**DO NOT BEGIN THIS EXAM UNTIL TOLD TO START**

**Name: ________________________________________**

**PeopleSoft ID: __________________________________**

# ECE 3317

**Applied Electromagnetic Waves**
**April 8, 2009**

1. This exam is closed book and closed notes. No additional material may be used for this exam except for a calculator (no computers), a ruler, and a compass.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily legible**.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructors. You will not be given credit for work that is based on a wrong assumption.
10. Make sure you sign the academic honesty statement on the next page.

## Academic Honesty Statement

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including getting an F in the class and getting expelled from the University.

______________________________________________

Signature

**FORMULA SHEET**

$$\nabla\times\underline{\mathscr{E}}=-\frac{\partial\underline{\mathscr{B}}}{\partial t}$$

$$\nabla\times\underline{\mathscr{H}}=\underline{\mathscr{J}}+\frac{\partial\underline{\mathscr{D}}}{\partial t}$$

$$\nabla\cdot\underline{\mathscr{B}}=0$$

$$\nabla\cdot\underline{\mathscr{D}}=\rho_v$$

$$\nabla\times\underline{E}=-j\omega\,\underline{B}$$

$$\nabla\times\underline{H}=\underline{J}+j\omega\,\underline{D}$$

$$\nabla\cdot\underline{B}=0$$

$$\nabla\cdot\underline{D}=\rho_v$$

$$\nabla\times\underline{V}=\underline{\hat{x}}\left(\frac{\partial V_z}{\partial y}-\frac{\partial V_y}{\partial z}\right)+\underline{\hat{y}}\left(\frac{\partial V_x}{\partial z}-\frac{\partial V_z}{\partial x}\right)+\underline{\hat{z}}\left(\frac{\partial V_y}{\partial x}-\frac{\partial V_x}{\partial y}\right)$$

$$\nabla\times\underline{V}=\underline{\hat{\rho}}\left(\frac{1}{\rho}\frac{\partial V_z}{\partial\phi}-\frac{\partial V_\phi}{\partial z}\right)+\underline{\hat{\phi}}\left(\frac{\partial V_\rho}{\partial z}-\frac{\partial V_z}{\partial\rho}\right)+\underline{\hat{z}}\frac{1}{\rho}\left(\frac{\partial\left(\rho V_\phi\right)}{\partial\rho}-\frac{\partial V_\rho}{\partial\phi}\right)$$

$$\nabla\times\underline{V}=\underline{\hat{r}}\frac{1}{r\sin\theta}\left[\frac{\partial\left(V_\phi\sin\theta\right)}{\partial\theta}-\frac{\partial V_\theta}{\partial\phi}\right]+\underline{\hat{\theta}}\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial V_r}{\partial\phi}-\frac{\partial\left(rV_\phi\right)}{\partial r}\right]+\underline{\hat{\phi}}\frac{1}{r}\left[\frac{\partial\left(rV_\theta\right)}{\partial r}-\frac{\partial V_r}{\partial\theta}\right]$$

$$c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$\varepsilon_0 \doteq 8.8541878\times10^{-12}\ \text{[F/m]}$$

$$\mu_0=4\pi\times10^{-7}\ \text{[H/m]}$$

$$\oint_S \left(\underline{\mathscr{E}} \times \underline{\mathscr{H}}\right) \cdot \hat{n}\, dS = -\int_V \sigma \left|\underline{\mathscr{E}}\right|^2 dV - \int_V \frac{\partial}{\partial t}\left(\frac{1}{2}\mu \left|\underline{\mathscr{H}}\right|^2\right) dV - \int_V \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon \left|\underline{\mathscr{E}}\right|^2\right) dV$$

$$\underline{\mathscr{S}} = \underline{\mathscr{E}} \times \underline{\mathscr{H}}$$

$$\underline{S} \equiv \frac{1}{2}\left(\underline{E} \times \underline{H}^*\right)$$

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\text{F/m}]$$

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) \quad [\text{H/m}]$$

$$G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)} \quad [\text{S/m}]$$

$$R = \frac{1}{\sigma_m \delta}\left(\frac{1}{2\pi a} + \frac{1}{2\pi b}\right) \quad [\Omega/\text{m}]$$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma_m}}$$

$$\frac{\partial v}{\partial z} = -Ri - L\frac{\partial i}{\partial t}$$

$$\frac{\partial i}{\partial z} = -Gv - C\frac{\partial v}{\partial t}$$

$$v(z,t) = f\left(z - c_d t\right) + g\left(z + c_d t\right)$$

$$i(z,t) = \frac{1}{Z_0}\left[ f(z - c_d t) - g(z + c_d t) \right]$$

$$v(z,t) = v_g(t - z / c_d)$$

$$\Gamma_g = \left( \frac{R_g - Z_0}{R_g + Z_0} \right) \qquad \Gamma_L = \left( \frac{R_L - Z_0}{R_L + Z_0} \right)$$

$$V^+ = \left( \frac{Z_0}{R_g + Z_0} \right) V_0$$

$$\Gamma_L(t) = 1 - 2\,e^{-(t-T)/\tau}, \quad t \geq T \qquad \tau = Z_0 C_L$$

$$\Gamma_L(t) = -1 + 2\,e^{-(t-T)/\tau}, \quad t \geq T \qquad \tau = L_L / Z_0$$

$$V(z) = Ae^{-\gamma z} + B\,e^{+\gamma z}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$LC = \mu\varepsilon = \frac{1}{c_d^2}$$

$$\gamma = \alpha + j\beta$$

$$k_z = -j\gamma = \beta - j\alpha$$

$$v_p = \frac{\omega}{\beta}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\text{attenuation} = \left(\frac{20}{\ln 10}\right)\alpha = (8.6859)\alpha \quad \text{[dB/m]}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$V(z) = Ae^{-\gamma z} + B\,e^{+\gamma z}$$

$$I(z) = \left(\frac{1}{Z_0}\right)\left[Ae^{-\gamma z} - B\,e^{+\gamma z}\right]$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$V(z) = A\left(e^{-\gamma z} + \Gamma_L\,e^{+\gamma z}\right)$$

$$I(z) = \frac{1}{Z_0}A\left(e^{-\gamma z} - \Gamma_L\,e^{+\gamma z}\right)$$

$$Z_{in}(z) = \frac{V(z)}{I(z)}$$

$$Z_{in}(z) = Z_0\left(\frac{1 + \Gamma_L\,e^{+2\gamma z}}{1 - \Gamma_L\,e^{+2\gamma z}}\right)$$

$$Z_{in} = Z_0\left(\frac{Z_L + Z_0\tanh(\gamma l)}{Z_0 + Z_L\tanh(\gamma l)}\right)$$

$$Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan\left(\beta l\right)}{Z_0 + jZ_L \tan\left(\beta l\right)} \right)$$

$$Z_{in}\left(z\right) = Z_0 \left( \frac{1+\Gamma\left(z\right)}{1-\Gamma\left(z\right)} \right)$$

$$Z_{in} = jZ_0 \tan\left(\beta l\right)$$

$$Z_{in} = -jZ_0 \cot\left(\beta l\right)$$

$$\Gamma_L = \left|\Gamma_L\right| e^{j\phi}$$

$$\left|V(z)\right| = \left|A\right| \left|1 + \left|\Gamma_L\right| e^{+j\left(\phi + 2\beta z\right)}\right|$$

$$\left|\frac{V(z)}{V^+}\right| = \left|1 + \left|\Gamma_L\right| e^{+j\left(\phi + 2\beta z\right)}\right| = \left|1 + \Gamma\left(z\right)\right|$$

$$\Gamma\left(z\right) = \Gamma_L \, e^{j2\beta z} = \Gamma_L \, e^{-j2\beta l}$$

$$\text{VSWR} \equiv \frac{V_{\max}}{V_{\min}}$$

$$\text{VSWR} \equiv \frac{1+\left|\Gamma_L\right|}{1-\left|\Gamma_L\right|}$$

$$\text{SWR} = \max\left(\frac{R_L}{Z_0}, \frac{Z_0}{R_L}\right)$$

$$Z_{in}^{N}(z) = \frac{1+\Gamma_L \, e^{+2j\beta z}}{1-\Gamma_L \, e^{+2j\beta z}}$$

$$Z_{in} = \frac{Z_{0\mathrm{T}}^2}{Z_L}$$

$$Z_{0\mathrm{T}} = \sqrt{Z_0 R_L}$$

## Problem 1

Assume that we have phasor-domain fields in free space given as

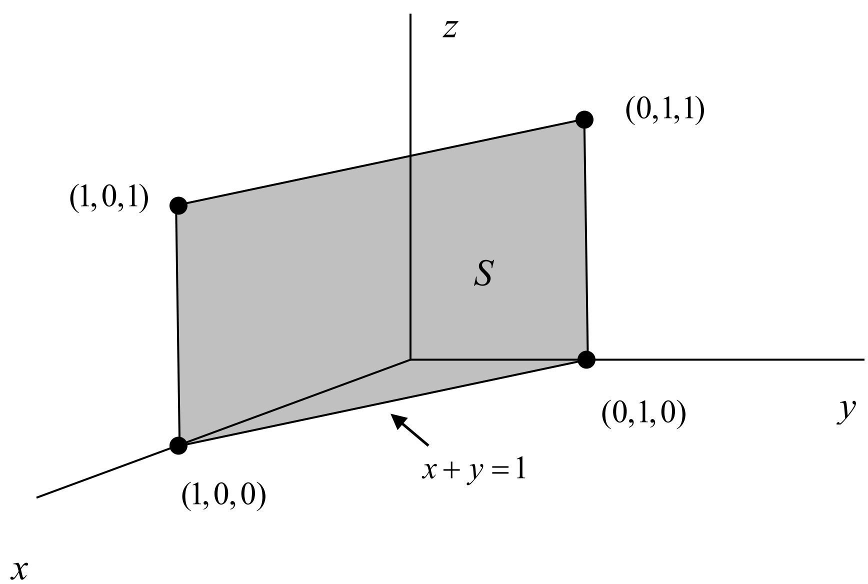
$$\underline{E} = \underline{\hat{x}}\, e^{+jky}$$
$$\underline{H} = (0.002654)\, \underline{\hat{z}}\, e^{+jky},$$

where $k$ is a real number. The radian frequency is $\omega$.

a) Find the complex Poynting vector.

b) Find the instantaneous Poynting vector.

c) Find the time-average power flowing through the surface $S$ shown below (crossing the surface from the back side to the front side, where the front side is the side facing outward into the first octant where $x$, $y$, and $z$ are positive).



## Room for extra work

Answer

$\underline{S} = -\underline{\hat{y}}(0.001327)$ [VA/m$^2$]

In the time domain we have:

$$\underline{\mathcal{E}} = \underline{\hat{x}}\cos(\omega t + ky)$$
$$\underline{\mathcal{H}} = \underline{\hat{z}}(0.002654)\cos(\omega t + ky)$$

Hence we have

$\underline{\mathcal{S}} = -\underline{\hat{y}}(0.002654)\cos^2(\omega t + ky)$ [W/m$^2$].

The time-average power crossing the surface is

$$P_{ave} = \langle \mathcal{P}(t) \rangle = \mathrm{Re}\int_S \underline{S}\cdot\underline{\hat{n}}\, dS\,.$$

The unit normal is

$$\underline{\hat{n}} = \frac{\underline{\hat{x}} + \underline{\hat{y}}}{\sqrt{2}}\,.$$

Hence we have

$$P_{ave} = \mathrm{Re}\int_S -\underline{\hat{y}}(0.001327)\cdot\left(\frac{\underline{\hat{x}} + \underline{\hat{y}}}{\sqrt{2}}\right) dS\,.$$

This gives us

$$P_{ave} = \mathrm{Re}\int_S -(0.001327)\left(\frac{1}{\sqrt{2}}\right) dS$$

or

$$P_{ave} = -(0.001327)\left(\frac{1}{\sqrt{2}}\right) A\,,$$

where the area $A$ is

$$A = (1)\left(\sqrt{2}\right).$$

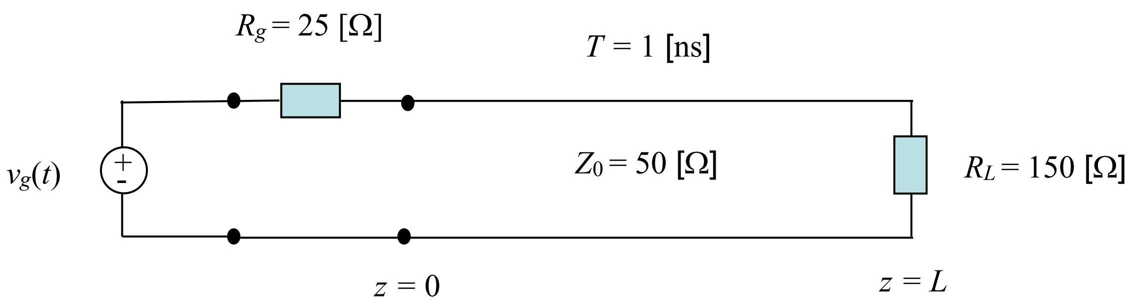
Hence, we have $P_{ave} = -(0.001327)$ [W].

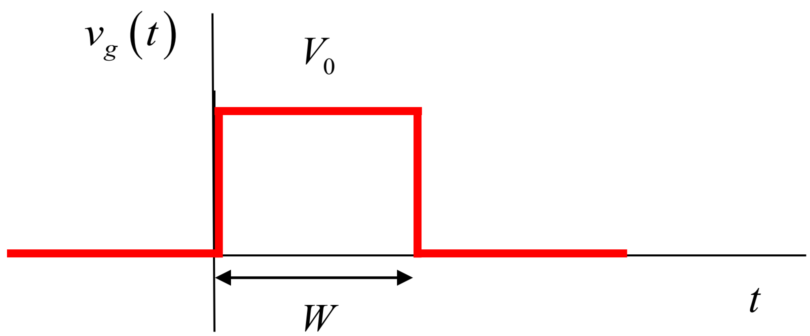
## Problem 2

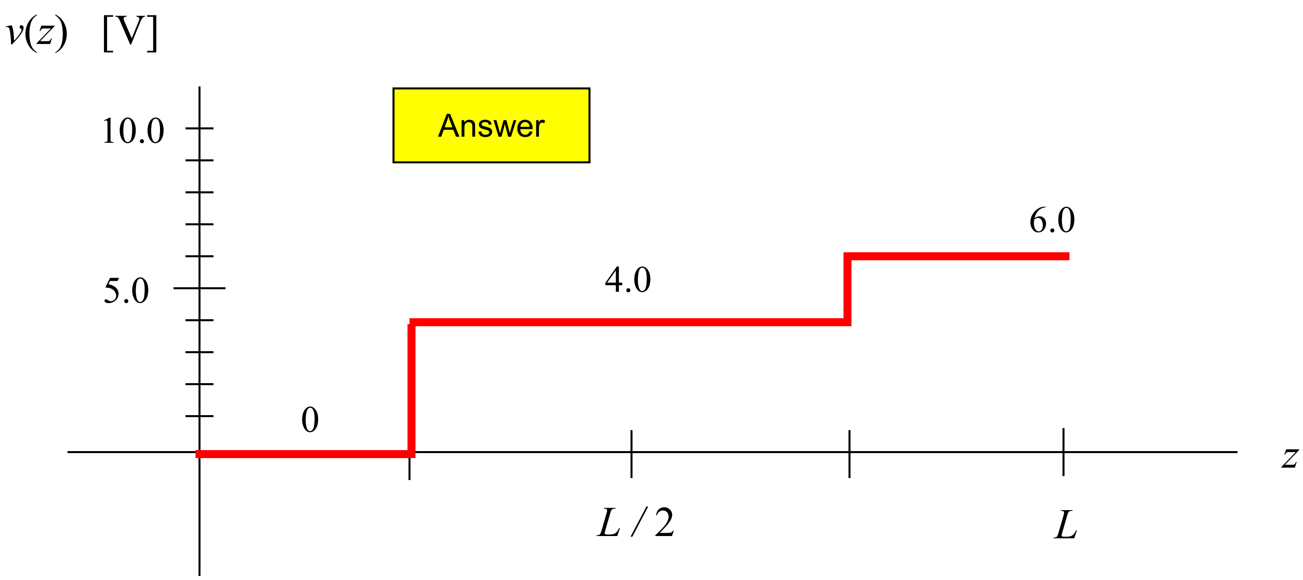
A digital pulse of amplitude $V_0 = 6.0$ [V] and duration $W = 1.0$ [ns] is applied at the input to the transmission line circuit shown below.

Construct a bounce diagram and from this make an accurate snapshot of the voltage on the line at the time $t = 1.25$ [ns]. Make your plot on the graph shown below.

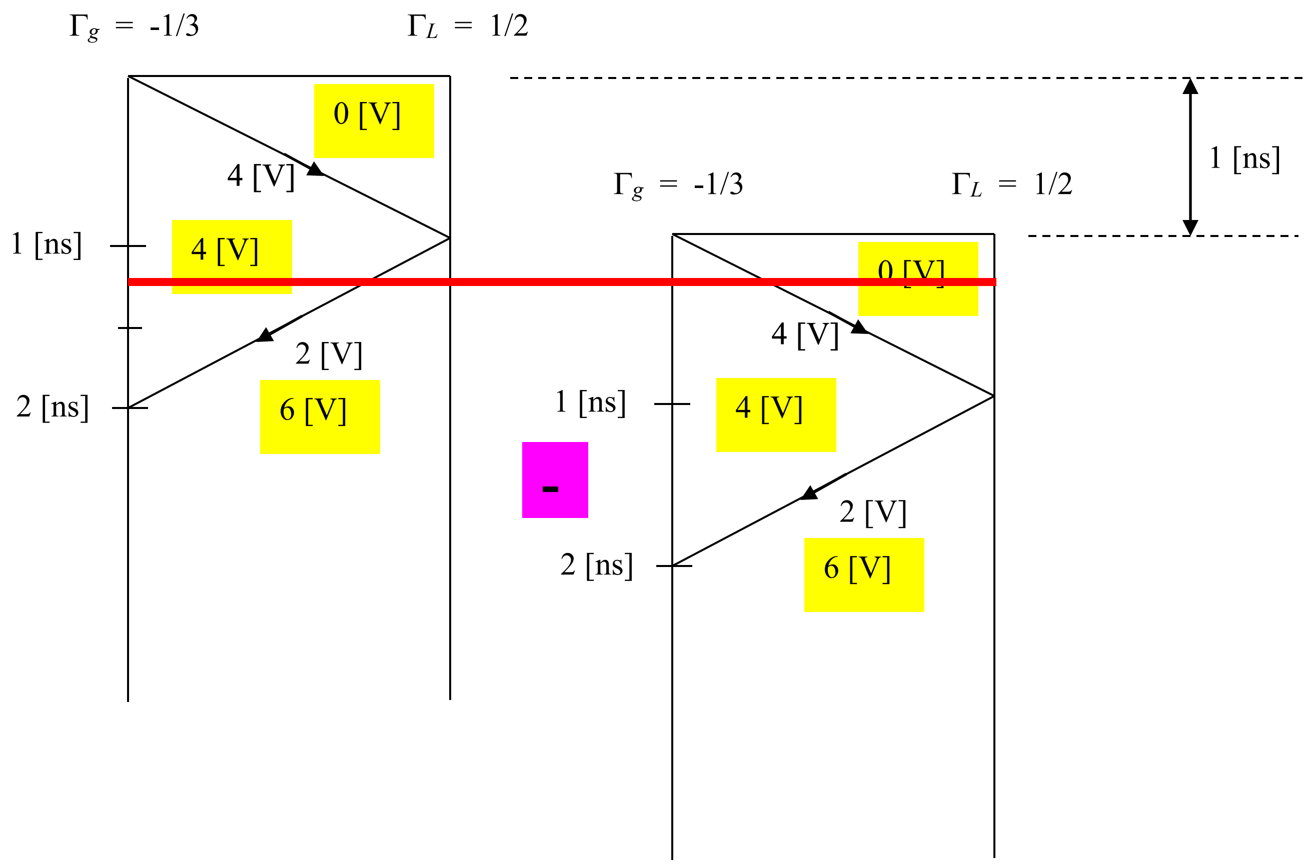
(The bounce diagram is on the next page.)



Room for extra work



## Problem 3

A transmission line has a load impedance of $Z_L$ = 50 + $j$ 50 [Ω]. The transmission line has a characteristic impedance of 50 [Ω]. Find the stub length $l_s$ (in terms of $\lambda$) of a short-circuited stub to be placed at the load, and the characteristic impedance $Z_{0T}$ of a quarter-wave transmission line connected to the load, which will provide a quarter-wave transformer matching system for the line. Assume that the characteristic impedance of the stub line is also 50 [Ω].

Do all calculations analytically – do not use the Smith chart.

Answer

The admittance of the load is

$$Y_L = \frac{1}{50(1+j)} = 0.02\left(\frac{1-j}{2}\right) = 0.01(1-j) = 0.01 - j(0.01) \text{ [S]}.$$

The stub admittance is thus

$$Y_s = j(0.01) \text{ [S]}.$$

Hence we have

$$-jY_0 \cot(\beta\ell) = j(0.01)$$

or

$$-Y_0 \cot(\beta\ell) = (0.01)$$

or

$$-(0.02)\cot(\beta\ell) = 0.01 .$$

The solution is

$$\beta\ell = -1.107 .$$

Since the length must be positive, we use

$$\beta\ell = \pi - 1.107 = 2.035 .$$

This gives us

$$\frac{\ell}{\lambda_0} = 0.324.$$

For the characteristic impedance of the quarter-wave transformer, we have

$$Z_{0T} = \sqrt{50(1/0.01)}.$$

Hence

$$Z_{0T} = 70.71[\Omega].$$

Room for extra work

## Problem 4

A transmission line has a load impedance of $Z_L$ = 25 + $j$ 10 [Ω]. The transmission line has a characteristic impedance of 50 [Ω]. Find the stub position $d$ and the stub length $l_s$ (in terms of $\lambda$) in order to provide a single short-circuit stub matching system for the line. Assume that the characteristic impedance of the stub line is also 50 [Ω]. Use the solution that will give the shortest possible distance $d$.

Use the Smith chart for all of the calculations. Show all of your work on the Smith chart that is attached here.

Please see the Smith chart that is attached.

Answer

From the Smith chart we have that the normalized input admittance just to the right of the stub is

$$Y_{in}^N = 1 - j(0.8).$$

Hence the normalized stub admittance is

$$Y_s^N = j(0.8).$$

From the Smith chart, the distance $d$ at which to place the stub is

$$d = (0.346 - 0.290)\lambda_0 .$$

Hence we have

$$d = 0.056\lambda_0 .$$

Also, from the Smith chart, the length of the stub is

$$\ell_s = 0.107\lambda_0 + 0.25\lambda_0 .$$

Hence we have

$$\ell_s = 0.357\lambda_0 .$$