# ECE 3317
## Applied Electromagnetic Waves

## Final Exam

## Dec. 9, 2024

**Name:** ______SOLUTION________________________________

**General Information:**

The exam is open-book and open-notes. You are not allowed to use any device that has communication functionality (laptop, cell phone, ipad, etc.).

**Instructions:**

- Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
- Write neatly. You will not be given credit for work that is not easily legible.
- Leave answers in terms of the parameters given in the problem.
- Show units in all of your final answers.
- Circle your final answers.
- Double-check your answers. For simpler problems, partial credit may not be given.
- If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
- Make sure you sign the academic honesty statement below.

## Academic Honesty Statement

By taking this exam, you agree to abide by the UH Academic Honesty Policy during this exam. You understand and agree that the punishment for violating this policy will be most severe, including getting an F in the class and getting expelled from the University.

Signature: ______________________________________

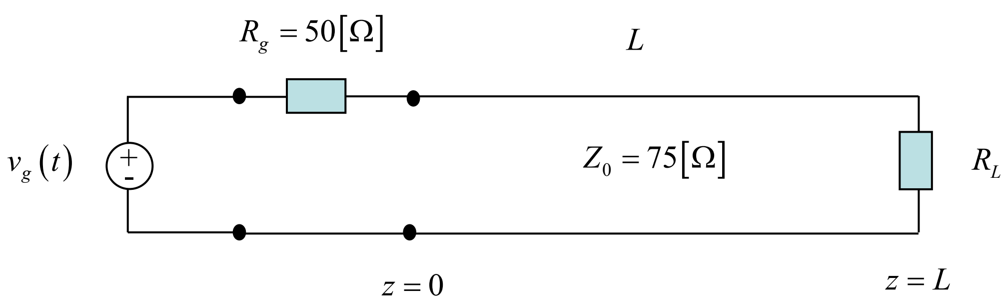
## Problem 1 (25 pts)

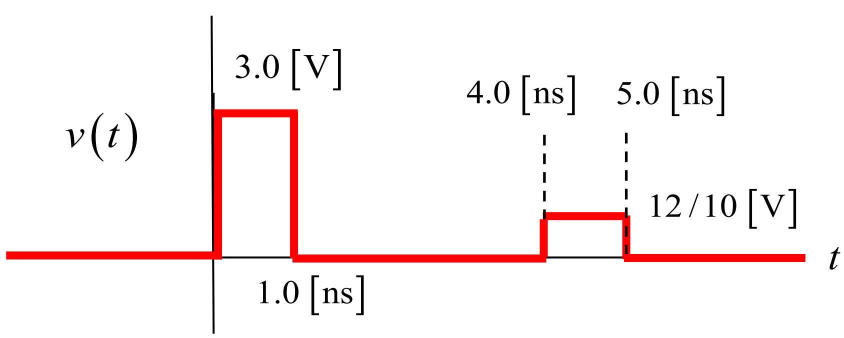
A generator voltage $v_g(t)$ puts out a rectangular pulse function with an amplitude of $V_0 = 5$ volts and a width of 1.0 [nS]. The generator has a 50 [Ω] Thévenin impedance. The generator is connected to a transmission line having a characteristic impedance of $Z_0 = 75$ [Ω]. The line is a coaxial cable of length $L$ that is filled with Teflon, having $\varepsilon_r = 2.1$. At the end of the line there is a load $R_L$. The voltage $v(t)$ at $z = 0$ is measured, and this is shown in the plot below.

(a) What is the length of the line $L$ in meters?

(b) What is the load resistance $R_L$?

Make a bounce diagram to show how you are getting your answers.



## Solution

### Part (a)

The distance travelled by the pulse in going down the line and bouncing back to the generator is $2L$. We therefore have

$$2T = \frac{2L}{v_p} = \frac{2L}{c/\sqrt{\varepsilon_r}}.$$

We then have

$$4.0\times 10^{-9} = \frac{2L}{2.99792458\times 10^{8}/\sqrt{2.1}}.$$

This gives us

$L = 0.4137\ [\mathrm{m}]$.

### Part (b)

The pulse travels down the line, reflects from the load, and then travels back to the generator and reflects from the generator. We see from the ounce diagram that for a step function, the voltage would jump at 4 [ns] to a value of

$$V_s = V^{+}(1+\Gamma_L+\Gamma_L\Gamma_g).$$

For the rectangular pulse, we subtract from the step response a delayed step response. We then see that the voltage would jump to a value of

or

$$V_p = V^{+}(1+\Gamma_L+\Gamma_L\Gamma_g) - V^{+}$$

We then have

$$V_p = V^{+}\Gamma_L(1+\Gamma_L\Gamma_g).$$

This gives us

$$\frac{12}{10} = 3.0\left(\Gamma_L\left(1-\frac{1}{5}\right)\right).$$

From this we have

$$\Gamma_L = \frac{1}{2}.$$

We then use

$$\Gamma_L = \frac{1}{2} = \frac{R_L - 75}{R_L + 75}.$$

We then have

$$R_L = 225[\Omega].$$

## Problem 2 (25 pts)

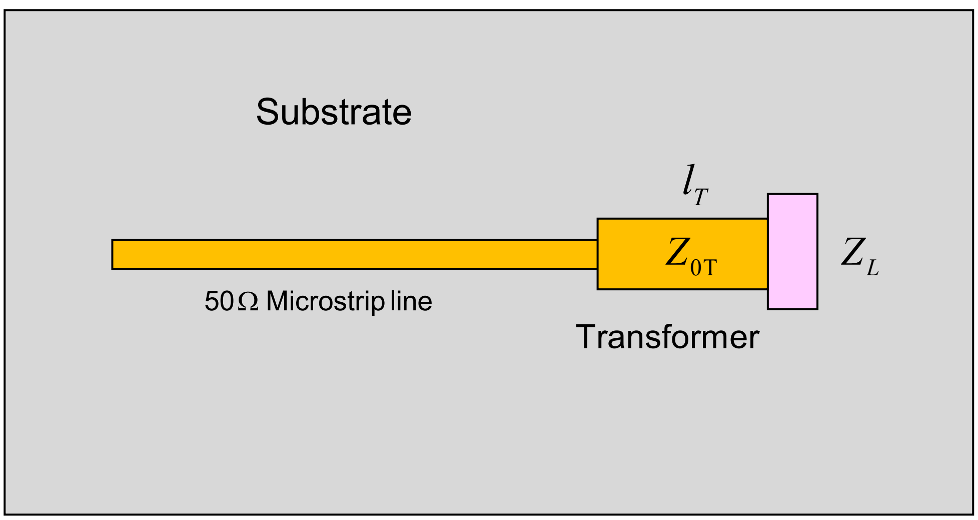
A device (load) on a printed circuit board has an input impedance of $Z_L = 100$ [Ω]. The load is connected to a quarter-wave transformer that is then connected to a 50 [Ω] microstrip line. The frequency is 3 [GHz] and the effective relative permittivity of the transformer line is 1.5.

a) What is the characteristic impedance of the transformer line?

b) What is the length of the transformer line, in cm?

c) Find the input impedance seen by the 50 [Ω] microstrip line looking into the transformer at a frequency of 4 GHz, using the Smith chart.

d) Find the SWR on the 50 [Ω] microstrip line at 4 GHz using the Smith chart.



## Solution

### Part (a)

The transformer impedance formula is

$$Z_{0\mathrm{T}} = \sqrt{Z_0 R_L}\ .$$

Using the transformer impedance formula, we have

$$Z_{0\mathrm{T}} = 70.71\ [\Omega].$$

### Part (b)

We have

$$l_{\mathrm{T}} = \frac{\lambda_g}{4} = \frac{\lambda_0 / \sqrt{\varepsilon_r^{\mathrm{eff}}}}{4} = \frac{(c/f)/\sqrt{\varepsilon_r^{\mathrm{eff}}}}{4} = \frac{c}{4f\sqrt{\varepsilon_r^{\mathrm{eff}}}}\ .$$

This gives us

$$l_{\mathrm{T}} = 2.040\ [\mathrm{cm}].$$

### Part (c)

We have

$$Z_L^{\mathrm{N}} = \frac{Z_L}{70.71} = 1.415\,.$$

At 3.0 [GHz] we are going 0.25 $\lambda_g$ on the Smith chart to get the input impedance. At 4.0 GHz we are moving 0.25 $\lambda_g$ (4/3) = 0.333 $\lambda_g$ = 0.25 $\lambda_g$ + 0.08333 $\lambda_g$. This gives us (from the first Smith chart)

$$Z_{\mathrm{in}}^{\mathrm{N}} = 0.77 + j(0.27)\,,$$

and therefore (multiplying by 70.71)

$$Z_{\mathrm{in}} = 54 + j(19)\ [\Omega].$$

### Part (d)

Using the Smith chart to find the SWR, we first renormalize to 50 Ω.
This gives us

$$Z_{\text{in}}^{\text{N}} = 1.08 + j(0.38).$$

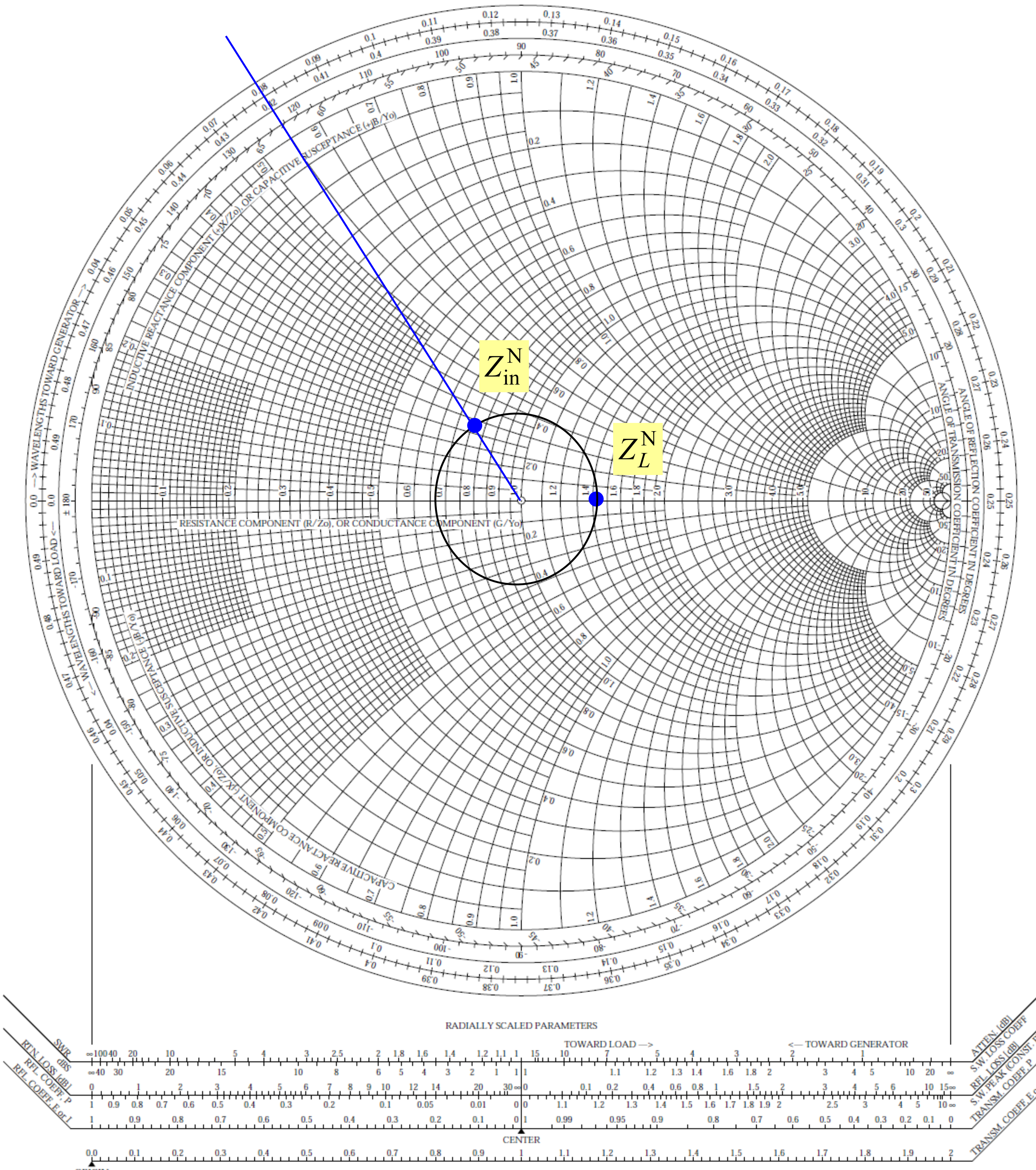
We then read the SWR from the positive real axis. The result is (from the second Smith chart)

$\text{SWR} = 1.38$.

# The Complete Smith Chart

Black Magic Design

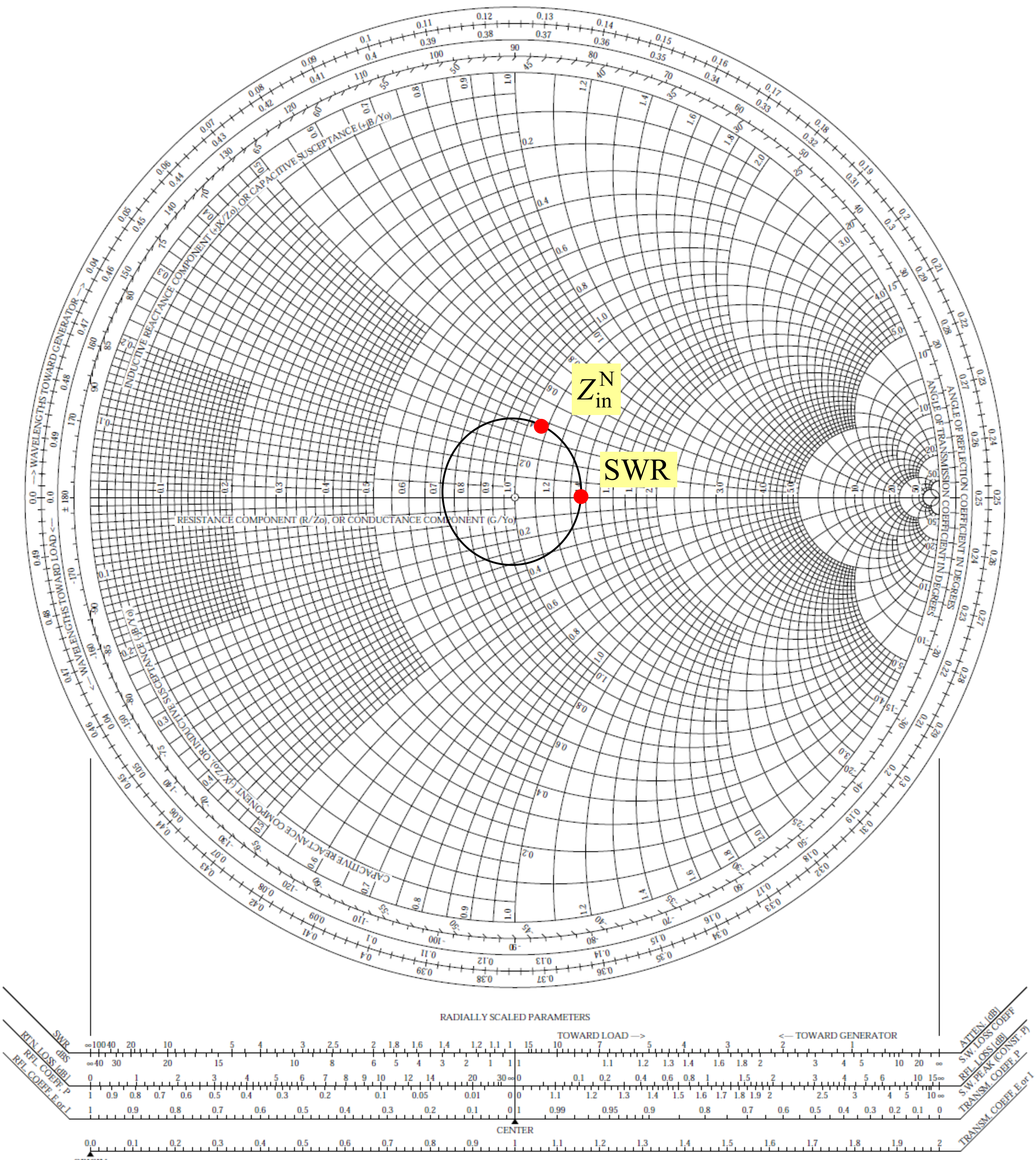
Part (a)



# The Complete Smith Chart

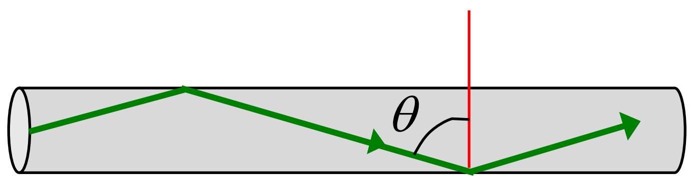
Black Magic Design

Part (b)



## Problem 3 (25 pts)

A laser beam inside of a fiber optic rod is bouncing at an angle of $\theta = 60^{\circ}$ as shown below. Assume that the glass fiber has an index of refraction of $n_1 = 1.5$ and the surrounding air region has an index of refraction of $n_2 = 1.0$. At the operating frequency the wavelength in free space is 1.3 [μm]. (1 μm = $10^{-6}$ meters.) How many dB of attenuation is there in the optical signal at a distance of 1 [μm] from the surface of the fiber in the air region?



## Solution

Since we are beyond the critical angle, the attenuation is given by

$$\alpha_{zt} = k_0 \sqrt{n_1^2 \sin^2 \theta_i - n_2^2} \, ,$$

with

$$\theta_i = 60^{\circ}$$

$$k_0 = \frac{2\pi}{\lambda_0} = 4.833 \times 10^6 \, [\text{rad/m}] .$$

This gives us

$$\alpha_{zt} = 4.01 \times 10^6 \, [\text{np/m}] .$$

Multiplying by 8.686, we have

$$\text{Attenuation} = 3.48 \times 10^7 \, [\text{dB/m}] .$$

Hence, the total attenuation is

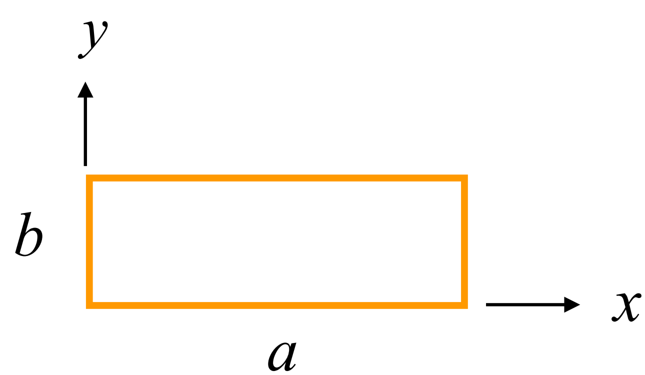
$$\text{Attenuation} = 34.8 \, [\text{dB}] .$$

This is a lot of attenuation, only 1 micron away from the surface of the fiber!

## Problem 4 (25 pts)

a) Design the dimensions $a$ and $b$ of an air-filled rectangular waveguide that is to be used for transmission of electromagnetic power at an operating frequency of 8.0 [GHz]. Assume that we choose $b = a/2$. The operating frequency of 8.0 GHz should be at the center of the frequency band that is between the cutoff frequencies of the $TE_{10}$ mode and the $TE_{20}$ mode.

b) Now assume that $a = 1.5$ [cm] and $b = a/2$. At a frequency of 5.0 GHz, how many dB of attenuation is there in the $TE_{10}$ mode after a distance of 5.0 cm?



## Solution

### Part (a)

We wish to have

$$f_0 = \frac{1}{2}\left(f_c^{\mathrm{TE}_{10}} + f_c^{\mathrm{TE}_{20}}\right)$$

and therefore

$$f_0 = \frac{1}{2}\left(\frac{c}{2a} + 2\left(\frac{c}{2a}\right)\right) = 1.5\left(\frac{c}{2a}\right).$$

This gives us

$$a = 0.0281\left[\mathrm{m}\right] = 2.81\left[\mathrm{cm}\right].$$

We then have

$$b = 0.0141\left[\mathrm{m}\right] = 1.41\left[\mathrm{cm}\right].$$

### Part (b)

We have

$$\alpha = \sqrt{k_c^2 - k_0^2} = \sqrt{\left(\frac{\pi}{a}\right)^2 - k_0^2}\ .$$

At 5.0 GHz we have

$$k_0 = \frac{2\pi}{\lambda_0} = 104.79\ \left[\mathrm{rad/m}\right].$$

We then have

$$\alpha = 181.34\ \left[\mathrm{np/m}\right].$$

Multiplying by 8.686, we have 1575.1 [dB/m].

The total attenuation after 5 [cm] (0.05 [m]) is then

$\alpha = 78.8\left[\mathrm{dB}\right]$.

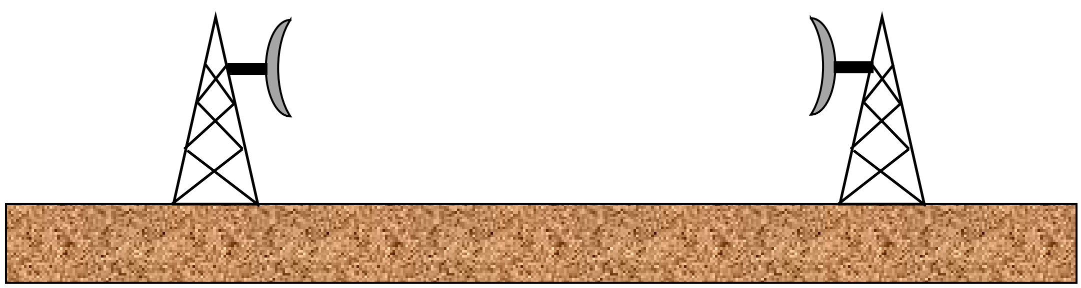
## Bonus Problem (30 pts)

A parabolic reflector (dish) antenna has a diameter of 1.0 [m] and an aperture efficiency of 75%. The frequency is 12 [GHz].

a) What is the gain of the dish antenna?

b) Now assume that the dish antenna radiates a power of 10 [W] and the gain of the dish is $1.0\times10^4$. (Assume this gain number, regardless of what you got for your answer to part (a).) What is the power density [W/m$^2$] that is radiated at a distance of 10.0 [km] from the dish, in the direction of the main beam?

c) At a distance of 10.0 [km] from the transmitting dish antenna that is transmitting 10 [W] of power, a second identical dish antenna is placed to receive the signal, as shown below. How much power will the receive dish antenna be able to pick up and deliver to a matched load? Assume that both dishes have a gain of $1.0\times10^4$.



**Solution**

**Part (a)**

We use

$$A_{\text{eff}} = G\left(\frac{\lambda_0^2}{4\pi}\right),$$

where

$$A_{\text{eff}} = Ae_{\text{ap}} = \left(\pi a^2\right)e_{\text{ap}} = \left(\pi\left(D/2\right)^2\right)e_{\text{ap}}.$$

and therefore

$$A_{\text{eff}} = 0.58905\left[\text{m}^2\right]$$

The gain is then

$G = 11{,}859.9$.

**Part (b)**

We use

$$P_d^{\text{inc}} = \left(\frac{P_{\text{rad}}}{4\pi r^2}\right)G_t,$$

This gives us

$P_d^{\text{inc}} = 7.958\times10^{-5}\ \left[\text{W/m}^2\right]$.

**Part (c)**

We use

$$P_{\text{rec}} = P_d^{\text{inc}}A_{\text{eff}}.$$

If we assume a gain of $1.0\times10^4$, the effective area is then

$$A_{\text{eff}} = 0.4967 \left[ \text{m}^2 \right].$$

We then have

$$P_{\text{rec}} = 3.953 \times 10^{-5} \left[ \text{W} \right].$$