

DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

Name: _____ **Solution** _____

PeopleSoft ID: _____

ECE 3317
Applied Electromagnetic Waves
March 22, 2012

1. This exam is open book and open notes. However, you are not allowed to use a computer or any electronic device other than a calculator. Any devices that may be used to communicate are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily legible**.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Make sure you sign the academic honesty statement on the next page.

Academic Honesty Statement

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including getting an F in the class and getting expelled from the University.

Signature

Problem 1 (30 pts)

Consider a cubical surface shown below. All of the six faces (of edge length a) of the cube are perfectly conducting except the top face, which has fields on it. The top face at $z = a$ is described by $0 < x < a$ and $0 < y < a$. On the top face (at $z = a$) there exists the following set of fields:

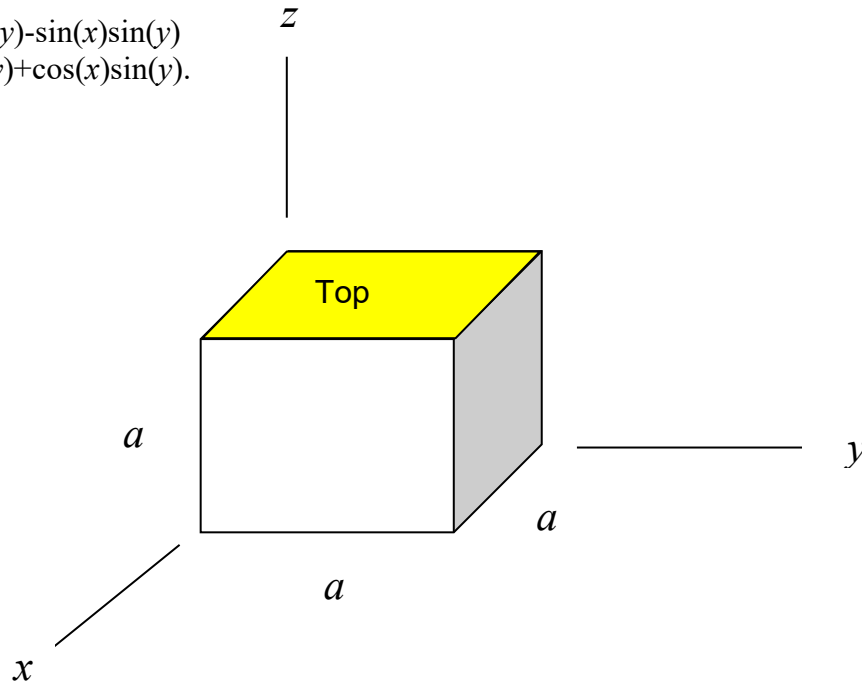
$$\underline{\mathcal{E}} = \underline{\hat{x}} \cos(\omega t) \sin\left(\frac{\pi x}{a}\right) \quad [\text{V/m}]$$

$$\underline{\mathcal{H}} = \underline{\hat{y}} \sin(\omega t + \pi/4) \sin\left(\frac{\pi x}{a}\right) \quad [\text{A/m}].$$

- Find the instantaneous power $\mathcal{P}(t)$ entering the cube through the top face.
- Find the complex power P entering the cube through the top face.
- Find the time-average power entering the cube through the top face by using the complex power P .
- Find the VARS entering the cube through the top face by using the complex power P .
- Find the time-average power entering the cube through the top face by directly taking the time average of your answer from part (a).

Helpful identities:

$$\begin{aligned} \cos(x+y) &= \cos(x)\cos(y) - \sin(x)\sin(y) \\ \sin(x+y) &= \sin(x)\cos(y) + \cos(x)\sin(y). \end{aligned}$$



Answers

(a)

$$\begin{aligned}
 \mathcal{P}(t) &= \int_{Top} (\underline{E} \times \underline{H}) \cdot (-\underline{\hat{z}}) dS \\
 &= \int_{Top} \left(\underline{\hat{z}} \cos(\omega t) \sin(\omega t + \pi/4) \sin^2\left(\frac{\pi x}{a}\right) \right) \cdot (-\underline{\hat{z}}) dS \\
 &= -\cos(\omega t) \sin(\omega t + \pi/4) \int_0^a \int_0^a \sin^2\left(\frac{\pi x}{a}\right) dx dy \\
 &= -\cos(\omega t) \sin(\omega t + \pi/4) (a) \int_0^a \sin^2\left(\frac{\pi x}{a}\right) dx \\
 &= -\cos(\omega t) \sin(\omega t + \pi/4) (a) \left(\frac{a}{2}\right) \\
 &= -\frac{a^2}{2} \cos(\omega t) \sin(\omega t + \pi/4) \quad [\text{W}]
 \end{aligned}$$

(b)

$$\begin{aligned}
 P &= \int_{Top} \frac{1}{2} (\underline{E} \times \underline{H}^*) \cdot (-\underline{\hat{z}}) dS \\
 &= \int_{Top} \frac{1}{2} \left(\underline{\hat{z}} e^{j\pi/4} \sin^2\left(\frac{\pi x}{a}\right) \right) \cdot (-\underline{\hat{z}}) dS \\
 &= -\frac{1}{2} e^{j\pi/4} \int_{Top} \left(\sin^2\left(\frac{\pi x}{a}\right) \right) dS \\
 &= -\frac{1}{2} e^{j\pi/4} \int_0^a \int_0^a \sin^2\left(\frac{\pi x}{a}\right) dx dy \\
 &= -\frac{1}{2} e^{j\pi/4} (a) \int_0^a \sin^2\left(\frac{\pi x}{a}\right) dx \\
 &= -\frac{1}{2} e^{j\pi/4} (a) \left(\frac{a}{2}\right) \\
 &= -\frac{a^2}{4} e^{j\pi/4} \quad [\text{VA}]
 \end{aligned}$$

$$(c) \quad \langle \mathcal{P}(t) \rangle = \operatorname{Re} P = -\frac{a^2}{4\sqrt{2}} \quad [\text{W}]$$

$$(d) \quad \text{VAR S} = \operatorname{Im} P = -\frac{a^2}{4\sqrt{2}} \quad [\text{VA}]$$

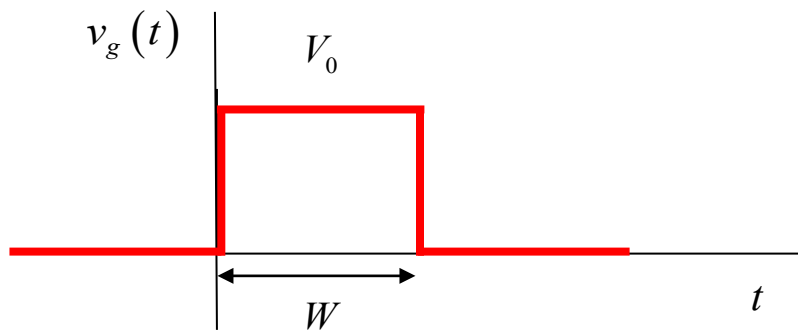
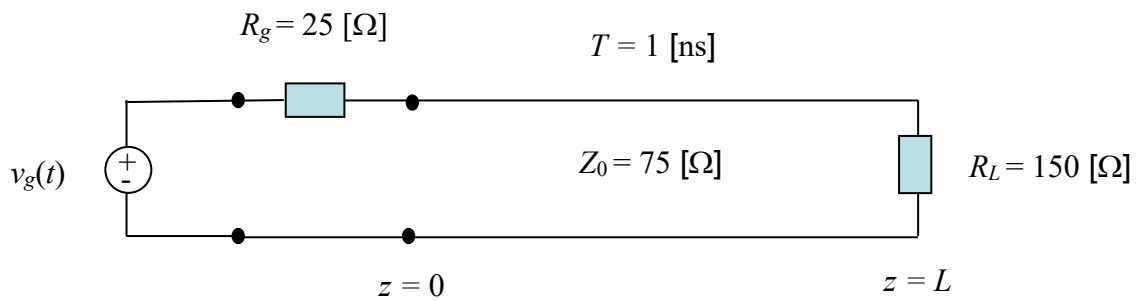
(e)

$$\begin{aligned} \langle \mathcal{P}(t) \rangle &= \left\langle -\frac{a^2}{2} \cos(\omega t) \sin(\omega t + \pi/4) \right\rangle \\ &= -\frac{a^2}{2} \langle \cos(\omega t) \cos(\omega t - \pi/4) \rangle \\ &\quad - \frac{a^2}{2} \langle \cos^2(\omega t) \cos(\pi/4) + \cos(\omega t) \sin(\omega t) \sin(\pi/4) \rangle \\ &= -\frac{a^2}{2} \langle \cos^2(\omega t) \cos(\pi/4) \rangle \\ &= -\frac{a^2}{4} \cos(\pi/4) \\ &= -\frac{a^2}{4\sqrt{2}} \quad [\text{W}] \end{aligned}$$

Problem 2 (40 pts)

A digital pulse of amplitude $V_0 = 4.0$ [V] and duration $W = 0.5$ [ns] is applied at the input to the transmission line circuit shown below.

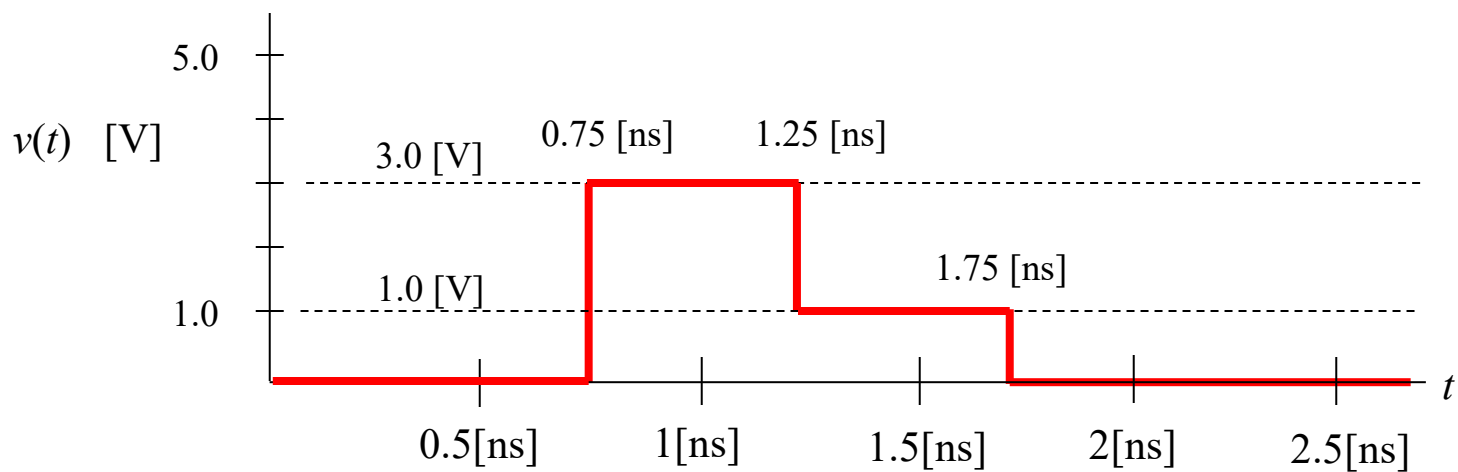
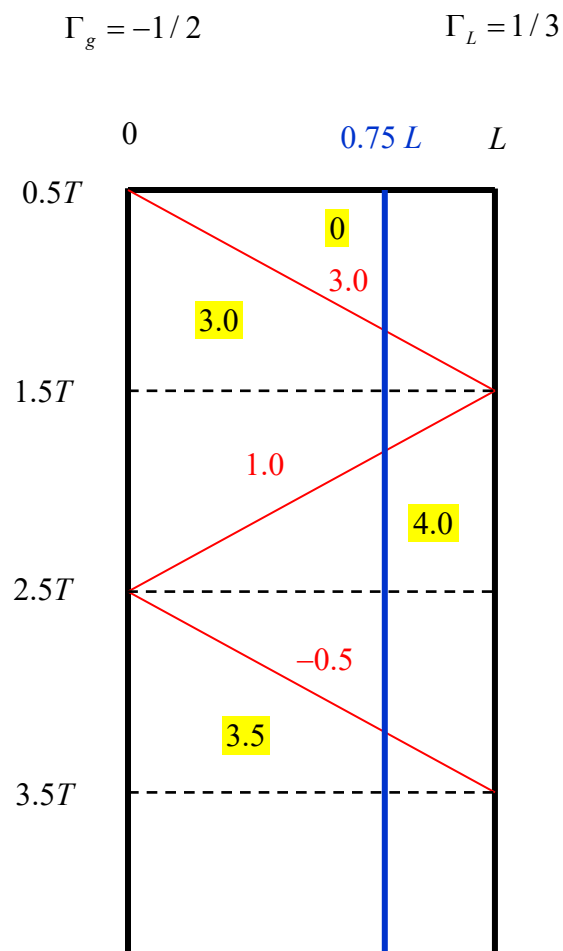
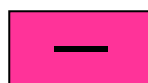
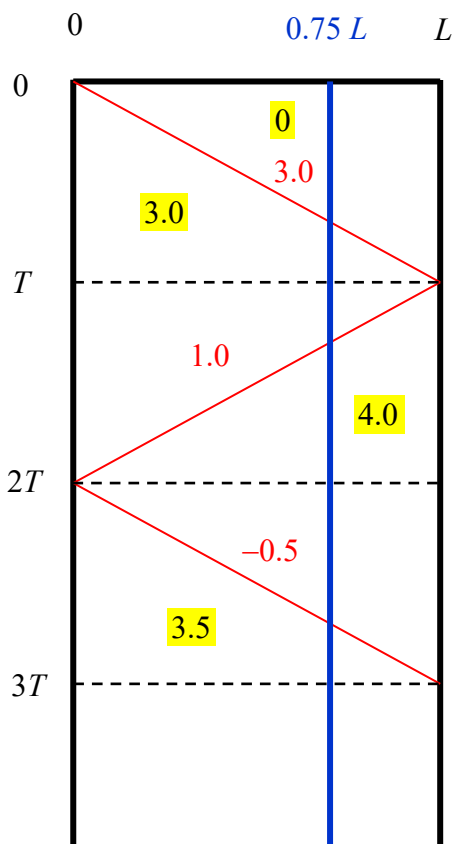
- Construct a bounce diagram that extends to a time of $3T$. (Make your bounce diagram on the next page.)
- Make an accurate oscilloscope trace of the voltage on the line at $z = 0.75 L$. Make your oscilloscope trace on the graph shown below, plotting out to 2.5 [ns]. Label all voltage values on your plot as well as all time values at which the voltage on your plot changes.



Answers

$$\Gamma_g = -1/2$$

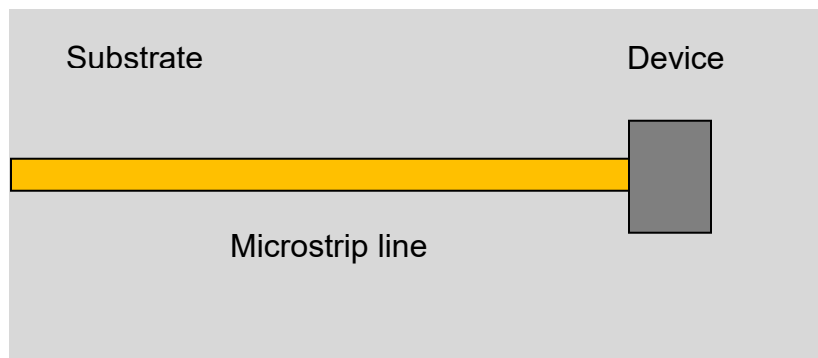
$$\Gamma_L = 1/3$$



Problem 3 (30 pts.)

A transmission line is connected to a certain device on a printed circuit board, operating at 10.0 GHz. (A top view is shown below.) The microstrip line has a characteristic impedance of $50\ \Omega$. The effective permittivity of the microstrip line is 3.0. The voltage on the line has a maximum magnitude of 1.6 volts and a minimum magnitude of 0.4 volts. A voltage minimum occurs at a distance of 1.0 [cm] from the device.

Determine the input impedance of the device. Do the calculation exactly – do not use the Smith chart.



Answers

$$SWR = \frac{1.6}{0.4} = 4.0$$

$$|\Gamma_L| = \frac{SWR - 1}{SWR + 1} = 0.6$$

$$\phi + 2\beta z_{\min} = \pi + 2\pi n \quad (\text{choose } n = 0)$$

$$\beta = k_0 \sqrt{\epsilon_r^{\text{eff}}} = k_0 \sqrt{3.0}$$

$$\beta z_{\min} = \left(\frac{2\pi}{\lambda_0} \sqrt{3.0} \right) z_{\min} = 2\pi \sqrt{3.0} \left(\frac{z_{\min}}{\lambda_0} \right)$$

where

$$\lambda_0 = \frac{c}{f} = 2.99792458 \text{ [cm]}.$$

Hence we have

$$\beta z_{\min} = 2\pi \sqrt{3.0} \left(\frac{-1.0}{2.99792458} \right) = -3.6301 \text{ [rad]}.$$

Hence

$$\phi = 10.402 \text{ [rad]} \quad (\text{or } 4.119 \text{ [rad]}, -2.164 \text{ [rad]}, \text{ etc.})$$

Therefore

$$\Gamma_L = |\Gamma_L| e^{j\phi} = 0.6 e^{j(10.402)}.$$

We then have

$$Z_L = Z_0 \left(\frac{1 + \Gamma_L}{1 - \Gamma_L} \right) = 50 \left(\frac{1 + 0.6 e^{j(10.402)}}{1 - 0.6 e^{j(10.402)}} \right)$$

$$Z_L = 29.111 e^{j(-0.999)} [\Omega] = 29.111 \angle -57.24^\circ [\Omega] = 15.75 + j(-24.48) [\Omega]$$