#### ECE 3318

#### Applied Electricity and Magnetism

**Exam 2**

#### April 30, 2024

**Name: \_\_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Instructions**

1. This exam is open-book and open-notes.
2. Cell phones, laptops, ipads, and any other devices that have communication functionality are not allowed during the exam.
3. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
4. Write neatly. You will not be given credit for work that is not easilylegible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Remember the UH Academic Honesty Policy. You must not receive or give assistance to anyone else during the exam, or communicate with anyone other than the instructor during the exam.

**TABLE OF INTEGRALS**

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**Problem 1 (35 pts.)**

An engineer measures the voltages *v*1, *v*2, *v*3, *v*4 in the circuit below. The engineer expects that the sum of these four voltages *v*sum will be zero, by the KVL law. However, there is a magnetic field in the room, and the magnetic field is given by

,

where

.

(a) Find a formula for what the sum of the four voltages *v*sum will be.

(b) Assume that each of the four elements in the loop are resistors. Their resistances are 1 [MΩ], 2 [MΩ], 3 [MΩ], and 4 [MΩ] (for elements 1, 2, 3, and 4, respectively). Find a formula for the voltage *v*1 (as a function of time).

**SOLUTION**

**Part (a)**

Let’s define a closed path *C* that goes counterclockwise around the loop path. From Faraday’s law (and the right-hand rule) we have

,

where  is the magnetic flux through the loop in the positive *z* direction and  is the sum of the four voltages as defined in the figure. The magnetic flux through the loop is



Hence, we have

.

We thus have

.

**Part (b)**

The current flowing counterclockwise in the loop is called . We have



.

We also have

.

We thus have

.

Hence, we have

.

**Problem 2 (35 pts.)**

A spherical shell of uniform volume charge density  exists in the region . The total charge is denoted as *Q*.

a) Assume that the reference point is chosen as the origin, where we have zero volts. Find the potential (voltage) function in all three regions: *r* < *a*, *a* < *r* < *b*, *r* > *b*. You may leave *Q* in your answer, as long as you relate it to .

b) Find the stored energy in the system. You may leave *Q* in your answer, as long as you relate it to . For this part of the problem, you do not need to carry out the calculus to evaluate any integrals in *r*. That is, you may leave integrals in *r* in your answer.



**SOLUTION**

**Part (a)**

We use

.

Using a radial path in the direction, we have

.

Since  (zero volts at the reference point) we have

.

From Gauss’s law we have

,

,

,

where

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We then have for 



or

.

For  we have

.

This gives us



or



or



or

.

We then have

.

For  we have

.

Hence, we have

.

We then have

.

**Part (b)**

If we use the stored energy formula that has charge density in it, then we have

.

We then have

.

For our problem we then have

.

Hence, we have

.

**Problem 3 (30 pts.)**

A power line has a radius *a* and the center of the line is located at a height of *h* above the earth. The voltage of the power line (with respect to the earth) is

,

where  denotes the RMS value of the line voltage.

a) Find a formula for the maximum RMS voltage  that can be put on the line before corona discharge takes place on the line, assuming that the surrounding air has a breakdown field strength of . (We wish to avoid corona discharge taking place at any point during the voltage waveform.) You may assume that .

b) Find a formula for the RMS current that flows from a length *L* of line down to the ground due to the capacitance that exists between the line and the ground. Put your answer in terms of all given parameters. (The air is assumed to be a perfect insulator in this problem, but there is a capacitance between the line and the ground.)

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**SOLUTION**

**Part (a)**

We use

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We set

.

We have



And therefore

.

Hence,

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We then have the maximum RMS voltage as

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**Part (b)**

We have

.

We also have



and

.

We then have

.

**ROOM FOR WORK**