

DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

Name: _____

PeopleSoft ID: _____

ECE 3317
Applied Electromagnetic Waves

Exam II
April 25, 2013

1. This exam is open book and open notes. However, you are not allowed to use a computer or any electronic device other than a calculator. Any devices that may be used to communicate are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily legible**.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Make sure you sign the academic honesty statement on the next page.

Academic Honesty Statement

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including getting an F in the class and getting expelled from the University.

Signature

FORMULA SHEET

$$\nabla \times \underline{\mathcal{E}} = -\frac{\partial \underline{\mathcal{B}}}{\partial t}$$

$$\nabla \times \underline{\mathcal{H}} = \underline{\mathcal{J}} + \frac{\partial \underline{\mathcal{Q}}}{\partial t}$$

$$\nabla \cdot \underline{\mathcal{B}} = 0$$

$$\nabla \cdot \underline{\mathcal{Q}} = \rho_v$$

$$\nabla \times \underline{E} = -j\omega \underline{B}$$

$$\nabla \times \underline{H} = \underline{J} + j\omega \underline{D}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \cdot \underline{D} = \rho_v$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\varepsilon_0 \doteq 8.8541878 \times 10^{-12} \text{ [F/m]}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ [H/m]}$$

$$\oint_s (\underline{\mathcal{E}} \times \underline{\mathcal{H}}) \cdot \hat{n} dS = - \int_V \sigma |\underline{\mathcal{E}}|^2 dV - \int_V \frac{\partial}{\partial t} \left(\frac{1}{2} \mu |\underline{\mathcal{H}}|^2 \right) dV - \int_V \frac{\partial}{\partial t} \left(\frac{1}{2} \varepsilon |\underline{\mathcal{E}}|^2 \right) dV$$

$$\underline{\mathcal{J}} = \underline{\mathcal{E}} \times \underline{\mathcal{H}}$$

$$\underline{S} \equiv \frac{1}{2} (\underline{E} \times \underline{H}^*)$$

$$C = \frac{2\pi\epsilon_0\epsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\text{F/m}]$$

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \quad [\text{H/m}]$$

$$G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)} \quad [\text{S/m}]$$

$$R = \frac{1}{\sigma_m\delta} \left(\frac{1}{2\pi a} + \frac{1}{2\pi b} \right) \quad [\Omega/\text{m}]$$

$$\frac{\partial v}{\partial z} = -Ri - L \frac{\partial i}{\partial t}$$

$$\frac{\partial I}{\partial z} i = -Gv - C \frac{\partial v}{\partial t}$$

$$v(z,t) = f(z - c_d t) + g(z + c_d t)$$

$$i(z,t) = \frac{1}{Z_0} [f(z - c_d t) - g(z + c_d t)]$$

$$v(z,t) = v_g(t - z / c_d)$$

$$\Gamma_g = \left(\frac{R_g - Z_0}{R_g + Z_0} \right) \qquad \Gamma_L = \left(\frac{R_L - Z_0}{R_L + Z_0} \right)$$

$$V^+ = \left(\frac{Z_0}{R_g + Z_0} \right) V_0$$

$$\Gamma_L(t)=1-2e^{-(t-T)/\tau}, \quad t \geq T \qquad \tau = Z_0 C_L$$

$$\Gamma_L(t)=-1+2e^{-(t-T)/\tau}, \quad t \geq T \qquad \tau = L_L / Z_0$$

$$V(z)=Ae^{-\gamma z}+Be^{+\gamma z}$$

$$\gamma=\sqrt{(R+j\omega L)(G+j\omega C)}$$

$$LC=\mu\varepsilon=\frac{1}{c_d^2}$$

$$\gamma=\alpha+j\beta$$

$$k_z=-j\gamma=\beta-j\alpha$$

$$v_p=\frac{\omega}{\beta}$$

$$\beta=\frac{2\pi}{\lambda}$$

$$\text{attenuation}=\left(\frac{20}{\ln 10}\right)\alpha=(8.6859)\alpha \quad [\text{dB/m}]$$

$$Z_0=\sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$i(z) = \left(\frac{1}{Z_0} \right) \left[A e^{-\gamma z} - B e^{+\gamma z} \right]$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$V(z) = A \left(e^{-\gamma z} + \Gamma_L e^{+\gamma z} \right)$$

$$I(z) = \frac{1}{Z_0} A \left(e^{-\gamma z} - \Gamma_L e^{+\gamma z} \right)$$

$$Z_{in}(z) = \frac{V(z)}{I(z)}$$

$$Z_{in}(z) = Z_0 \left(\frac{1 + \Gamma_L e^{+2\gamma z}}{1 - \Gamma_L e^{+2\gamma z}} \right)$$

$$Z_{in} = Z_0 \left(\frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} \right)$$

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right)$$

$$Z_{in}(z) = Z_0 \left(\frac{1 + \Gamma(z)}{1 - \Gamma(z)} \right)$$

$$Z_{in} = jZ_0 \tan(\beta l)$$

$$Z_{in} = -jZ_0 \cot(\beta l)$$

$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

$$|V(z)| = |A| \left| 1 + |\Gamma_L| e^{+j(\phi+2\beta z)} \right|$$

$$\left| \frac{V(z)}{V^+} \right| = \left| 1 + |\Gamma_L| e^{+j(\phi+2\beta z)} \right| = \left| 1 + \Gamma(z) \right|$$

$$\Gamma(z) = \Gamma_L e^{j2\beta z} = \Gamma_L e^{-j2\beta l}$$

$$\text{VSWR} \equiv \frac{V_{\max}}{V_{\min}}$$

$$\text{VSWR} \equiv \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$$\text{SWR} = \max\left(\frac{R_L}{Z_0}, \frac{Z_0}{R_L}\right)$$

$$Z_{in}^N(z) = \frac{1 + \Gamma_L e^{+2j\beta z}}{1 - \Gamma_L e^{+2j\beta z}}$$

$$Z_{in} = \frac{Z_{0T}^2}{Z_L}$$

$$Z_{0T} = \sqrt{Z_0 R_L}$$

$$k=\omega\sqrt{\mu\varepsilon}$$

$$\eta=\sqrt{\frac{\mu}{\varepsilon}}$$

$$\eta_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}\doteq 376.730313\;[\Omega]$$

$$\underline{S}=\underline{\hat{z}}\frac{\left|E_0\right|^2}{2\eta}$$

$$v_p=\frac{\omega}{k}$$

$$\lambda=\frac{2\pi}{k}$$

$$\lambda=\frac{c_d}{f}$$

$$E_x=E_0\,e^{-jkz}$$

$$H_y=\frac{1}{\eta}E_0\,e^{-jkz}$$

$$\varepsilon_c=\varepsilon-j\left(\frac{\sigma}{\omega}\right)$$

$$k=k'-jk''$$

$$\lambda = \frac{2\pi}{k'}$$

$$d_p = 1 / k''$$

$$\tan \delta = \frac{\varepsilon_c''}{\varepsilon_c'} = \frac{\sigma}{\omega \varepsilon}$$

$$\delta = d_p = \sqrt{\frac{2}{\omega \mu \sigma}}$$

$$k' \approx k'' \approx \frac{1}{\delta}$$

$$Z_s \equiv \frac{E_{x0}}{J_{sx}}$$

$$R_s = \frac{1}{\sigma \delta} = \sqrt{\frac{\omega \mu}{2 \sigma}}$$

$$X_s = R_s$$

$$R=X=R_s\left(\frac{l}{2\pi a}\right)$$

$$R=R_s\left(\frac{1}{2\pi a}+\frac{1}{2\pi b}\right)$$

$$\underline{E}(z) = (\hat{x} \, E_x + \hat{y} \, E_y) e^{-jkz}$$

$$(a) \quad 0 < \beta < \pi \quad \text{LHEP}$$

$$(b) \quad -\pi < \beta < 0 \quad \text{RHEP}$$

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$\Gamma_{TM} = \frac{Z_2^{TM} - Z_1^{TM}}{Z_2^{TM} + Z_1^{TM}}$$

$$\Gamma_{TE} = \frac{Z_2^{TE} - Z_1^{TE}}{Z_2^{TE} + Z_1^{TE}}$$

$$Z_1^{TE} = \left(\frac{\omega \mu_1}{k_{zi}} \right) \quad Z_2^{TE} = \left(\frac{\omega \mu_2}{k_{zt}} \right)$$

$$Z_1^{TM} = \left(\frac{k_{zi}}{\omega \epsilon_1} \right) \quad Z_2^{TM} = \left(\frac{k_{zt}}{\omega \epsilon_2} \right)$$

$$\theta_i = \theta_b = \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$k_z = k \sqrt{1 - (f_c / f)^2}$$

$$f_c = \frac{c_d}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\beta = k \sqrt{1 - (f_c / f)^2}, \quad f > f_c$$

$$\alpha = k_c \sqrt{1 - (f / f_c)^2}, \quad f < f_c$$

$$H_z(x, y, z) = A_{10} \cos\left(\frac{\pi x}{a}\right) e^{-jk_z z}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1 - (f_c / f)^2}}$$

$$v_p = \frac{c_d}{\sqrt{1 - (f_c / f)^2}}$$

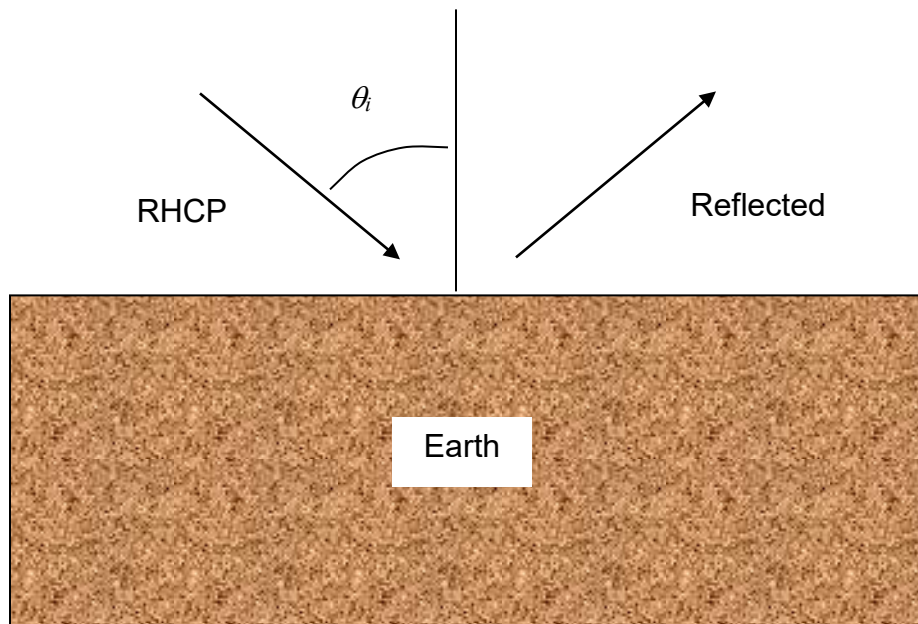
$$v_g = \frac{d\omega}{d\beta} = \frac{1}{\frac{d\beta}{d\omega}}$$

$$v_g = c_d \sqrt{1 - (f_c / f)^2}$$

Problem 1 (40 pts)

A RHCP plane wave at 1.575 GHz from a GPS satellite is incident on the earth. The earth has $\epsilon_r = 4.0$ and is taken as lossless. The angle of incidence is $\theta_i = 45^\circ$.

Calculate the total power density in the reflected plane wave relative to that of the incident plane wave.



Room for work

Solution

The incident power divides equally between TE_z and TM_z parts. Therefore, if there is an incident power density of one $[W/m^2]$, then the incident power density in the TE_z and TM_z parts is 0.5 $[W/m^2]$. The power density in the reflected wave is therefore

$$P_r = \frac{1}{2} |\Gamma_{TM}|^2 + \frac{1}{2} |\Gamma_{TE}|^2.$$

From Snells' law, the transmitted angle is

$$\theta_t = 20.705^\circ.$$

We then have

$$Z_1^{TM} = \eta_0 (0.7071)$$

$$Z_1^{TE} = \eta_0 (1.4142)$$

$$Z_2^{TM} = \eta_0 (0.4677)$$

$$Z_2^{TE} = \eta_0 (0.5345).$$

This gives us

$$\Gamma_{TM} = -0.2038$$

$$\Gamma_{TE} = -0.4514.$$

We then have

$$P_r = 0.1227$$

Problem 2 (30 pts)

A coaxial cable has an inner radius of 1 [mm] and an outer radius of 5 [mm]. The inner wire is made of copper with a conductivity of 5.8×10^7 [S/m]. The outer conductor is a braided aluminum conductor that has an effective conductivity of 2.0×10^7 [S/m]. Both the copper and the aluminum are nonmagnetic. The filling material is Teflon which has a relative permittivity of 2.1 and a loss tangent of 0.001.

Calculate the attenuation in [dB/m] on the coaxial cable at a frequency of 1.0 [GHz].

Room for work

Solution

The propagation constant is given by

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

where

$$C = \frac{2\pi\epsilon_0\epsilon_r}{\ln\left(\frac{b}{a}\right)}$$

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)}$$

$$R = \left(\frac{R_{sa}}{2\pi a} + \frac{R_{sb}}{2\pi b} \right)$$

and

$$R_{sa} = \frac{1}{\sigma_a \delta_a}$$

$$R_{sb} = \frac{1}{\sigma_b \delta_b}.$$

We then use

$$\text{Attenuation} = \alpha \left(\frac{20}{\ln(10)} \right) [\text{dB/m}].$$

The results are

$$C = 7.259 \cdot 10^{-11} \text{ [F/m]}$$

$$L = 3.219 \cdot 10^{-7} \text{ [H/m]}$$

$$G = 4.561 \cdot 10^{-4} \text{ [S/m]}$$

$$R = 1.76 \text{ } [\Omega/\text{m}]$$

This gives us

$$\gamma = 0.028403 + j(30.37168) \text{ [1/m]}.$$

From this we have

$$\text{Attenuation} = (0.028403) \left(\frac{20}{\ln(10)} \right) \text{ [dB/m]}.$$

This gives us

$$\text{Attenuation} = 0.246705 \text{ [dB/m]}.$$

Problem 3 (30 pts)

An air-filled rectangular waveguide has $b < a$. For a certain application, we want to make sure that all of the modes are attenuated sufficiently fast inside the waveguide, so that the entire field attenuates very fast. Note that all modes will attenuate at least as fast as the dominant mode, if it attenuates.

What is the largest value that the dimension a can be, so that the dominant mode is attenuated at a rate of at least A [dB/m] at a given frequency f ?

Your answer should be a formula that has the result for the dimension a in terms of A and f .

Room for work

Solution

In order to have the dominant more decay fast enough, we require

$$\text{Attenuation [dB/m]} = \alpha \left(\frac{20}{\ln(10)} \right) = A.$$

Hence we have

$$k_c \sqrt{1 - \left(\frac{f}{f_c} \right)^2} \left(\frac{20}{\ln(10)} \right) = A.$$

This gives us

$$\left(\frac{\pi}{a} \right) \sqrt{1 - \left(\frac{f}{\left(\frac{c}{2a} \right)} \right)^2} \left(\frac{20}{\ln(10)} \right) = A.$$

Squaring both sides, we have

$$\left(\frac{\pi}{a} \right)^2 \left(1 - \left(\frac{(2a)f}{c} \right)^2 \right) \left(\frac{20}{\ln(10)} \right)^2 = A^2.$$

Multiplying by a^2 , we have

$$\pi^2 \left(1 - \left(\frac{(2a)f}{c} \right)^2 \right) \left(\frac{20}{\ln(10)} \right)^2 = A^2 a^2.$$

Collecting terms, we have

$$a^2 \left(A^2 + \frac{4\pi^2 f^2}{c^2} \left(\frac{20}{\ln(10)} \right)^2 \right) = \pi^2 \left(\frac{20}{\ln(10)} \right)^2.$$

This gives us the result for the maximum allowed value of a , which is

$$a = \left(\frac{\frac{20\pi}{\ln(10)}}{\sqrt{A^2 + \frac{4\pi^2 f^2}{c^2} \left(\frac{20}{\ln(10)} \right)^2}} \right).$$

This may also be written as

$$a = \left(\frac{\pi}{\sqrt{A^2 \left(\frac{\ln(10)}{20} \right)^2 + \frac{4\pi^2 f^2}{c^2}}} \right).$$