#### ECE 3318

#### Applied Electricity and Magnetism

**Exam 2**

#### April 29, 2025

**Name: SOLUTION**

**Instructions**

1. This exam is open-book and open-notes.
2. Cell phones, laptops, ipads, and any other devices that have communication functionality are not allowed during the exam.
3. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
4. Write neatly. You will not be given credit for work that is not easilylegible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Remember the UH Academic Honesty Policy. You must not receive or give assistance to anyone else during the exam or communicate with anyone other than the instructor during the exam.

**TABLE OF INTEGRALS**

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**Problem 1 (30 pts.)**

A vertical piece of a cylindrical transformer core of radius *a* has a relative permeability of  and a conductivity of *σ*. (A cross-sectional view is shown below.) Inside the core there is a magnetic field given by

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where *A* is a constant and *ρ* is the usual distance in cylindrical coordinates.

Find an expression for the eddy current *J* that flows inside the transformer core.

**Solution**

From Faraday’s law we have

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The electric field is in the  direction.

Let’s assume that the path *C* is a circle of radius *ρ*, and choose the path to run counterclockwise. The unit normal to the path (from the right-hand rule) is then in the positive  direction. We then have



or

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The flux is given by

.

We then have

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Hence,

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Evaluating the integral, we have

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We then have

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From Ohm’s law the eddy current is then

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In vector form, the final answer is

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**Problem 2 (40 pts.)**

A hollow spherical conducting (PEC) shell of radius *a* has a charge of *Q* coulombs on it. The conducting shell is surrounded by a dielectric shell of inner radius *a* and outer radius *b*. The dielectric shell has a relative permittivity of . The potential is taken to be zero volts at infinity.

1. Find the electric field vector in all three regions: *r* < *a*, *a* < *r* < *b*, *r* > *b*.
2. Find the stored energy in the system.
3. Find voltage of the conducting spherical shell.
4. Find the maximum charge *Q* (called *Q*max) that can be put on the sphere before the air breaks down, assuming that the breakdown field strength of the air is *Ec*.











**Solution**

**Part (a)**

From Gauss’s law we have

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Therefore, we have:







**Part (b)**

We have

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Hence, we have

.

This gives us

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We then have

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This gives us

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Simplifying, we have

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**Part (c)**

We have

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Hence, we have

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Thus, we have



So that

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We then have

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**Part (d)**

We set

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This gives us

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**Problem 3 (30 pts.)**

A three-phase power line consists of three lines, each of radius *a*, with the center of each line located at a height of , , andabove the earth, for lines 1, 2, and 3, respectively. The earth may be modelled as a perfect conductor at the power line frequency. The voltage of each line (with respect to the earth, which is at zero volts) is:



1. Find a formula for the electric field vector (as a function of time) at the surface of the earth (*y* = 0) directly below the three lines (*x* = 0) in terms of the given parameters. Assume that the charge density on each of the three lines is the same as what it would be if the other two lines were not there.
2. Find the electric field vector at the same point as above in the phasor domain.
3. What would the phasor domain electric field vector be at the same point as above if we had ?

You may use any formula that you wish from the class notes to help you.

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**Solution**

**Part (a)**

For a single line, we have from the class notes that



where

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Using superposition, we have



where

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**Part (b)**

In the phasor domain we have



where

.

**Part (c)**

In the phasor domain we have

.

We have

.

Hence, we have

.

In practice, we will never have the heights of the three lines exactly the same. But the interesting conclusion here is that as the three phase lines get closer together, the field that they produce becomes less. That is, there is a cancellation effect since the three phasor voltages all add up to zero.