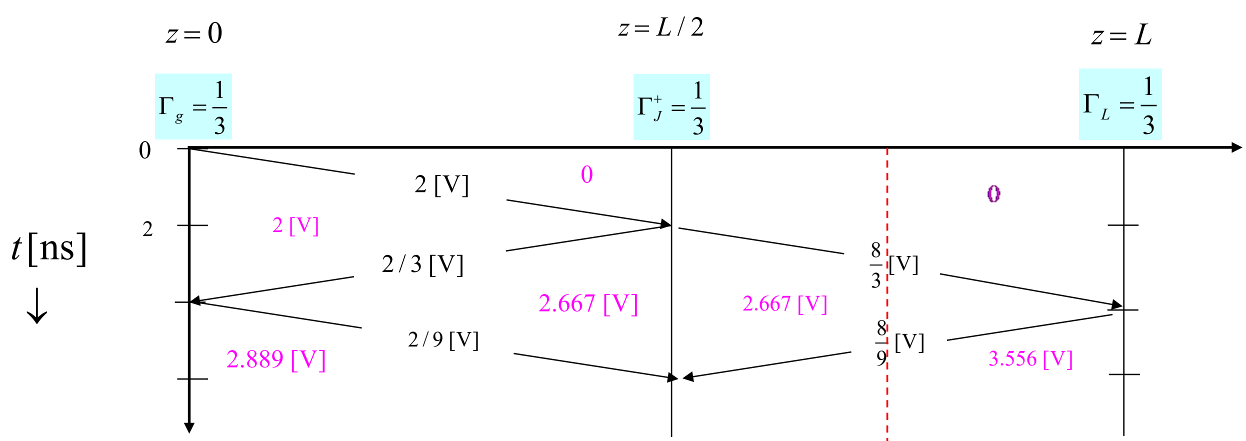
Part (c)

The plot below shows bounce diagram and the shifted bounce diagram, from which we get the plot below for the snapshot. Note that for the shifted bounce diagram, the voltage stays zero on the line.

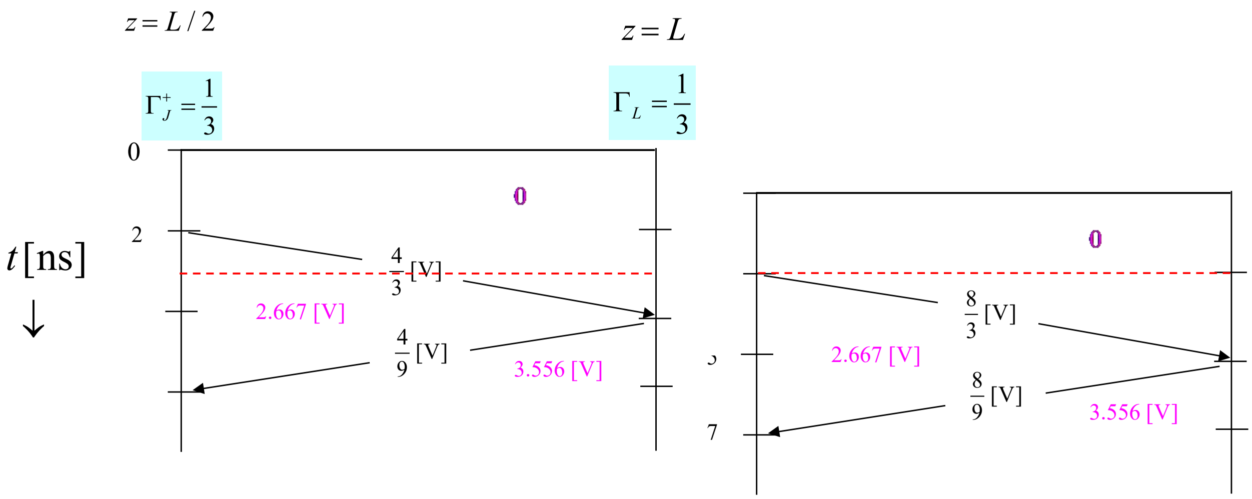
Make your bounce diagram here (part (a):



Part (b)



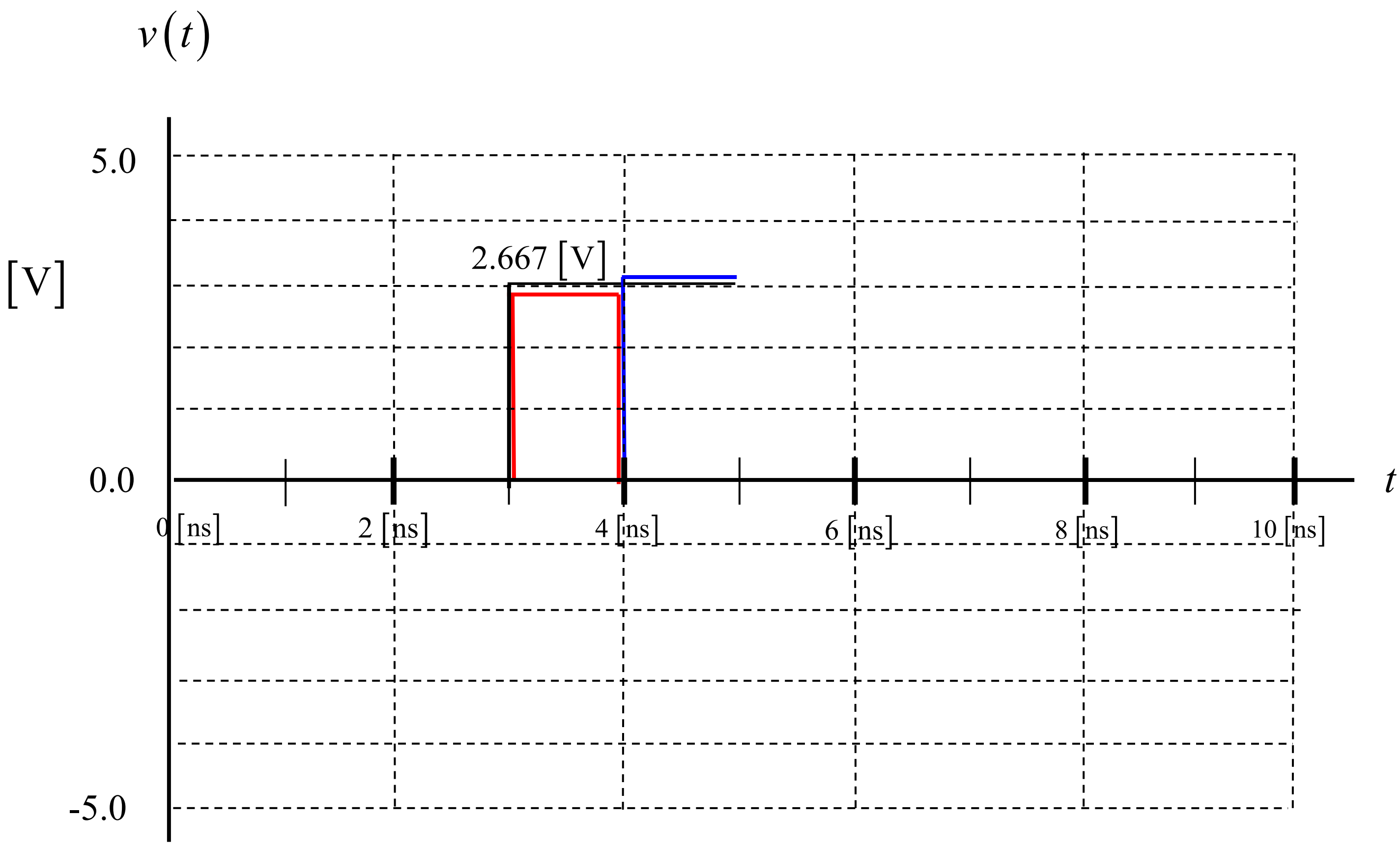
Part (c)



$z = L/2$
$z = L$
$\Gamma_J^+ = \frac{1}{3}$
$\Gamma_L = \frac{1}{3}$
0
2
t [ns]
0
$\frac{4}{3}$ [V]
2.667 [V]
$\frac{4}{9}$ [V]
3.556 [V]
0
$\frac{8}{3}$ [V]
2.667 [V]
$\frac{8}{9}$ [V]
3.556 [V]
7

Make your plots here:

Part (b)



Make your plots here:

Part (c)