FORMULA SHEET FOR EXAM 1

$$dS = dx\,dy$$
$$dV = dx\,dy\,dz$$

$$dS = \rho\, d\phi\, dz$$
$$dV = \rho\, d\rho\, d\phi\, dz$$

$$dS = r^2 \sin\theta\, d\theta\, d\phi$$
$$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$d\underline{r} = \underline{\hat{x}}\, dx + \underline{\hat{y}}\, dy + \underline{\hat{z}}\, dz$$
$$d\underline{r} = \underline{\hat{z}}\, dz + \underline{\hat{\rho}}\, d\rho + \underline{\hat{\phi}}\, \rho\, d\phi$$
$$d\underline{r} = \underline{\hat{r}}\, dr + \underline{\hat{\phi}}\, r \sin\theta\, d\phi + \underline{\hat{\theta}}\, r\, d\theta$$

$$\nabla \cdot \underline{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\nabla \cdot \underline{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho D_\rho\right) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

$$\nabla \cdot \underline{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(D_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}$$

$$i = \frac{dq}{dt}$$

$$i = \int_S \underline{J} \cdot \underline{\hat{n}}\; dS$$

$$\underline{J} = \rho_v\, \underline{v}$$

$$\underline{J} = \sigma \underline{E}$$

$$\underline{F} = q\underline{E}$$

$$V_{AB} = \int_{\underline{A}}^{\underline{B}} \underline{E} \cdot d\underline{r}$$

$$\mathrm{PE}\left(\underline{r}\right) = q\,\Phi\left(\underline{r}\right) + C$$

$$\underline{E} = \int_V \frac{\rho_v}{4\pi\varepsilon_0 R^2}\,\hat{\underline{R}}\,dV'$$

$$\underline{E} = \int_S \frac{\rho_s}{4\pi\varepsilon_0 R^2}\,\hat{\underline{R}}\,dS'$$

$$\underline{E} = \int_V \frac{\rho_l}{4\pi\varepsilon_0 R^2}\,\hat{\underline{R}}\,dl'$$

$$\underline{R} = \underline{r} - \underline{r}' = \hat{\underline{x}}\left(x - x'\right) + \hat{\underline{y}}\left(y - y'\right) + \underline{z}\left(z - z'\right)$$

$$R = \left|\underline{R}\right|$$

$$\oint_S \underline{D} \cdot \hat{\underline{n}}\,dS = Q_{\text{encl}}$$

$$\nabla \cdot \underline{D} = \rho_v$$

$$\psi \equiv \int_S \underline{D} \cdot \hat{\underline{n}}\ dS$$

$$\psi_l \equiv \int_C \underline{D} \cdot \hat{\underline{n}}\ dl$$

$$\psi \propto N_S$$

$$\psi_l \propto N_C$$

For PEC:

$\underline{E} = \underline{0}$ (inside)

$\Phi =$ constant (inside and on surface)

$\underline{E} = \underline{0}$ (inside)

$\rho_v = 0$ (inside)

TABLE OF INTEGRALS

$$\int \frac{dx}{\left(x^2+a^2\right)^{1/2}} = \ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int \frac{dx}{\left(x^2+a^2\right)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$

$$\int \frac{x\,dx}{\left(x^2+a^2\right)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}}$$

$$\int \frac{x\,dx}{\left(x^2+a^2\right)^{1/2}} = \sqrt{x^2+a^2}$$

$$\int \frac{x^2\,dx}{\left(x^2+a^2\right)^{3/2}} = \frac{-x}{\sqrt{x^2+a^2}} + \ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int \frac{x^2\,dx}{\left(x^2+a^2\right)^{1/2}} = \frac{x\sqrt{x^2+a^2}}{2} - \frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int x\left(x^2+a^2\right)^{3/2}dx = \frac{1}{5}\left(x^2+a^2\right)^{5/2}$$

$$\int \left(x^2+a^2\right)^{3/2}dx = \frac{x\left(x^2+a^2\right)^{3/2}}{4} + \frac{3a^2x\sqrt{x^2+a^2}}{8} + \frac{3}{8}a^4\ln\left(x+\sqrt{x^2+a^2}\right)$$

TABLE OF COORDINATE SYSTEM FORMULAS

$$x = \rho \cos\phi$$
$$y = \rho \sin\phi$$
$$z = z$$

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$\rho = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}\left(y/x\right)$$
$$z = z$$

$$\rho = r\sin\theta$$
$$z = r\cos\theta$$
$$\phi = \phi$$

$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \cos^{-1}\left(z/\sqrt{x^2 + y^2 + z^2}\right)$$
$$\phi = \tan^{-1}\left(y/x\right)$$

$$r = \sqrt{\rho^2 + z^2}$$
$$\theta = \tan^{-1}\left(\rho/z\right)$$
$$\phi = \phi$$

$$\underline{\hat{r}} \cdot \underline{\hat{x}} = \sin\theta\cos\phi = x/r$$
$$\underline{\hat{r}} \cdot \underline{\hat{y}} = \sin\theta\sin\phi = y/r$$
$$\underline{\hat{r}} \cdot \underline{\hat{z}} = \cos\theta = z/r$$

$$\underline{\hat{\rho}} \cdot \underline{\hat{x}} = \cos\phi$$
$$\underline{\hat{\phi}} \cdot \underline{\hat{x}} = -\sin\phi$$

$$\underline{\hat{\rho}} = \underline{\hat{x}}\cos\phi + \underline{\hat{y}}\sin\phi$$
$$\underline{\hat{\phi}} = \underline{\hat{x}}\left(-\sin\phi\right) + \underline{\hat{y}}\cos\phi$$
$$\underline{\hat{z}} = \underline{\hat{z}}$$

$$\underline{\hat{x}} = \underline{\hat{\rho}}\cos\phi + \underline{\hat{\phi}}\left(-\sin\phi\right)$$
$$\underline{\hat{y}} = \underline{\hat{\rho}}\sin\phi + \underline{\hat{\phi}}\cos\phi$$
$$\underline{\hat{z}} = \underline{\hat{z}}$$

$$\underline{\hat{r}} = \underline{\hat{x}}\sin\theta\cos\phi + \underline{\hat{y}}\sin\theta\sin\phi + \underline{\hat{z}}\cos\theta$$
$$\underline{\hat{\theta}} = \underline{\hat{x}}\cos\theta\cos\phi + \underline{\hat{y}}\cos\theta\sin\phi + \underline{\hat{z}}\left(-\sin\theta\right)$$
$$\underline{\hat{\phi}} = \underline{\hat{x}}\left(-\sin\phi\right) + \underline{\hat{y}}\cos\phi$$

$$\underline{\hat{x}} = \underline{\hat{r}}\sin\theta\cos\phi + \underline{\hat{\theta}}\cos\theta\cos\phi + \underline{\hat{\phi}}\left(-\sin\phi\right)$$
$$\underline{\hat{y}} = \underline{\hat{r}}\sin\theta\sin\phi + \underline{\hat{\theta}}\cos\theta\sin\phi + \underline{\hat{\phi}}\cos\phi$$
$$\underline{\hat{z}} = \underline{\hat{r}}\cos\theta + \underline{\hat{\theta}}\left(-\sin\theta\right)$$