

DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

Name: _____

PeopleSoft ID: _____

ECE 3317
Applied Electromagnetic Waves
March 19, 2013

1. This exam is open book and open notes. However, you are not allowed to use a computer or any electronic device other than a calculator. Any devices that may be used to communicate are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily legible**.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Make sure you sign the academic honesty statement on the next page.

Academic Honesty Statement

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including getting an F in the class and getting expelled from the University.

Signature

FORMULA SHEET

$$\nabla \times \underline{\mathcal{E}} = -\frac{\partial \underline{\mathcal{B}}}{\partial t}$$

$$\nabla \times \underline{\mathcal{H}} = \underline{\mathcal{J}} + \frac{\partial \underline{\mathcal{Q}}}{\partial t}$$

$$\nabla \cdot \underline{\mathcal{B}} = 0$$

$$\nabla \cdot \underline{\mathcal{Q}} = \rho_v$$

$$\nabla \times \underline{E} = -j\omega \underline{B}$$

$$\nabla \times \underline{H} = \underline{J} + j\omega \underline{D}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \cdot \underline{D} = \rho_v$$

$$\nabla \times \underline{V} = \hat{x} \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} \right) + \hat{y} \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \right) + \hat{z} \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right)$$

$$\nabla \times \underline{V} = \hat{\rho} \left(\frac{1}{\rho} \frac{\partial V_z}{\partial \phi} - \frac{\partial V_\phi}{\partial z} \right) + \hat{\phi} \left(\frac{\partial V_\rho}{\partial z} - \frac{\partial V_z}{\partial \rho} \right) + \hat{z} \frac{1}{\rho} \left(\frac{\partial (\rho V_\phi)}{\partial \rho} - \frac{\partial V_\rho}{\partial \phi} \right)$$

$$\nabla \times \underline{V} = \hat{r} \frac{1}{r \sin \theta} \left[\frac{\partial (V_\phi \sin \theta)}{\partial \theta} - \frac{\partial V_\theta}{\partial \phi} \right] + \hat{\theta} \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial V_r}{\partial \phi} - \frac{\partial (r V_\phi)}{\partial r} \right] + \hat{\phi} \frac{1}{r} \left[\frac{\partial (r V_\theta)}{\partial r} - \frac{\partial V_r}{\partial \theta} \right]$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\epsilon_0 \doteq 8.8541878 \times 10^{-12} \text{ [F/m]}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ [H/m]}$$

$$\oint_S (\underline{\mathcal{E}} \times \underline{\mathcal{H}}) \cdot \hat{n} \, dS = - \int_V \sigma |\underline{\mathcal{E}}|^2 \, dV - \int_V \frac{\partial}{\partial t} \left(\frac{1}{2} \mu |\underline{\mathcal{H}}|^2 \right) dV - \int_V \frac{\partial}{\partial t} \left(\frac{1}{2} \varepsilon |\underline{\mathcal{E}}|^2 \right) dV$$

$$\underline{\mathcal{S}} = \underline{\mathcal{E}} \times \underline{\mathcal{H}}$$

$$\underline{S} \equiv \frac{1}{2} \big(\underline{E} \times \underline{H}^* \big)$$

$$C=\frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)}\quad\left[\text{F/m}\right]$$

$$L=\frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)\quad\left[\text{H/m}\right]$$

$$G=\frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)}\quad\left[\text{S/m}\right]$$

$$R=\frac{1}{\sigma_m\delta}\left(\frac{1}{2\pi a}+\frac{1}{2\pi b}\right)\quad\left[\Omega/\text{m}\right]$$

$$\delta=\sqrt{\frac{2}{\omega\mu\sigma_m}}$$

$$\frac{\partial v}{\partial z}=-Ri-L\frac{\partial i}{\partial t}$$

$$\frac{\partial i}{\partial z}=-Gv-C\frac{\partial v}{\partial t}$$

$$v(z,t)=f(z-c_d t)+g(z+c_d t)$$

$$i(z,t)=\frac{1}{Z_0}\big[f(z-c_d t)-g(z+c_d t)\big]$$

$$v(z,t) \, = \, v_g(t-z/c_d)$$

$$\Gamma_g=\left(\frac{R_g-Z_0}{R_g+Z_0}\right) \qquad \Gamma_L=\left(\frac{R_L-Z_0}{R_L+Z_0}\right)$$

$$V^+=\left(\frac{Z_0}{R_g+Z_0}\right)V_0$$

$$\Gamma_L(t)=1-2e^{-(t-T)/\tau},\,\,t\geq T\qquad \tau=Z_0C_L$$

$$\Gamma_L(t)=-1+2e^{-(t-T)/\tau},\,\,t\geq T\qquad \tau=L_L/Z_0$$

$$V(z)=Ae^{-\gamma z}+Be^{+\gamma z}$$

$$\gamma=\sqrt{(R+j\omega L)(G+j\omega C)}$$

$$LC=\mu\varepsilon=\frac{1}{c_d^2}$$

$$\gamma=\alpha+j\beta$$

$$k_z=-j\gamma=\beta-j\alpha$$

$$v_p=\frac{\omega}{\beta}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\text{attenuation} = \left(\frac{20}{\ln 10} \right) \alpha = (8.6859) \alpha \quad [\text{dB/m}]$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$V(z) = A e^{-\gamma z} + B e^{+\gamma z}$$

$$I(z) = \left(\frac{1}{Z_0} \right) \left[A e^{-\gamma z} - B e^{+\gamma z} \right]$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$V(z) = A \left(e^{-\gamma z} + \Gamma_L e^{+\gamma z} \right)$$

$$I(z) = \frac{1}{Z_0} A \left(e^{-\gamma z} - \Gamma_L e^{+\gamma z} \right)$$

$$Z_{in}(z) = \frac{V(z)}{I(z)}$$

$$Z_{in}(z) = Z_0 \left(\frac{1 + \Gamma_L e^{+2\gamma z}}{1 - \Gamma_L e^{+2\gamma z}} \right)$$

$$Z_{in} = Z_0 \left(\frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} \right)$$

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right)$$

$$Z_{in}(z) = Z_0 \left(\frac{1 + \Gamma(z)}{1 - \Gamma(z)} \right)$$

$$Z_{in} = jZ_0 \tan(\beta l)$$

$$Z_{in} = -jZ_0 \cot(\beta l)$$

$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

$$|V(z)| = |A| \left| 1 + |\Gamma_L| e^{+j(\phi+2\beta z)} \right|$$

$$\left| \frac{V(z)}{V^+} \right| = \left| 1 + |\Gamma_L| e^{+j(\phi+2\beta z)} \right| = \left| 1 + \Gamma(z) \right|$$

$$\Gamma(z) = \Gamma_L e^{j2\beta z} = \Gamma_L e^{-j2\beta l}$$

$$\text{VSWR} \equiv \frac{V_{\max}}{V_{\min}}$$

$$\text{VSWR} \equiv \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$$\text{SWR} = \max \left(\frac{R_L}{Z_0}, \frac{Z_0}{R_L} \right)$$

$$Z_{in}^N(z) = \frac{1 + \Gamma_L e^{+2j\beta z}}{1 - \Gamma_L e^{+2j\beta z}}$$

$$Z_{in} = \frac{Z_{0T}^2}{Z_L}$$

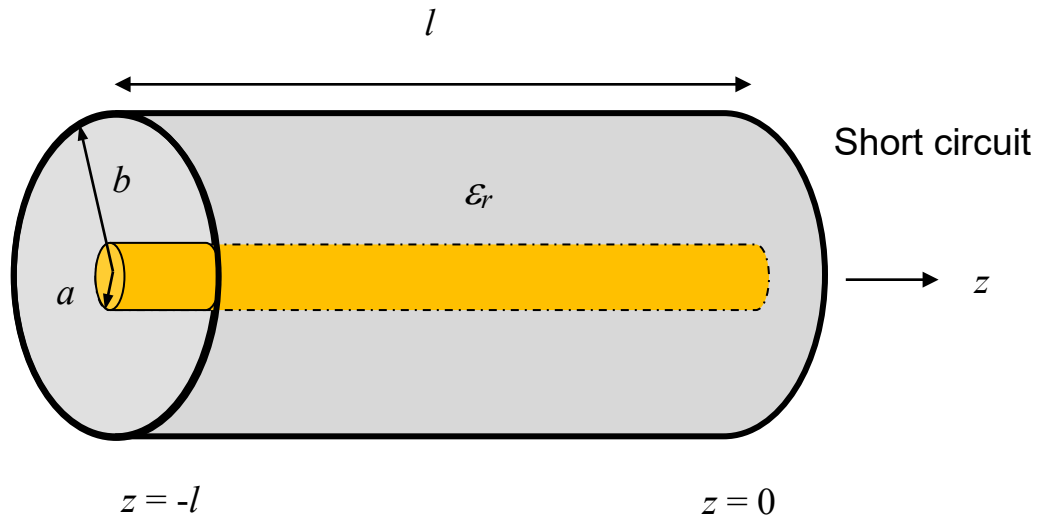
$$Z_{0T} = \sqrt{Z_0 R_L}$$

Problem 1 (30 pts)

A lossless coaxial cable transmission line of length l is short circuited at the end ($z = 0$) with a short-circuit conducting plate. The electric field inside the coax ($a < \rho < b$) is given in the phasor domain by

$$\underline{E} = \hat{\rho} \left(\frac{1}{\rho} \right) \left(\frac{-V_0}{\sin(kl) \ln(b/a)} \right) \sin(kz) \quad [\text{V/m}], \quad -l < z < 0.$$

- Find the magnetic field inside the coax (in the phasor domain).
- Find the complex power entering into the coax.
- Find the electric and magnetic fields inside the coax in the time domain.



Room for work

Part (a)

From Ampere's law, we have

$$\nabla \times \underline{E} = -j\omega \underline{B}.$$

Hence, we have

$$\underline{H} = -\frac{1}{j\omega\mu_0} \nabla \times \underline{E}.$$

Taking the curl in cylindrical coordinates, we have

$$\underline{H} = -\frac{1}{j\omega\mu_0} \left(\hat{\rho} \left(\frac{1}{\rho} \frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} \right) + \hat{\phi} \left(\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho} \right) + \hat{z} \frac{1}{\rho} \left(\frac{\partial(\rho E_\phi)}{\partial \rho} - \frac{\partial E_\rho}{\partial \phi} \right) \right)$$

For the coax we then have

$$\underline{H} = -\frac{1}{j\omega\mu_0} \hat{\phi} \left(\frac{\partial E_\rho}{\partial z} \right).$$

Hence,

$$\underline{H} = -\frac{1}{j\omega\mu_0} \hat{\phi} \left(\frac{1}{\rho} \right) \left(\frac{-V_0 k}{\sin(kl) \ln(b/a)} \right) \cos(kz) \quad [\text{A/m}].$$

Part (b)

The complex power entering into the coax is

$$P_c = \frac{1}{2} \int_0^{2\pi} \int_a^b E_\rho H_\phi^* \rho d\rho d\phi = \pi \int_a^b E_\rho H_\phi^* \rho d\rho.$$

This gives us

$$P_c = \pi \int_a^b \left(\frac{1}{\rho} \right) \left(\frac{-V_0}{\sin(kl) \ln(b/a)} \right) \sin(k(-l)) \left(-\frac{1}{j\omega\mu_0} \left(\frac{1}{\rho} \right) \left(\frac{-V_0 k}{\sin(kl) \ln(b/a)} \right) \cos(k(-l)) \right)^* d\rho.$$

or

$$P_c = +j\pi \left(\frac{-V_0}{\sin(kl) \ln(b/a)} \right)^2 \left(\frac{k}{\omega\mu_0} \right) \sin(kl) \cos(kl) \int_a^b \left(\frac{1}{\rho} \right) d\rho.$$

Hence, we have

$$P_c = j\pi \left(\frac{V_0}{\sin(kl) \ln(b/a)} \right)^2 \left(\frac{1}{\eta_0} \right) \sin(kl) \cos(kl) \ln\left(\frac{b}{a}\right) \quad [\text{VARs}].$$

Note: There is purely reactive power entering the coax, since it is a lossless system that is terminated by a lossless (short-circuit) plate.

Part (c)

In the time domain we have

$$\underline{\mathcal{E}} = \hat{\rho} \left(\frac{1}{\rho} \right) \left(\frac{-V_0}{\sin(kl) \ln(b/a)} \right) \sin(kz) \cos(\omega t) \quad [\text{V/m}].$$

The magnetic field is

$$\underline{\mathcal{H}} = -j \frac{1}{\omega\mu_0} \hat{\phi} \left(\frac{1}{\rho} \right) \left(\frac{V_0 k}{\sin(kl) \ln(b/a)} \right) \cos(kz) \quad [\text{A/m}].$$

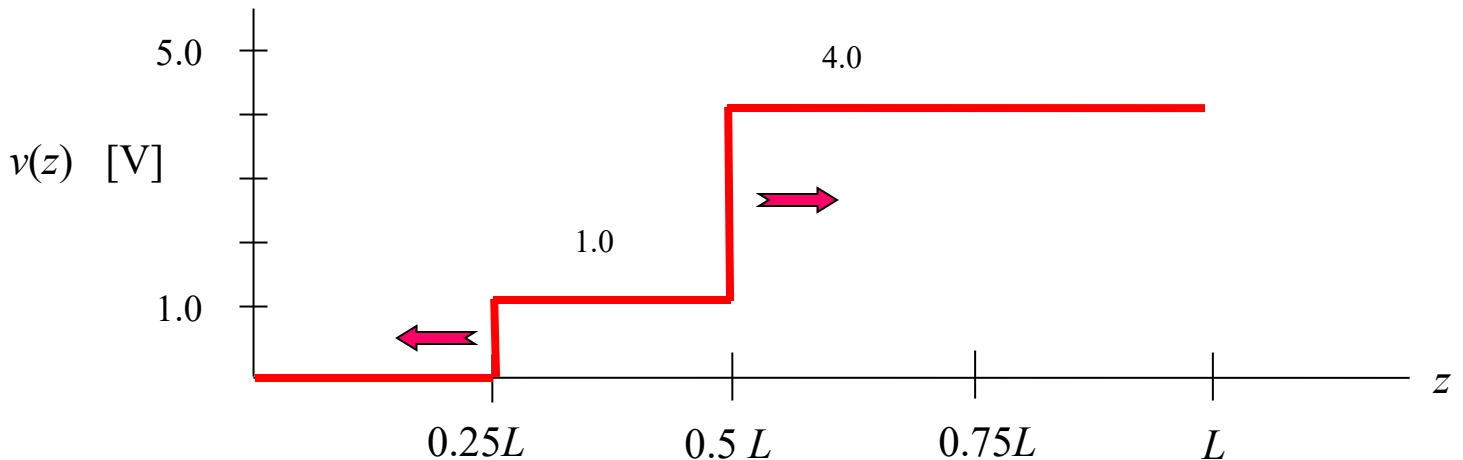
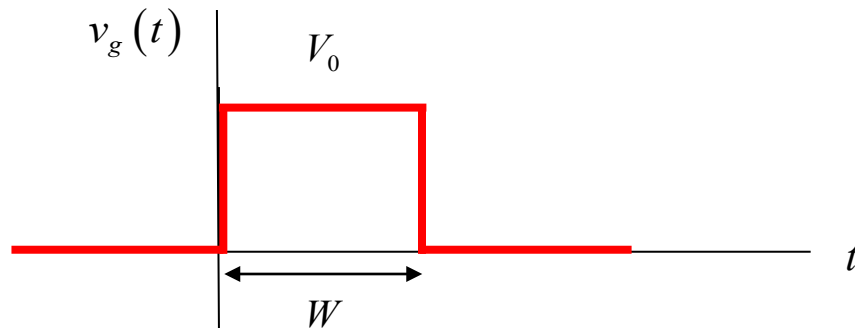
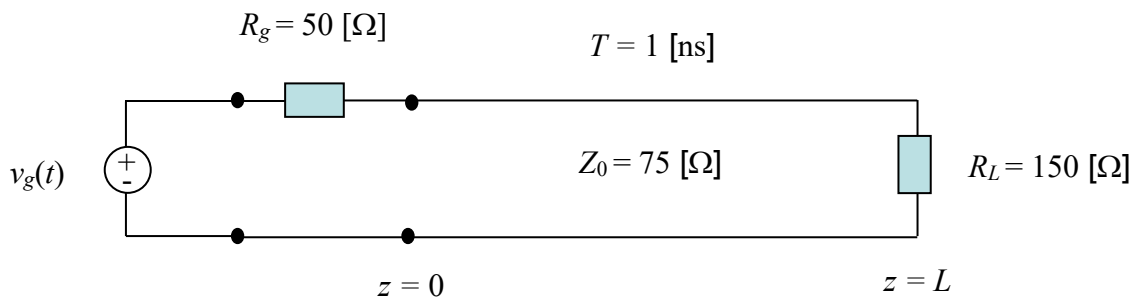
In the time domain this is

$$\underline{\mathcal{H}} = \frac{1}{\omega\mu_0} \hat{\phi} \left(\frac{1}{\rho} \right) \left(\frac{V_0 k}{\sin(kl) \ln(b/a)} \right) \cos(kz) \sin(\omega t) \quad [\text{A/m}].$$

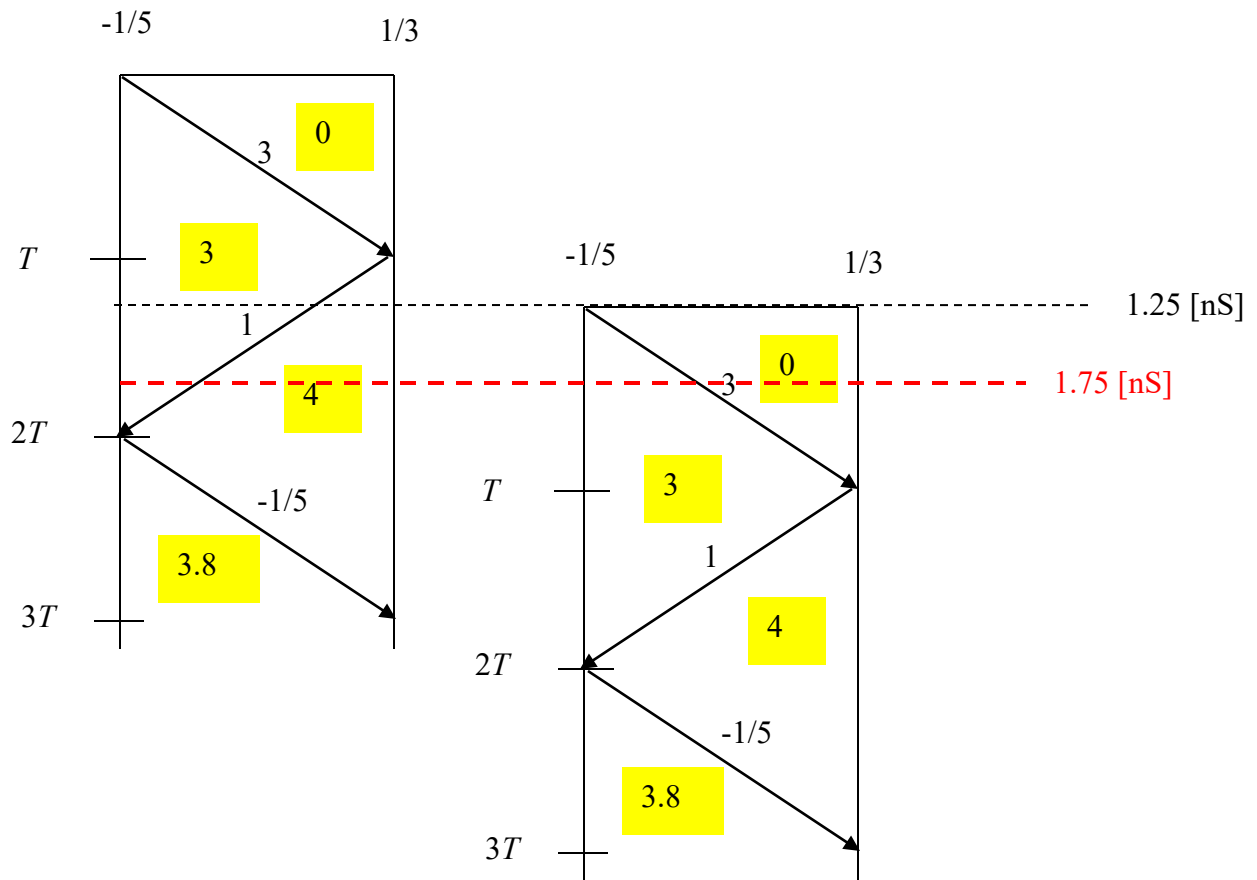
Problem 2 (30 pts)

A digital pulse of amplitude $V_0 = 5.0$ [V] and duration $W = 1.25$ [ns] is applied at the input to the transmission line circuit shown below.

- Construct a bounce diagram for this problem that extends to a time of $3T$. (Make your bounce diagram on the next page.)
- Make an accurate “snapshot” plot of the voltage on the line at $t = 1.75$ [ns]. Make your plot on the graph shown below. Label all voltage values on your plot. Also indicate with arrows in which direction all “wavefronts” (points of voltage discontinuity) are moving (to the left or to the right).



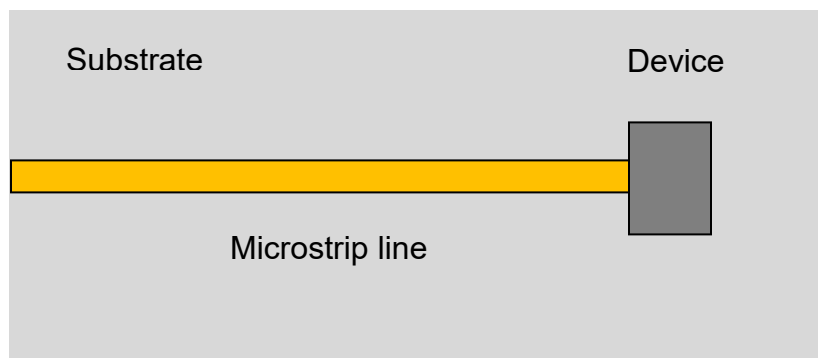
Room for work



Problem 3 (40 pts.)

A transmission line is connected to a certain device on a printed circuit board, operating at 10.0 GHz (a top view is shown below). The device is located at $z = 0$. The microstrip line has a characteristic impedance of $50\ [\Omega]$. The effective permittivity of the microstrip line is 1.5.

- By probing the line, it is found that the voltage on the line has a maximum magnitude of 1.5 volts and a minimum magnitude of 0.5 volts. A voltage minimum occurs at a distance of 0.75 [cm] from the device. Determine the input impedance of the device. Do the calculation exactly (do not use the Smith chart).
- Assume now that a new device is connected to the end of the line, which has an input impedance of $Z_{in} = 100 + j100\ [\Omega]$. An open-circuited stub line having characteristic impedance of $50\ [\Omega]$ and an effective permittivity of 1.5 is added at a distance d from the load. Find the distance d and the length of the open-circuited stub line (in cm) to obtain a perfect match seen by an incoming wave that arrives from a generator on the left (not shown). Use the shortest distance d possible. Use the Smith chart to do all calculations. (A Smith chart is included at the end of this problem.)
- As a continuation of part (b), what is the SWR on the main line between the device and the open-circuited stub? What is the SWR on the open-circuited stub line? What is the SWR on the main line to the left of the open-circuited stub? Do the calculations exactly (do not use the Smith chart).



Room for work

At 10.0 GHz we have

$$\lambda_0 = 2.9979 \text{ [cm]}.$$

$$\lambda_{eff} = \frac{\lambda_0}{\sqrt{\epsilon_r^{eff}}} = 2.4478 \text{ [cm]}$$

Part (a)

We have

$$Z_{in} = Z_0 \left(\frac{1 + \Gamma_L}{1 - \Gamma_L} \right)$$

with

$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

and

$$|\Gamma_L| = \frac{SWR - 1}{SWR + 1}$$

$$\phi + 2\beta z_{min} = \pi + n\pi .$$

We thus have

$$|\Gamma_L| = \frac{3-1}{3+1} = \frac{1}{2} .$$

Also, we have

$$\phi + 2 \left(2\pi \frac{z_{min}}{\lambda_{eff}} \right) = \pi + n\pi$$

or

$$\phi + 2 \left(2\pi \frac{z_{min}}{\lambda_{eff}} \right) = \pi + n\pi .$$

This gives us

$$\phi = 0.70875 \text{ [rad]}.$$

Hence

$$\Gamma_L = 0.5 e^{j(0.70875)}.$$

This gives us

$$Z_{in} = 50 \left(\frac{1 + 0.5 e^{j(0.70875)}}{1 - 0.5 e^{j(0.70875)}} \right).$$

The final result is

$$Z_{in} = 76.402 + j(66.305) \text{ } [\Omega].$$

Part (b)

The Smith chart is omitted here. The solution is

$$d = 0.215 \lambda_{eff} = 0.526 \text{ [cm]}$$

$$l = 0.34 \lambda_{eff} = 0.278 \lambda_0 = 0.832 \text{ [cm]}$$

Part (c)

On the main line between the device and the stub, we have

$$\Gamma_L = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = 0.538 + j(0.308)$$

Using

$$\text{SWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|},$$

we have

$$\text{SWR} = 4.26.$$

On the open-circuited stub we have

$$\text{SWR} = \infty.$$

On the main line to the left of the stub we have a perfect match, so we have

$$\text{SWR} = 1.0.$$

