DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

Name: ___________SOLUTION______________________

PeopleSoft ID: ___________________________________

ECE 3317

Applied Electromagnetic Waves

FINAL EXAM

Dec. 12, 2018

1. This exam is open book and open notes. However, you are not allowed to use a computer or any electronic device other than a calculator. Any devices that may be used to communicate are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily legible**.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Make sure you sign the academic honesty statement on the next page.

Academic Honesty Statement

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including getting an F in the class and getting expelled from the University.

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Signature

$$\nabla \times \underline{\mathscr{E}} = -\frac{\partial \underline{\mathscr{B}}}{\partial t}$$

$$\nabla \times \underline{\mathscr{H}} = \underline{\mathscr{J}} + \frac{\partial \underline{\mathscr{D}}}{\partial t}$$

$$\nabla \cdot \underline{\mathscr{B}} = 0$$

$$\nabla \cdot \underline{\mathscr{D}} = \rho_v$$

$$\nabla \times \underline{E} = -j\omega \underline{B}$$

$$\nabla \times \underline{H} = \underline{J} + j\omega \underline{D}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \cdot \underline{D} = \rho_v$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\varepsilon_0 \doteq 8.8541878 \times 10^{-12} \text{ [F/m]}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ [H/m]}$$

$$\oint_S \left(\underline{\mathscr{E}} \times \underline{\mathscr{H}} \right) \cdot \hat{n}\, dS = -\int_V \sigma \left| \underline{\mathscr{E}} \right|^2 dV - \int_V \frac{\partial}{\partial t} \left(\frac{1}{2} \mu \left| \underline{\mathscr{H}} \right|^2 \right) dV - \int_V \frac{\partial}{\partial t} \left(\frac{1}{2} \varepsilon \left| \underline{\mathscr{E}} \right|^2 \right) dV$$

$$\underline{\mathscr{S}} = \underline{\mathscr{E}} \times \underline{\mathscr{H}}$$

$$\underline{S} \equiv \frac{1}{2} \left(\underline{E} \times \underline{H}^* \right)$$

$$C = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln\left(\dfrac{b}{a} \right)} \quad \left[\text{F/m} \right]$$

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a} \right) \quad \left[\text{H/m} \right]$$

$$G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)} \quad [\text{S/m}]$$

$$R = \frac{1}{\sigma_m \delta}\left(\frac{1}{2\pi a} + \frac{1}{2\pi b}\right) \quad [\Omega/\text{m}]$$

$$\frac{\partial v}{\partial z} = -Ri - L\frac{\partial i}{\partial t}$$

$$\frac{\partial I}{\partial z} i = -Gv - C\frac{\partial v}{\partial t}$$

$$v(z,t) = f(z - c_d t) + g(z + c_d t)$$

$$i(z,t) = \frac{1}{Z_0}\left[f(z - c_d t) - g(z + c_d t)\right]$$

$$v(z,t) = v_g(t - z/c_d)$$

$$\Gamma_g = \left(\frac{R_g - Z_0}{R_g + Z_0}\right) \qquad \Gamma_L = \left(\frac{R_L - Z_0}{R_L + Z_0}\right)$$

$$V^+ = \left(\frac{Z_0}{R_g + Z_0}\right)V_0$$

$$\Gamma_L(t) = 1 - 2e^{-(t-T)/\tau}, \quad t \ge T \qquad \tau = Z_0 C_L$$

$$\Gamma_L(t) = -1 + 2e^{-(t-T)/\tau}, \quad t \ge T \qquad \tau = L_L / Z_0$$

$$V(z) = Ae^{-\gamma z} + B\,e^{+\gamma z}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$LC = \mu\varepsilon = \frac{1}{c_d^2}$$

$$\gamma = \alpha + j\beta$$

$$k_z = -j\gamma = \beta - j\alpha$$

$$v_p = \frac{\omega}{\beta}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\text{attenuation} = \left(\frac{20}{\ln 10}\right)\alpha = \left(8.6859\right)\alpha \qquad \text{[dB/m]}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$i(z) = \left(\frac{1}{Z_0}\right)\left[Ae^{-\gamma z} - B\,e^{+\gamma z}\right]$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$V(z) = A\left(e^{-\gamma z} + \Gamma_L\,e^{+\gamma z}\right)$$
$$I(z) = \frac{1}{Z_0}A\left(e^{-\gamma z} - \Gamma_L\,e^{+\gamma z}\right)$$

$$Z_{in}\left(z\right) = \frac{V\left(z\right)}{I\left(z\right)}$$

$$Z_{in}\left(z\right) = Z_0\left(\frac{1 + \Gamma_L\,e^{+2\gamma z}}{1 - \Gamma_L\,e^{+2\gamma z}}\right)$$

$$Z_{in} = Z_0\left(\frac{Z_L + Z_0\tanh\left(\gamma l\right)}{Z_0 + Z_L\tanh\left(\gamma l\right)}\right)$$

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right)$$

$$Z_{in}(z) = Z_0 \left(\frac{1+\Gamma(z)}{1-\Gamma(z)} \right)$$

$$Z_{in} = jZ_0 \tan(\beta l)$$

$$Z_{in} = -jZ_0 \cot(\beta l)$$

$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

$$|V(z)| = |A| \left| 1 + |\Gamma_L| e^{+j(\phi+2\beta z)} \right|$$

$$\left| \frac{V(z)}{V^+} \right| = \left| 1 + |\Gamma_L| e^{+j(\phi+2\beta z)} \right| = \left| 1 + \Gamma(z) \right|$$

$$\Gamma(z) = \Gamma_L \, e^{j2\beta z} = \Gamma_L \, e^{-j2\beta l}$$

$$\text{VSWR} \equiv \frac{V_{\max}}{V_{\min}}$$

$$\text{VSWR} \equiv \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$\text{SWR} = \max\left(\frac{R_L}{Z_0}, \frac{Z_0}{R_L} \right)$$

$$Z_{in}^N(z) = \frac{1+\Gamma_L \, e^{+2j\beta z}}{1-\Gamma_L \, e^{+2j\beta z}}$$

$$Z_{in} = \frac{Z_{0\text{T}}^2}{Z_L}$$

$$Z_{0\mathrm{T}} = \sqrt{Z_0 R_L}$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \doteq 376.730313\ [\Omega]$$

$$\underline{S} = \underline{\hat{z}}\,\frac{\left|E_0\right|^2}{2\eta}$$

$$v_p = \frac{\omega}{k}$$

$$\lambda = \frac{2\pi}{k}$$

$$\lambda = \frac{c_d}{f}$$

$$E_x = E_0\, e^{-jkz}$$

$$H_y = \frac{1}{\eta} E_0\, e^{-jkz}$$

$$\varepsilon_c = \varepsilon - j\left(\frac{\sigma}{\omega}\right)$$

$$k = k' - jk''$$

$$\lambda = \frac{2\pi}{k'}$$

$$d_p = 1/k''$$

$$\tan\delta = \frac{\varepsilon_c''}{\varepsilon_c'} = \frac{\sigma}{\omega\varepsilon}$$

$$\delta = d_p = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$k' \approx k'' \approx \frac{1}{\delta}$$

$$Z_s \equiv \frac{E_{x0}}{J_{sx}}$$

$$R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\omega\mu}{2\sigma}}$$

$$X_s = R_s$$

$$R = X = R_s\left(\frac{l}{2\pi a}\right)$$

$$R = R_s\left(\frac{1}{2\pi a} + \frac{1}{2\pi b}\right)$$

$$\underline{E}(z) = (\underline{\hat{x}}\, E_x + \underline{\hat{y}}\, E_y)\, e^{-jkz}$$

(a) $0 < \beta < \pi$ LHEP

(b) $-\pi < \beta < 0$ RHEP

$$\gamma = \tan^{-1}\left(\frac{b}{a}\right)$$

$$0 \le \gamma \le 90^{\circ}$$

$$\mathrm{AR} = |\cot \xi|$$

$\xi > 0$: LHEP

$\xi < 0$: RHEP

where

$$\sin 2\xi = \sin 2\gamma \sin \beta$$

$$-45^{\circ} \le \xi \le +45^{\circ}$$

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$n_1 \sin \theta_c = n_2 \sin 90^{\circ}$$

$$\theta_i = \theta_b = \tan^{-1} \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$Z_1^{TE} = \left(\frac{\omega\mu_1}{k_{zi}}\right) \qquad Z_2^{TE} = \left(\frac{\omega\mu_2}{k_{zt}}\right)$$

$$Z_1^{TM} = \left(\frac{k_{zi}}{\omega\varepsilon_1}\right) \qquad Z_2^{TM} = \left(\frac{k_{zt}}{\omega\varepsilon_2}\right)$$

$$k_z = k\sqrt{1-\left(f_c / f\right)^2}$$

$$f_c = \frac{c_d}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\beta = k\sqrt{1-\left(f_c / f\right)^2}, \qquad f > f_c$$

$$\alpha = k_c\sqrt{1-\left(f / f_c\right)^2}, \qquad f < f_c$$

$$H_z\left(x,y,z\right)=A_{10}\cos\left(\frac{\pi x}{a}\right)e^{-jk_z z}$$

$$\lambda_g=\frac{\lambda}{\sqrt{1-\left(f_c/f\right)^2}}$$

$$v_p=\frac{c_d}{\sqrt{1-\left(f_c/f\right)^2}}$$

$$v_g=\frac{d\omega}{d\beta}=\frac{1}{\frac{d\beta}{d\omega}}$$

$$v_g=c_d\sqrt{1-\left(f_c/f\right)^2}$$

$$\underline{S}=\underline{\hat{r}}\left(\frac{\left|\underline{E}\right|^2}{2\eta_0}\right)$$

$$\underline{E}\left(r,\theta,\phi\right)=\left(\frac{e^{-jk_0 r}}{r}\right)\underline{E}^F\left(\theta,\phi\right)$$

$$P_{rad}=\frac{1}{2\eta_0}\int_0^{2\pi}\int_0^{\pi}\left|\underline{E}^F\left(\theta,\phi\right)\right|^2\sin\theta\,d\theta d\phi$$

$$D\left(\theta,\phi\right)\equiv\frac{S_r\left(\theta,\phi\right)}{P_{rad}/\left(4\pi r^2\right)}\qquad r\to\infty$$

$$D\left(\theta,\phi\right)=\frac{4\pi\left|\underline{E}^F\left(\theta,\phi\right)\right|^2}{\int_0^{2\pi}\int_0^{\pi}\left|\underline{E}^F\left(\theta,\phi\right)\right|^2\sin\theta\,d\theta d\phi}$$

$$G\left(\theta,\phi\right)\equiv e_r\,D\left(\theta,\phi\right)$$

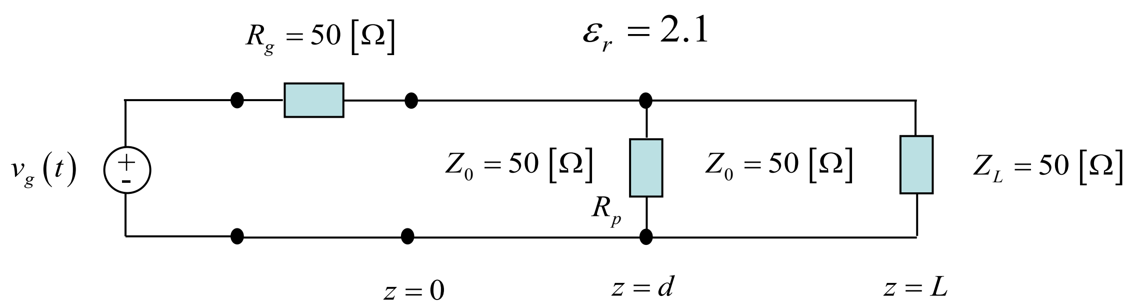
$$Z_{Th}=Z_{in}$$

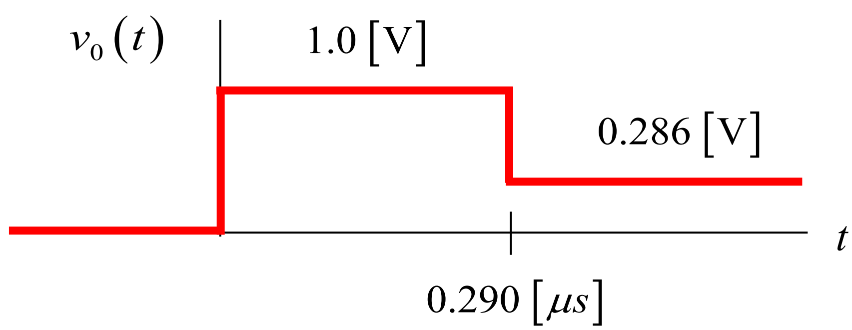
Problem 1 (20 pts.)

A Time Domain Reflectometer (TDR) is being used to determine a fault on a transmission line. The TDR has a voltage generator $v_g(t)$ that applies a voltage step function with an amplitude of $V_0 = 2.0$ [V] at the input to the transmission line circuit shown below. The transmission line has a relative permittivity of 2.1. The voltage waveform $v_0(t)$ that is recorded by the TDR at $z = 0$ is shown below. A partial short on the line (the fault) is modeled as a parallel resistance R_p as shown.

Determine the unknown resistance R_p and the distance d between the short and the TDR.

Support your answer by constructing a bounce diagram and using it to get the voltage $v_0(t)$ in terms of d and R_p.





Room for Work

$d = 30$ [m]

$R_p = 10$ [Ω]

Problem 2 (20 pts.)

A 75 [Ω] RG-59 coaxial cable has an outer radius of b = 1.85 [mm] and an inner radius of a = 0.292 [mm]. The coax is filled with Teflon (ε_r = 2.25) that has a loss tangent of 0.001. The conductors are made of copper, which is nonmagnetic and has a conductivity of $\sigma = 3.0\times10^8$ [S/m]. Calculate the attenuation in dB when a 500 [MHz] signal travels along the cable a distance of 20 meters.

$R = 1.619$ [Ω/m]

$G = 2.13\times10^{-4}$ [S/m]

$L = 3.692\times10^{-7}$ [H/m]

$C = 6.78\times10^{-11}$ [F/m]

$\gamma = 0.0188 + j(15.72)$ [1/m]

$\alpha = 0.0188$ [np/m]

$\alpha = 0.164$ [dB/m]

Attenuation = 3.27 [dB]

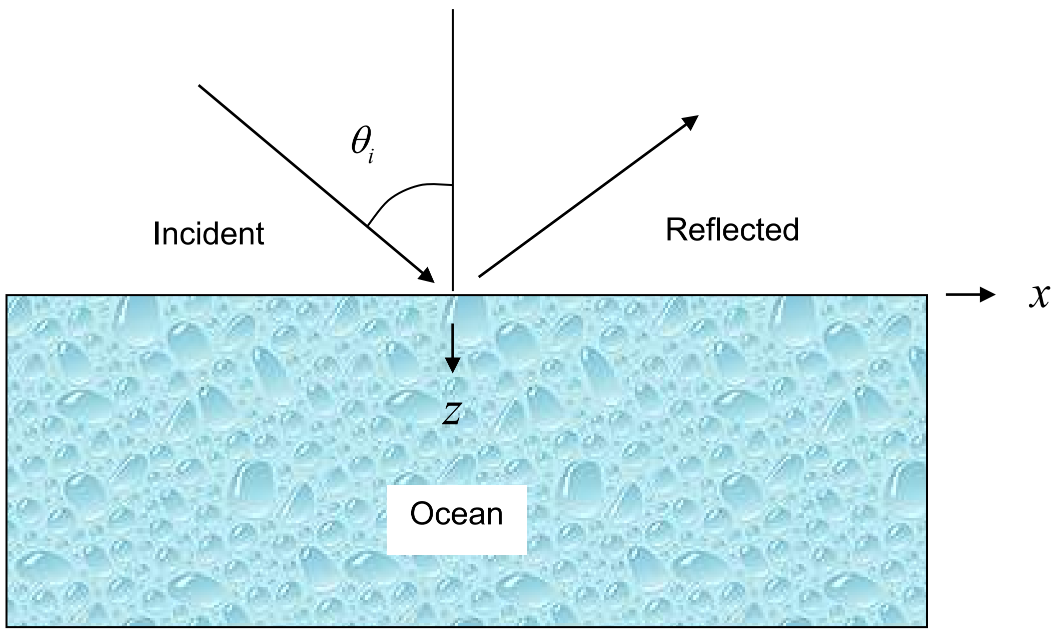
Room for Work

Problem 3 (20 pts.)

A TE_z plane wave at a frequency of 1.575 GHz (GPS frequency) is incident from the air onto the ocean (which is nonmagnetic). The ocean has a relative permittivity of $\varepsilon_r = 81$ and a conductivity of $\sigma = 4.0$ [S/m]. The angle of incidence is $\theta_i = 45^{\circ}$.

a) Find percentage of power that is reflected from the ocean.

b) Find the dB of attenuation in the plane wave as it travels from just below the surface down to a depth of 10 meters ($z = 10$ [m]).



Room for Work

$\varepsilon_{rc} = 81.0 - j(45.65)$

$Z_1^{TE} = 532.77 \; [\Omega]$

$Z_2^{TE} = 37.86 + j(9.99) \; [\Omega]$

$\Gamma_{TE} = -0.867 + j(0.033)$

$P_r^{\%} = 75.2$

$k_{zt} = 307.0 - j(81.0) \; [1/\mathrm{m}]$

$\alpha_{zt} = 81.0 \; [\mathrm{np/m}]$

$\alpha_{zt} = 703.6 \; [\mathrm{dB/m}]$

$\text{Attenuation} = 7036 \; [\mathrm{dB}]$

Problem 4 (20 pts.)

An air-filled rectangular waveguide is designed to transmit the dominant mode at an operating frequency of 8.0 [GHz]. The operating frequency should be halfway between the cutoff frequencies of the dominant mode and the next higher mode. Assume that $b = a/2$.

a) Determine the dimensions a and b of the waveguide.

b) Assume that we wish to have the dominant mode attenuated by at least 100 dB when the frequency is 5% below the cutoff frequency of the dominant mode. What is the minimum length of waveguide (in the z direction) required to do this?

Room for Work

$a = 0.02811 \ [\text{m}]$

$b = 0.01405 \ [\text{m}]$

$\alpha = 34.90 \ [\text{np/m}]$

$\alpha = 303.1 \ [\text{dB/m}]$

$l = 0.330 \ [\text{m}]$

Problem 5 (20 pts.)

NASA wants to use a large reflector antenna in the Deep Space Network (DSN) to communicate with a CubeSat satellite. A simple dipole antenna is used on the CubeSat. The frequency is 7.2 [GHz]. The transmit dish antenna in the DSN has a diameter of 34 [m]. Assume that the aperture efficiency of the dish is 70%.

a) If the transmitted power is 100 [W], find the power density at the cubesat location, assuming that the cubesat is located near Mars, at a distance of 75 million [km].

b) Find the power that the CubeSat antenna is able to deliver to a matched load, assuming ideal conditions (the polarization of the transmitted wave and the receive dipole antenna are perfectly aligned).

Room for Work

$A_e^{dish} = 635.54 \left[\mathrm{m}^2\right]$

$G^{dish} = 4.607 \times 10^6$

$P_d = 6.52 \times 10^{-15} \left[\mathrm{W}\right]$

$A_e^{dip} = 2.267 \times 10^{-4} \left[\mathrm{m}^2\right]$

$P_{rec} = 1.477 \times 10^{-18} \left[\mathrm{W}\right]$

Bonus (20 pts.)

An antenna radiates an electric field in the far field that is given by

$$\underline{E}^F(\theta,\phi) = \begin{cases} E_0\left(\hat{\underline{\theta}}\cos\theta\cos\phi + \hat{\underline{\phi}}\sin\phi\right), & \theta \le \pi/2 \\ \underline{0}, \quad \theta \ge \pi/2, \end{cases}$$

where E_0 is a constant.

Calculate the directivity $D(\theta, \phi)$. Make sure that you evaluate all integrals to get a closed-form result.

Room for Work

$$D(\theta,\phi)=\frac{4\pi\left(\cos^2\theta\cos^2\phi+\sin^2\phi\right)}{\int_0^{2\pi}\int_0^{\pi/2}\left(\cos^2\theta\cos^2\phi+\sin^2\phi\right)\sin\theta\,d\theta d\phi}$$

$$D(\theta,\phi)=\frac{4\pi\left(\cos^2\theta\cos^2\phi+\sin^2\phi\right)}{\pi\int_0^{\pi/2}\left(\cos^2\theta+1\right)\sin\theta\,d\theta}$$

$$D(\theta,\phi)=\frac{4\left(\cos^2\theta\cos^2\phi+\sin^2\phi\right)}{\int_0^{\pi/2}\left(\cos^2\theta+1\right)\sin\theta\,d\theta}$$

$$D(\theta,\phi)=\frac{4\left(\cos^2\theta\cos^2\phi+\sin^2\phi\right)}{\int_0^{\pi/2}\cos^2\theta\sin\theta\,d\theta+\int_0^{\pi/2}\sin\theta\,d\theta}$$

$$D(\theta,\phi)=\frac{4\left(\cos^2\theta\cos^2\phi+\sin^2\phi\right)}{\frac{1}{3}+1}$$

$$D(\theta,\phi)=3\left(\cos^2\theta\cos^2\phi+\sin^2\phi\right)$$