FORMULA SHEET

$$dS = dx\,dy$$
$$dV = dx\,dy\,dz$$

$$dS = \rho\,d\phi\,dz$$
$$dV = \rho\,d\rho\,d\phi\,dz$$

$$dS = r^2 \sin\theta\,d\theta\,d\phi$$
$$dV = r^2 \sin\theta\,dr\,d\theta\,d\phi$$

$$d\underline{r} = \underline{\hat{z}}\,dz + \underline{\hat{\rho}}\,d\rho + \underline{\hat{\phi}}\,\rho\,d\phi$$
$$d\underline{r} = \underline{\hat{r}}\,dr + \underline{\hat{\phi}}\,r\sin\theta\,d\phi + \underline{\hat{\theta}}\,r\,d\theta$$

$$\nabla\cdot\underline{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\nabla\cdot\underline{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho D_\rho\right) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$$

$$\nabla\cdot\underline{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(D_\theta\sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$$

$$\nabla\times\underline{V} = \underline{\hat{x}}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - \underline{\hat{y}}\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right) + \underline{\hat{z}}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

$$\nabla\times\underline{V} = \underline{\hat{\rho}}\left(\frac{1}{\rho}\frac{\partial V_z}{\partial\phi} - \frac{\partial V_\phi}{\partial z}\right) + \underline{\hat{\phi}}\left(\frac{\partial V_\rho}{\partial z} - \frac{\partial V_z}{\partial\rho}\right) + \underline{\hat{z}}\,\frac{1}{\rho}\left(\frac{\partial\left(\rho V_\phi\right)}{\partial\rho} - \frac{\partial V_\rho}{\partial\phi}\right)$$

$$\nabla\times\underline{V} = \underline{\hat{r}}\,\frac{1}{r\sin\theta}\left[\frac{\partial\left(V_\phi\sin\theta\right)}{\partial\theta} - \frac{\partial V_\theta}{\partial\phi}\right] + \underline{\hat{\theta}}\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial V_r}{\partial\phi} - \frac{\partial\left(rV_\phi\right)}{\partial r}\right] + \underline{\hat{\phi}}\frac{1}{r}\left[\frac{\partial\left(rV_\theta\right)}{\partial r} - \frac{\partial V_r}{\partial\theta}\right]$$

$$\nabla\Phi \equiv \underline{\hat{x}}\ \frac{\partial\Phi}{\partial x} + \underline{\hat{y}}\ \frac{\partial\Phi}{\partial y} + \underline{\hat{z}}\ \frac{\partial\Phi}{\partial z}$$

$$\nabla\Phi = \underline{\hat{\rho}}\frac{\partial\Phi}{\partial\rho} + \underline{\hat{\phi}}\frac{1}{\rho}\frac{\partial\Phi}{\partial\phi} + \underline{\hat{z}}\frac{\partial\Phi}{\partial z}$$

$$\nabla\Phi = \underline{\hat{r}}\ \frac{\partial\Phi}{\partial r} + \underline{\hat{\theta}}\frac{1}{r}\frac{\partial\Phi}{\partial\theta} + \underline{\hat{\phi}}\frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}$$

$$\nabla^2\Phi = \left(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2}\right)$$

$$\nabla^2\Phi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2}$$

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$$

$$\int_V \nabla\cdot\underline{V}\,dV = \oint_S \underline{V}\cdot\underline{\hat{n}}\ dS$$

$$\int_S \left(\nabla\times\underline{V}\right)\cdot\underline{\hat{n}}\,dS = \oint_C \underline{V}\cdot\underline{dr}$$

$$\nabla\times\underline{E} = -\frac{\partial\underline{B}}{\partial t} \qquad \oint_C \underline{E}\cdot dr = \int_S\left(-\frac{\partial\underline{B}}{\partial t}\right)\cdot\underline{\hat{n}}\,dS$$

$$\nabla\times\underline{H} = \underline{J} + \frac{\partial\underline{D}}{\partial t} \qquad \oint_C \underline{H}\cdot dr = i_{\text{encl}} + \int_S\left(-\frac{\partial\underline{D}}{\partial t}\right)\cdot\underline{\hat{n}}\,dS$$

$$\nabla\cdot\underline{D} = \rho_v \qquad \oint_S \underline{D}\cdot\underline{\hat{n}}\,dS = Q_{\text{encl}}$$

$$\nabla\cdot\underline{B} = 0 \qquad \oint_S \underline{B}\cdot\underline{\hat{n}}\,dS = 0$$

$$i = \frac{dq}{dt}$$

$$i = \int_S \underline{J} \cdot \underline{\hat{n}}\ dS$$

$$\underline{J} = \rho_v\, \underline{v}$$

$$\underline{J} = \sigma \underline{E}$$

$$\underline{F} = q\underline{E}$$

$$V_{AB} = \int_{\underline{A}}^{\underline{B}} \underline{E} \cdot d\underline{r}$$

$$\mathrm{PE}\left(\underline{r}\right) = q\,\Phi\left(\underline{r}\right) + C$$

$$\underline{E} = \int_V \frac{\rho_v}{4\pi\varepsilon_0 R^2}\, \underline{\hat{R}}\, dV'$$

$$\underline{E} = \int_S \frac{\rho_s}{4\pi\varepsilon_0 R^2}\, \underline{\hat{R}}\, dS'$$

$$\underline{E} = \int_V \frac{\rho_l}{4\pi\varepsilon_0 R^2}\, \underline{\hat{R}}\, dl'$$

$$\underline{R} = \underline{r} - \underline{r}' = \underline{\hat{x}}\left(x - x'\right) + \underline{\hat{y}}\left(y - y'\right) + \underline{z}\left(z - z'\right)$$

$$R = \left|\underline{R}\right|$$

$$\underline{\hat{R}} = \frac{\underline{R}}{R}$$

$$\oint_S \underline{D} \cdot \underline{\hat{n}}\, dS = Q_{\mathrm{encl}}$$

$$\psi \equiv \int_S \underline{D} \cdot \underline{\hat{n}}\ dS$$

$$\psi_l \equiv \int_C \underline{D} \cdot \underline{\hat{n}}\ dl$$

$$\psi \propto N_S$$

$$\psi_l \propto N_C$$

$$\Phi(\underline{r}) = \Phi(\underline{R}) - \int_{\underline{R}}^{r} \underline{E} \cdot d\underline{r}$$

$$\mathrm{PE}(\underline{r}) = q\Phi(\underline{r}) + C$$

$$\Phi = \int_V \frac{\rho_v}{4\pi\varepsilon_0 R} dV' \quad (\Phi(\infty) = 0)$$

$$\Phi = \int_S \frac{\rho_s}{4\pi\varepsilon_0 R} dS' \quad (\Phi(\infty) = 0)$$

$$\Phi = \int_C \frac{\rho_l}{4\pi\varepsilon_0 R} dl' \quad (\Phi(\infty) = 0)$$

$$\underline{E} = -\nabla\Phi$$

$$\nabla^2\Phi = -\frac{\rho_v}{\varepsilon}$$

$$\nabla^2\Phi = 0 \quad (\rho_v = 0)$$

Faraday's law: $\oint_C \underline{E} \cdot dr = -\frac{d\psi}{dt}$ (S is stationary)

$$\psi = \int_S \underline{B} \cdot \underline{\hat{n}}\, dS \quad \left(\underline{\hat{n}} \text{ from right - hand rule}\right)$$

$$v(t) = \frac{d\psi_z}{dt}$$

$$\psi_z \equiv \int_S B_z dS$$

$$\underline{D} \equiv \varepsilon_0 \underline{E} + \underline{P}$$

$$\underline{P} = \varepsilon_0 \chi_e \underline{E}$$

$$\varepsilon_r \equiv 1 + \chi_e$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$\underline{D} = \varepsilon \underline{E}$$

$$\underline{D} \cdot \underline{\hat{n}} = \rho_s$$

$$\underline{\hat{n}} \cdot \left(\underline{D}_1 - \underline{D}_2\right) = \rho_s$$

$$\underline{E}_{t1} = \underline{E}_{t2}$$

$$\Phi = \frac{\rho_{\ell 0}}{2\pi\varepsilon_0} \ln\left(\frac{b}{\rho}\right) \quad (\text{line charge})$$

$$\rho_{\ell 0} = \frac{2\pi\varepsilon_0 V_0}{\ln\left(\frac{2h-a}{a}\right)}$$

$$q' = -q\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)$$

$$q'' = q\left(\frac{2\varepsilon_r}{\varepsilon_r + 1}\right)$$

$$C = \frac{Q}{V} = \frac{Q_A}{V_{AB}}$$

$$C = \frac{2U_E}{V^2}$$

$$U_E = \frac{1}{2}\int_V \underline{D}\cdot\underline{E}\,dV$$

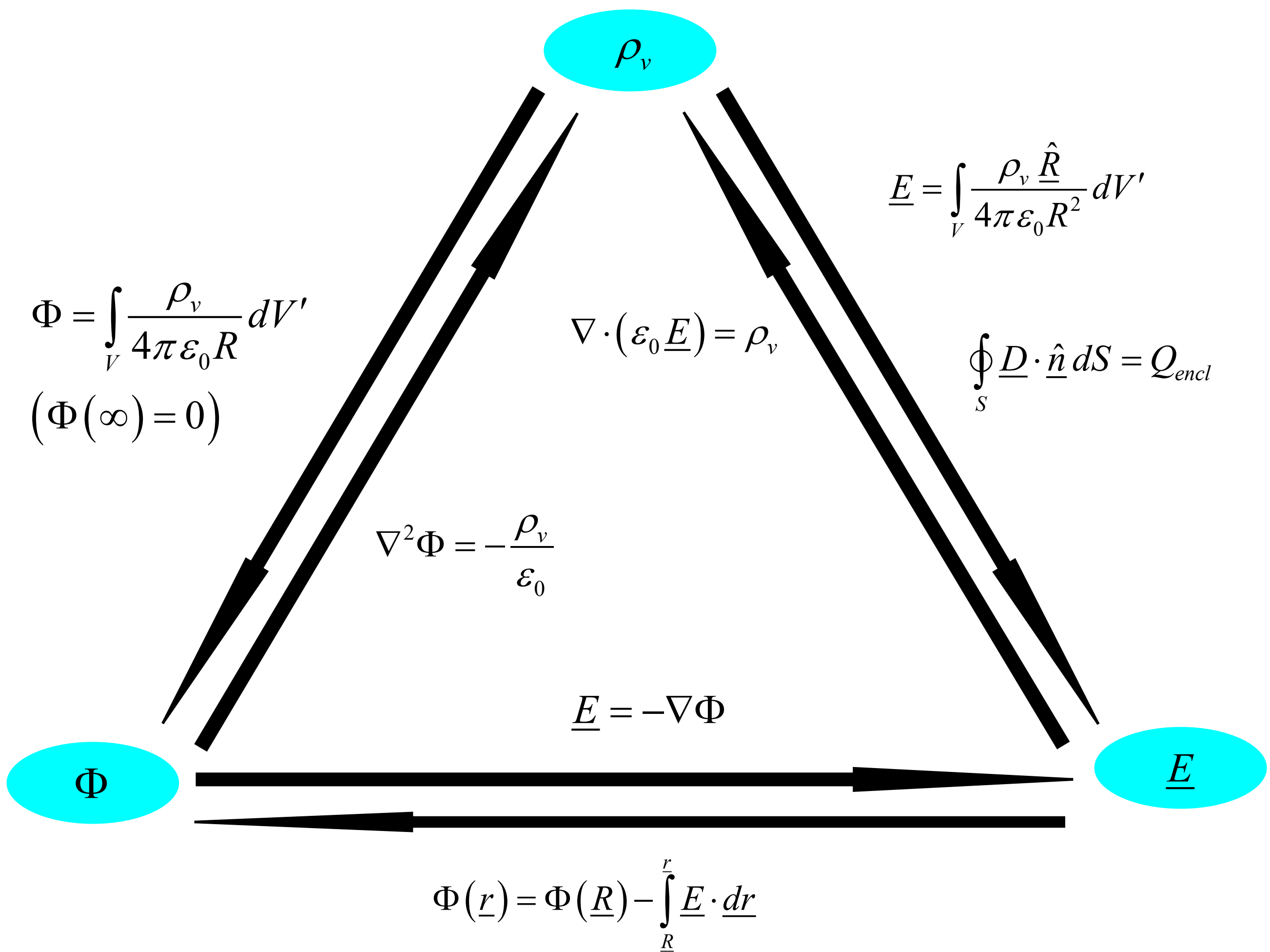
$$U_E = \frac{1}{2}\int_V \rho_v \Phi\,dV \quad \left(\Phi\left(\underline{\infty}\right) = 0\right)$$

$$U_E = \frac{1}{2}\int_S \rho_s \Phi\,dS \quad \left(\Phi\left(\underline{\infty}\right) = 0\right)$$

$$U_E = \frac{1}{2}CV^2$$

$$U_E = \frac{1}{2}QV$$

Electrostatic Triangle



$$R = \frac{L}{\sigma A}$$

$$P_d = \int_V \underline{J} \cdot \underline{E}\, dV = \int_V \sigma \left|\underline{E}\right|^2 dV$$

$$RC = \frac{\varepsilon}{\sigma}$$

$$\varepsilon \to \sigma,\ C \to G$$

$$\int_C \underline{H} \cdot \underline{dr} = I_{\text{encl}}$$

$$\underline{H} = \underline{\hat{z}}\left(nI\right)$$

$$\underline{H} = \int_C \frac{I\, \underline{d\ell}' \times \underline{\hat{R}}}{4\pi R^2}$$

$$\underline{B} = \mu \underline{H} = \mu_0 \mu_r \underline{H}$$

$$\underline{\hat{n}} \times \left(\underline{H}_1 - \underline{H}_2\right) = \underline{J}_s$$

$$\underline{\hat{n}} \cdot \left(\underline{B}_1 - \underline{B}_2\right) = 0$$

$$L \equiv \frac{\Lambda}{I} = \frac{N\psi}{I}$$

$$\psi = \int_S \underline{B} \cdot \underline{\hat{n}}\, dS$$

$$v = L\frac{di}{dt}$$

$$U_H = \frac{1}{2}\int_V \underline{B} \cdot \underline{H}\ dV$$

$$U_H = \frac{1}{2} L I^2$$

$$L = \frac{2U_H}{I^2}$$

$$\underline{F} = q\left(\underline{E} + \underline{v} \times \underline{B}\right)$$

$$\underline{F} = \int_C I \, \underline{d\ell} \times \underline{B}$$

$$\underline{T} = \underline{m} \times \underline{B}$$

$$\underline{m} = \underline{\hat{n}}\left(AI\right)N$$

$$M_{12} \equiv \frac{N_1 \psi_{12}}{I_2}$$

$$M_{21} \equiv \frac{N_2 \psi_{21}}{I_1}$$

$$\psi_{12} = \int_{S_1} \underline{B}_2 \cdot \underline{\hat{n}}_1 \, dS$$

$$\psi_{21} = \int_{S_2} \underline{B}_1 \cdot \underline{\hat{n}}_2 \, dS$$

$$M_{12} = M_{21} = M$$

$$R_m = \frac{L}{\mu_0 \mu_r A}$$

$$I_m = \psi$$

$$V_m = NI$$

TABLE OF INTEGRALS

$$\int \frac{dx}{\left(x^2+a^2\right)^{1/2}} = \ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int \frac{dx}{\left(x^2+a^2\right)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$

$$\int \frac{x\,dx}{\left(x^2+a^2\right)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}}$$

$$\int \frac{x\,dx}{\left(x^2+a^2\right)^{1/2}} = \sqrt{x^2+a^2}$$

$$\int \frac{x^2\,dx}{\left(x^2+a^2\right)^{3/2}} = \frac{-x}{\sqrt{x^2+a^2}} + \ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int \frac{x^2\,dx}{\left(x^2+a^2\right)^{1/2}} = \frac{x\sqrt{x^2+a^2}}{2} - \frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int x\left(x^2+a^2\right)^{3/2}dx = \frac{1}{5}\left(x^2+a^2\right)^{5/2}$$

$$\int \left(x^2+a^2\right)^{3/2}dx = \frac{x\left(x^2+a^2\right)^{3/2}}{4} + \frac{3a^2x\sqrt{x^2+a^2}}{8} + \frac{3}{8}a^4\ln\left(x+\sqrt{x^2+a^2}\right)$$

TABLE OF COORDINATE SYSTEM FORMULAS

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

$$\underline{\hat{r}} \cdot \underline{\hat{x}} = \sin\theta\cos\phi = x / r$$
$$\underline{\hat{r}} \cdot \underline{\hat{y}} = \sin\theta\sin\phi = y / r$$
$$\underline{\hat{r}} \cdot \underline{\hat{z}} = \cos\theta = z / r$$

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$\underline{\hat{\rho}} \cdot \underline{\hat{x}} = \cos\phi$$
$$\underline{\hat{\phi}} \cdot \underline{\hat{x}} = -\sin\phi$$

$$\rho = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}\left(y / x\right)$$
$$z = z$$

$$\rho = r\sin\theta$$
$$z = r\cos\theta$$
$$\phi = \phi$$

$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \cos^{-1}\left(z / \sqrt{x^2 + y^2 + z^2}\right)$$
$$\phi = \tan^{-1}\left(y / x\right)$$

$$r = \sqrt{\rho^2 + z^2}$$
$$\theta = \tan^{-1}\left(\rho / z\right)$$
$$\phi = \phi$$