

ECE 5317-6351

Microwave Engineering

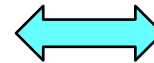
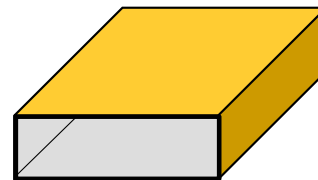
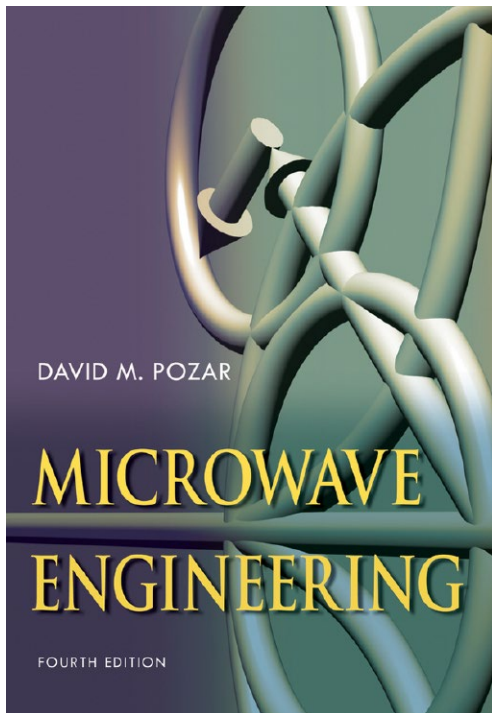
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Fall 2026

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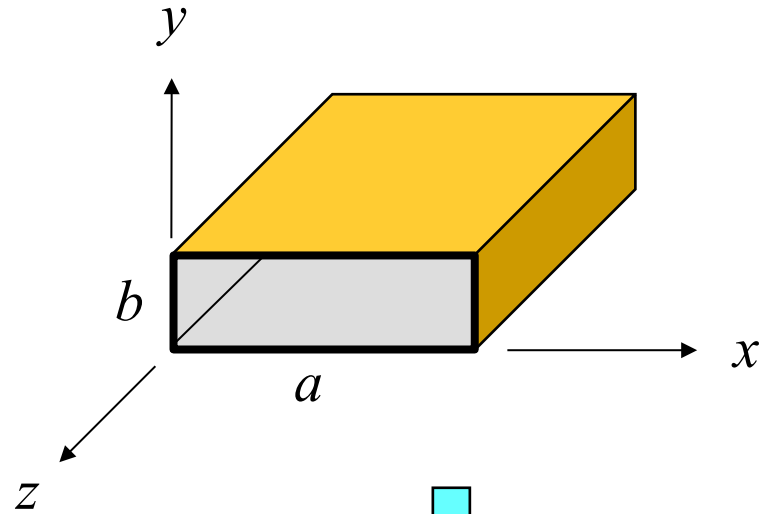
Notes 11

Waveguiding Structures Part 6: Transverse Equivalent Network (TEN)



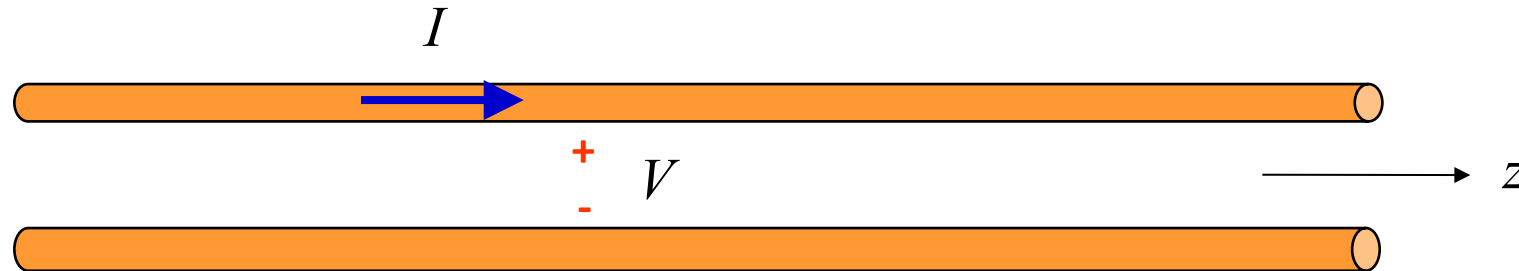
Waveguide Transmission Line Model

Our goal is to come up with a transmission line model for a waveguide mode.



Important point:

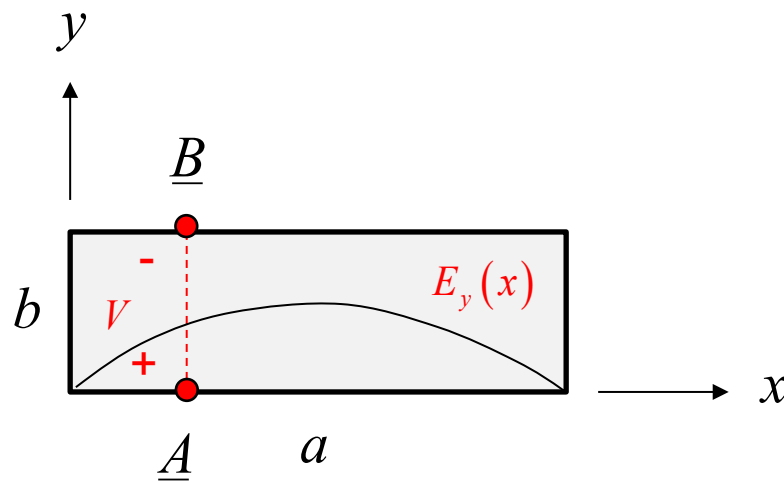
The waveguide mode is not a TEM mode, but it can be modeled as a wave on a transmission line.



“Transverse Equivalent Network” (TEN) model of TE_{10} waveguide mode

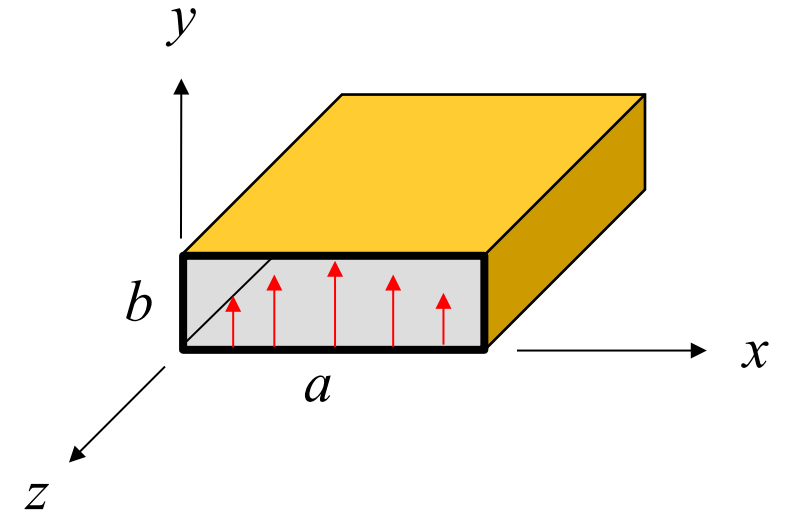
Waveguide Transmission Line Model (cont.)

For a waveguide mode, voltage and current are not uniquely defined.



TE₁₀ Mode

$$\underline{E} = \underline{\hat{y}} E_{10} \sin\left(\frac{\pi}{a} x\right) e^{-jk_z z}$$



$$V(x, z) = V_{AB}(x, z) = \int_A^B \underline{E} \cdot d\underline{r} = \int_0^b E_y dy = E_{10} b \sin\left(\frac{\pi}{a} x\right) e^{-jk_z z} = V_0 \sin\left(\frac{\pi}{a} x\right) e^{-jk_z z}$$

The voltage depends on x ! $V_0 = E_{10} b = V(a/2, 0)$

Waveguide Transmission Line Model (cont.)

Examine the transverse (x, y) fields of a waveguide mode:

Modal amplitudes

$$\underline{E}_t(x, y, z) = \underline{e}_t(x, y) \left(\textcolor{red}{A}^+ e^{-jk_z z} + \textcolor{red}{A}^- e^{+jk_z z} \right)$$

$$\underline{H}_t(x, y, z) = \underline{h}_t(x, y) \left(\textcolor{red}{A}^+ e^{-jk_z z} - \textcolor{red}{A}^- e^{+jk_z z} \right)$$

$$\underline{h}_t(x, y) = \frac{1}{Z_w} (\hat{\underline{z}} \times \underline{e}_t)$$

Note:

The shape function $\underline{e}_t(x, y)$ has an arbitrary amplitude normalization (usually chosen to be as simple as possible).

Example : TE₁₀ mode, choose $\underline{e}_t = \hat{y} \sin\left(\frac{\pi}{a}x\right)$

Note: The minus sign above arises from:

$$\underline{H}_t^\pm = \pm \frac{1}{Z_w} (\hat{\underline{z}} \times \underline{E}_t^\pm)$$



Wave impedance

$$Z_w = Z_{\text{TE}} \text{ or } Z_{\text{TM}}$$

$$Z_{\text{TE}} \equiv \frac{\omega\mu}{k_z} \quad Z_{\text{TM}} = \frac{k_z}{\omega\epsilon_c}$$

Waveguide Transmission Line Model (cont.)

Introduce a **defined voltage** into the field equations:

$$V_0^\pm \equiv C_1 A^\pm \quad (\text{The units of } C_1 \text{ are meters.})$$

We may use whatever definition of voltage we wish here. (In other words, C_1 is arbitrary.)

Examples (two common choices):

- Pick C_1 so that voltage V_0 is the actual voltage drop along the center of the waveguide.
- Pick C_1 so that the power flowing on the TEN is the actual power flowing in the waveguide*

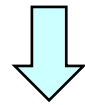
*This is done later.

Waveguide Transmission Line Model (cont.)

We then have:

$$\underline{E}_t(x, y, z) = \underline{e}_t(x, y) \left(\mathbf{A}^+ e^{-jk_z z} + \mathbf{A}^- e^{+jk_z z} \right)$$

$$\underline{H}_t(x, y, z) = \underline{h}_t(x, y) \left(\mathbf{A}^+ e^{-jk_z z} - \mathbf{A}^- e^{+jk_z z} \right)$$



$$V_0^\pm \equiv C_1 A^\pm$$

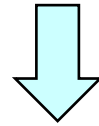
$$\underline{E}_t(x, y, z) = \frac{1}{C_1} \underline{e}_t(x, y) \left(\mathbf{V}_0^+ e^{-jk_z z} + \mathbf{V}_0^- e^{+jk_z z} \right)$$

$$\underline{H}_t(x, y, z) = \frac{1}{C_1} \underline{h}_t(x, y) \left(\mathbf{V}_0^+ e^{-jk_z z} - \mathbf{V}_0^- e^{+jk_z z} \right)$$

Waveguide Transmission Line Model (cont.)

Next, introduce a **characteristic impedance** (having an arbitrary value) into the equation for H_t :

$$\underline{H}_t(x, y, z) = \frac{Z_0}{C_1} \underline{h}_t(x, y) \left(\frac{V_0^+}{Z_0} e^{-jk_z z} - \frac{V_0^-}{Z_0} e^{+jk_z z} \right)$$



$$\underline{H}_t(x, y, z) = \frac{1}{C_2} \underline{h}_t(x, y) \left(\frac{V_0^+}{Z_0} e^{-jk_z z} - \frac{V_0^-}{Z_0} e^{+jk_z z} \right)$$

where

$$C_2 \equiv \frac{C_1}{Z_0}$$

Waveguide Transmission Line Model (cont.)

Summary of Fields

$$\underline{E}_t(x, y, z) = \frac{1}{C_1} \underline{e}_t(x, y) \overbrace{\left(V_0^+ e^{-jk_z z} + V_0^- e^{+jk_z z} \right)}^{V(z)}$$

$$\underline{H}_t(x, y, z) = \frac{1}{C_2} \underline{h}_t(x, y) \overbrace{\left(\frac{V_0^+}{Z_0} e^{-jk_z z} - \frac{V_0^-}{Z_0} e^{+jk_z z} \right)}^{I(z)}$$

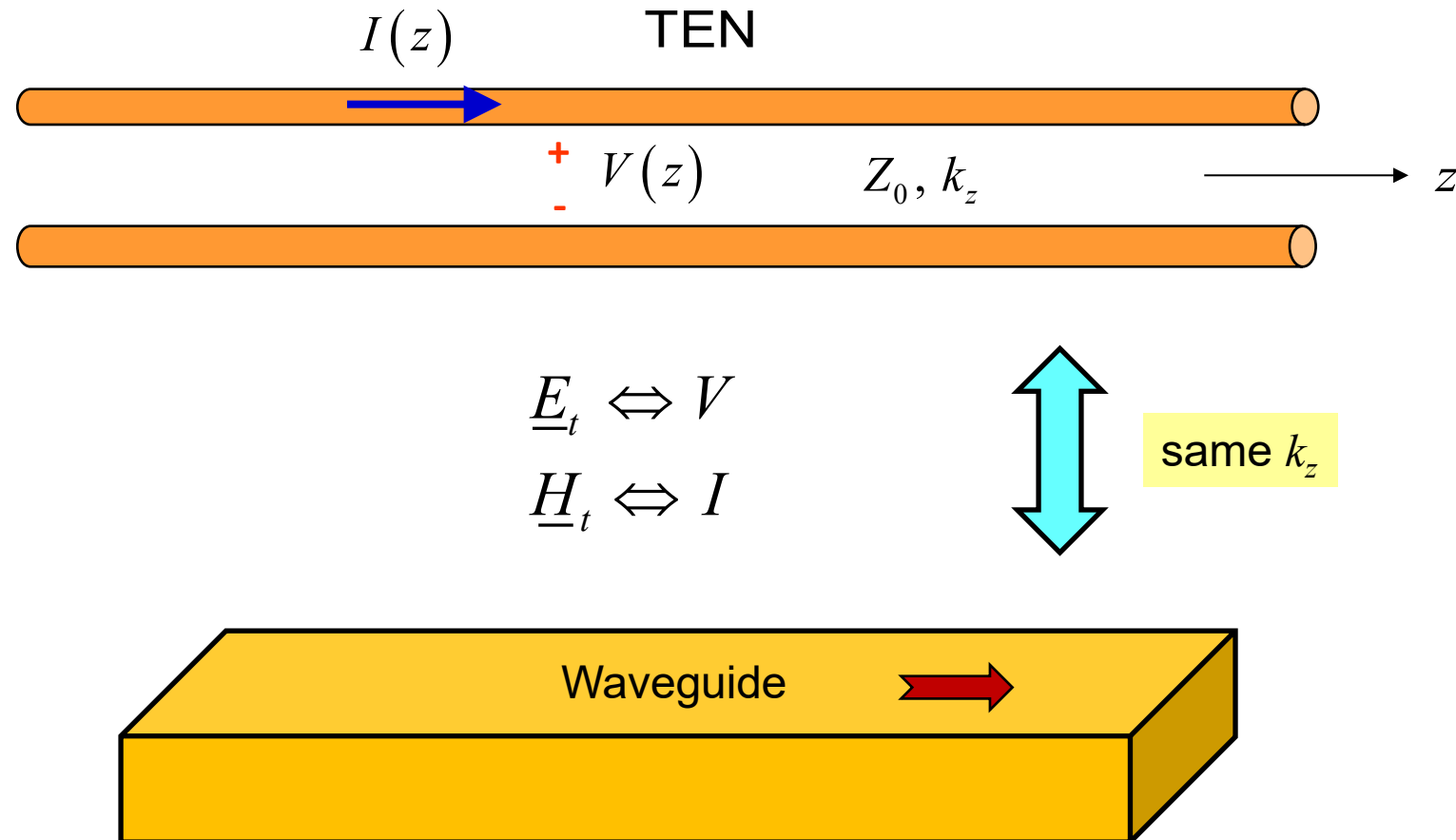
$$C_2 \equiv \frac{C_1}{Z_0}$$

The z dependence of the transverse fields behaves like voltage and current on a transmission line.

$k_z = k_z$ of waveguide mode

Waveguide Transmission Line Model (cont.)

The transmission-line model is called the Transverse Equivalent Network (TEN) model of the waveguide.



Waveguide Transmission Line Model (cont.)

Power flow down the waveguide (complex power):

$$\begin{aligned} P^{\text{WG}}(z) &= \frac{1}{2} \int_S (\underline{E}_t \times \underline{H}_t^*) \cdot \hat{z} dS \\ &= \frac{1}{2} V(z) I^*(z) \left[\frac{1}{C_1 C_2^*} \int_S (\underline{e}_t(x, y) \times \underline{h}_t^*(x, y)) \cdot \hat{z} dS \right] \end{aligned}$$

$$\begin{aligned} \underline{E}_t(x, y, z) &= \frac{1}{C_1} \underline{e}_t(x, y) V(z) \\ \underline{H}_t(x, y, z) &= \frac{1}{C_2} \underline{h}_t(x, y) I(z) \end{aligned}$$

$$P^{\text{WG}}(z) = P^{\text{TEN}}(z) \left[\frac{1}{C_1 C_2^*} \int_S (\underline{e}_t(x, y) \times \underline{h}_t^*(x, y)) \cdot \hat{z} dS \right]$$

Complex power flowing down the TEN transmission line.

Waveguide Transmission Line Model (cont.)

Assume we choose to have:

$$P^{\text{WG}}(z) = P^{\text{TEN}}(z)$$

Then we have the following constraint:

$$C_1 C_2^* = \int_S \left(\underline{e}_t(x, y) \times \underline{h}_t^*(x, y) \right) \cdot \underline{\hat{z}} dS$$

It is not necessary to make this assumption of equal powers, but it is a useful choice that can be made.

Waveguide Transmission Line Model (cont.)

Summary of Constants (assuming equal powers):

$$\frac{C_1}{C_2} = Z_0$$

$$C_1 C_2^* = \int_S \left(\underline{e}_t(x, y) \times \underline{h}_t^*(x, y) \right) \cdot \underline{\hat{z}} dS$$

Once we pick Z_0 , the constants are then determined.

The most common choice: $Z_0 = Z_w$

Waveguide Transmission Line Model (cont.)

Summary

$$\underline{E}_t(x, y, z) = \frac{1}{C_1} \underline{e}_t(x, y) \overbrace{\left(V_0^+ e^{-jk_z z} + V_0^- e^{+jk_z z} \right)}^{V(z)}$$

$$\underline{H}_t(x, y, z) = \frac{1}{C_2} \underline{h}_t(x, y) \overbrace{\left(\frac{V_0^+}{Z_0} e^{-jk_z z} - \frac{V_0^-}{Z_0} e^{+jk_z z} \right)}^{I(z)}$$

$$\frac{C_1}{C_2} = Z_0 \quad Z_0 = Z_w \quad (\text{most common choice})$$

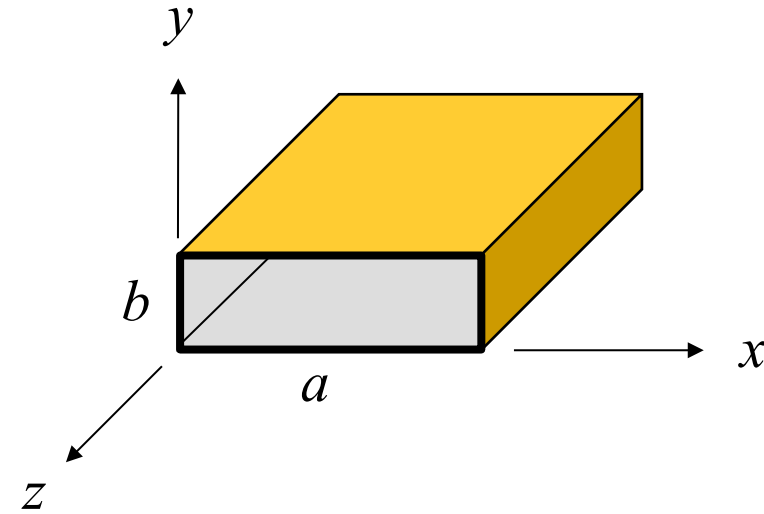
$$C_1 C_2^* = \int_S \left(\underline{e}_t(x, y) \times \underline{h}_t^*(x, y) \right) \cdot \hat{\underline{z}} dS \quad (\text{for equal power})$$

$$\underline{h}_t(x, y) = \frac{1}{Z_w} (\hat{\underline{z}} \times \underline{e}_t) \quad Z_w = Z_{\text{TE}} \text{ or } Z_{\text{TM}} \quad Z_{\text{TE}} \equiv \frac{\omega \mu}{k_z} \quad Z_{\text{TM}} = \frac{k_z}{\omega \epsilon_c}$$

TE₁₀ Mode of Rectangular Waveguide

We make the following choices:

- Choose $Z_0 = Z_{\text{TE}}$
- Assume power equality



$$\frac{C_1}{C_2} = Z_{\text{TE}}$$

$$C_1 C_2^* = \int_S \left(\underline{e}_t(x, y) \times \underline{h}_t^*(x, y) \right) \cdot \underline{\hat{z}} dS$$

$$Z_{\text{TE}} = Z_{\text{TE}}^{10} = \frac{\omega \mu_0}{k_z^{10}}$$

$$k_z^{10} = \sqrt{k^2 - \left(\frac{\pi}{a} \right)^2}$$

$$k = \omega \sqrt{\mu \epsilon_c}$$

$$\epsilon_c = \epsilon_r (1 - j \tan \delta) \quad (\epsilon_r \text{ denotes } \epsilon_r')$$

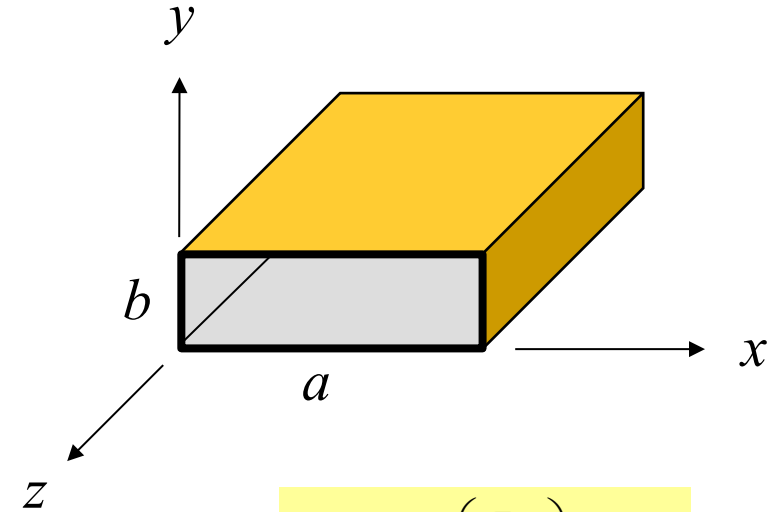
TE₁₀ Mode (cont.)

Calculate this term:

$$C_1 C_2^* = \int_S \left(\underline{e}_t(x, y) \times \underline{h}_t^*(x, y) \right) \cdot \hat{\underline{z}} dS$$

We have:

$$\begin{aligned} C_1 C_2^* &= \int_S \frac{1}{Z_{\text{TE}}^*} \sin^2 \left(\frac{\pi x}{a} \right) dS \\ &= \frac{1}{Z_{\text{TE}}^*} \int_0^a \int_0^b \sin^2 \left(\frac{\pi x}{a} \right) dy dx \\ &= \frac{1}{Z_{\text{TE}}^*} \left(\frac{ab}{2} \right) \end{aligned}$$



$$\begin{aligned} \underline{e}_t &= \hat{y} \sin \left(\frac{\pi}{a} x \right) \\ \underline{h}_t &= -\hat{x} \frac{1}{Z_{\text{TE}}} \sin \left(\frac{\pi}{a} x \right) \\ Z_0 &= Z_{\text{TE}} = \frac{\omega \mu}{k_z^{10}} \\ k_z^{10} &= \sqrt{k^2 - \left(\frac{\pi}{a} \right)^2} \end{aligned}$$

Note:

The transverse shape function $\underline{e}_t$ has been chosen (arbitrarily) to have a unit amplitude at the center of the waveguide, to keep it as simple as possible.

TE₁₀ Mode (cont.)

We then have:

$$C_1 C_2^* = \frac{1}{Z_{\text{TE}}^*} \left(\frac{ab}{2} \right)$$

$$\frac{C_1}{C_2} = Z_{\text{TE}}$$

Take the conjugate of the second one and then multiply the two equations together.

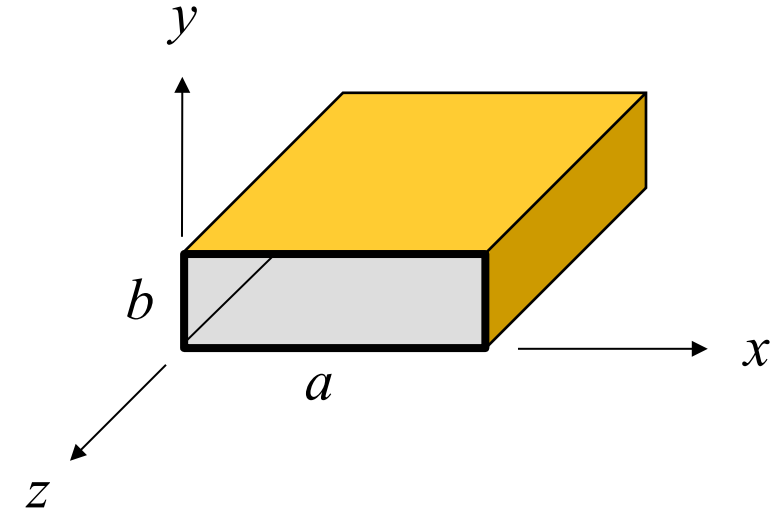
$$\Rightarrow |C_1|^2 = \frac{ab}{2}$$

Note:

The solution is unique to within a phase term (we choose the phase to be zero here).

Solution:

$$C_1 = \sqrt{\frac{ab}{2}}$$
$$C_2 = \frac{1}{Z_{\text{TE}}} \sqrt{\frac{ab}{2}}$$



$$\underline{e}_t = \underline{\hat{y}} \sin\left(\frac{\pi}{a}x\right)$$
$$\underline{h}_t = -\underline{\hat{x}} \frac{1}{Z_{\text{TE}}} \sin\left(\frac{\pi}{a}x\right)$$
$$Z_0 = Z_{\text{TE}} = \frac{\omega\mu}{k_z^{10}}$$
$$k_z^{10} = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}$$

TE₁₀ Mode (cont.)

Recall:

$$\underline{E}_t(x, y, z) = \frac{1}{C_1} \underline{e}_t(x, y) \overbrace{\left(V_0^+ e^{-jk_z z} + V_0^- e^{+jk_z z} \right)}^{V(z)}$$

$$\underline{H}_t(x, y, z) = \frac{1}{C_2} \underline{h}_t(x, y) \overbrace{\left(\frac{V_0^+}{Z_0} e^{-jk_z z} - \frac{V_0^-}{Z_0} e^{+jk_z z} \right)}^{I(z)}$$



Hence, we have:

$$\underline{E}_t(x, y, z) = \sqrt{\frac{2}{ab}} \left(\underline{\hat{y}} \sin\left(\frac{\pi}{a} x\right) \right) \overbrace{\left(V_0^+ e^{-jk_z z} + V_0^- e^{+jk_z z} \right)}^{V(z)}$$

$$\underline{H}_t(x, y, z) = \cancel{Z_{\text{TE}}} \sqrt{\frac{2}{ab}} \left(-\underline{\hat{x}} \frac{1}{\cancel{Z_{\text{TE}}}} \sin\left(\frac{\pi}{a} x\right) \right) \overbrace{\left(\frac{V_0^+}{Z_{\text{TE}}} e^{-jk_z z} - \frac{V_0^-}{Z_{\text{TE}}} e^{+jk_z z} \right)}^{I(z)}$$

$$C_1 = \sqrt{\frac{ab}{2}}$$

$$C_2 = \frac{1}{Z_{\text{TE}}} \sqrt{\frac{ab}{2}}$$

$$\underline{e}_t = \underline{\hat{y}} \sin\left(\frac{\pi}{a} x\right)$$

$$\underline{h}_t = -\underline{\hat{x}} \frac{1}{Z_{\text{TE}}} \sin\left(\frac{\pi}{a} x\right)$$

$$Z_0 = Z_{\text{TE}} = \frac{\omega\mu}{k_z^{10}}$$

$$k_z^{10} = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}$$

TE₁₀ Mode (cont.)

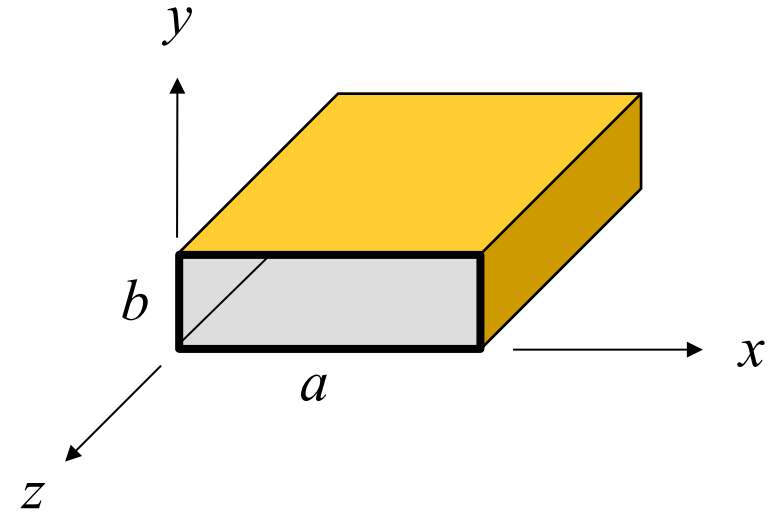
Summary for TE₁₀ mode

$$C_1 = \sqrt{\frac{ab}{2}}$$

$$C_2 = \frac{1}{Z_{\text{TE}}} \sqrt{\frac{ab}{2}}$$

$$Z_{\text{TE}} = \frac{\omega\mu}{k_z^{10}}$$

$$k_z^{10} = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}$$



$$\underline{E}_t(x, y, z) = \underline{\hat{y}} \sqrt{\frac{2}{ab}} \sin\left(\frac{\pi}{a}x\right) \overbrace{\left(V_0^+ e^{-jk_z z} + V_0^- e^{+jk_z z}\right)}^{V(z)}$$

$$\underline{H}_t(x, y, z) = -\underline{\hat{x}} \sqrt{\frac{2}{ab}} \sin\left(\frac{\pi}{a}x\right) \overbrace{\left(\frac{V_0^+}{Z_{\text{TE}}} e^{-jk_z z} - \frac{V_0^-}{Z_{\text{TE}}} e^{+jk_z z}\right)}^{I(z)}$$

$$\underline{e}_t = \underline{\hat{y}} \sin\left(\frac{\pi}{a}x\right)$$

$$\underline{h}_t = -\underline{\hat{x}} \frac{1}{Z_{\text{TE}}} \sin\left(\frac{\pi}{a}x\right)$$

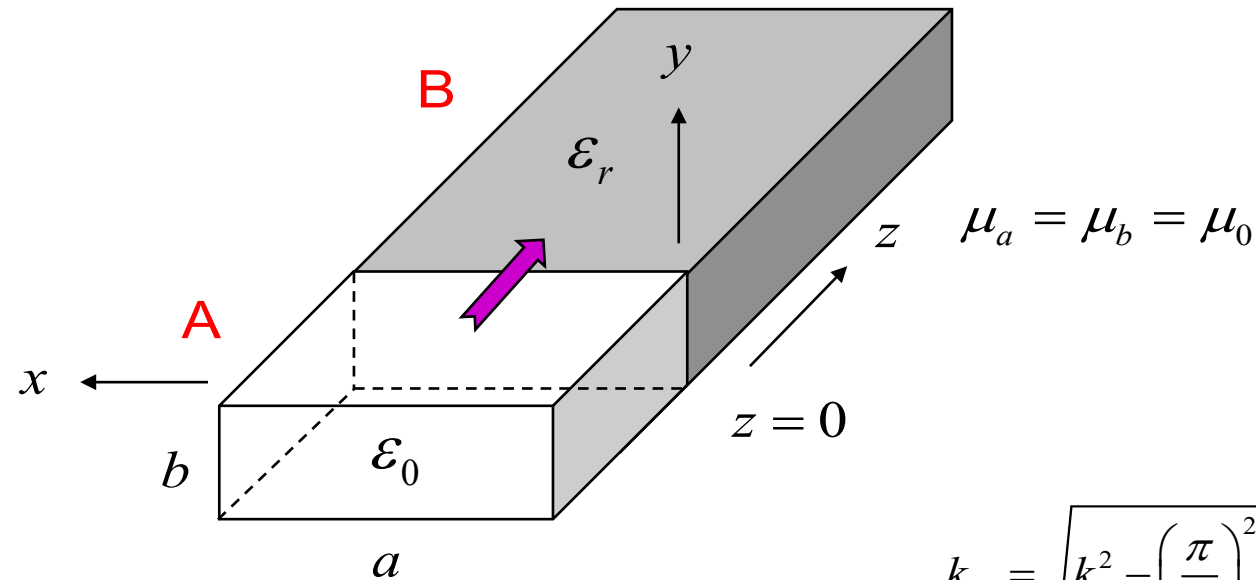
$$Z_0 = Z_{\text{TE}} = \frac{\omega\mu}{k_z^{10}}$$

$$k_z^{10} = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}$$

Example: Waveguide Discontinuity

For a 1 [V/m] (field at the center of the guide) incident TE₁₀ mode in guide A, find the TE₁₀ mode fields in both guides, and the reflected and transmitted powers.

$$E_y^{\text{inc}} = \sin\left(\frac{\pi}{a}x\right)e^{-jk_{za}z}$$



$$a = 2.2856 \text{ cm}$$

$$b = 1.016 \text{ cm}$$

$$\epsilon_r = 2.54$$

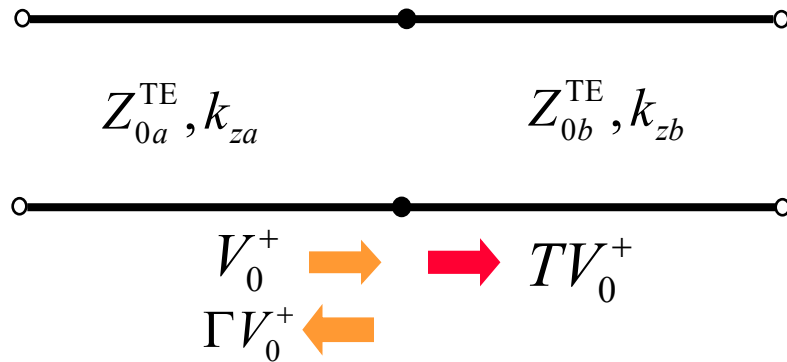
$$f = 10 \text{ GHz}$$

$$k_{za} = \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2} = 158.0 \text{ [rad / m]}$$

$$k_{zb} = \sqrt{\epsilon_r k_0^2 - \left(\frac{\pi}{a}\right)^2} = 304.1 \text{ [rad / m]}$$

Example (cont.)

TEN



$$C_1 = \sqrt{\frac{ab}{2}}$$

$$C_2 = \frac{1}{Z_{\text{TE}}} \sqrt{\frac{ab}{2}}$$

$$\underline{e}_t = \underline{\hat{y}} \sin\left(\frac{\pi}{a}x\right)$$

$$\underline{h}_t = -\underline{\hat{x}} \frac{1}{Z_{\text{TE}}} \sin\left(\frac{\pi}{a}x\right)$$

$$Z_0 = Z_{\text{TE}} = \frac{\omega\mu}{k_z^{10}}$$

$$k_z^{10} = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}$$

Convention:

- Choose $Z_0 = Z_{\text{TE}}$
- Assume power equality

$$Z_{0a} = Z_{\text{TE}}^a = \frac{\omega\mu}{k_{za}} = 499.7 \text{ } [\Omega]$$

$$Z_{0b} = Z_{\text{TE}}^b = \frac{\omega\mu}{k_{zb}} = 259.6 \text{ } [\Omega]$$

Note:

C_1 is the same for both guides, but C_2 is different.

$$\underline{E}_t^{\text{inc}}(x, y, z) = \underline{e}_t(x, y) \left(\textcolor{red}{A}^+ e^{-jk_z z} \right)$$

$A^+ = 1$ (since $\underline{e}_t(x, y)$ already has 1 [V/m] at the center)

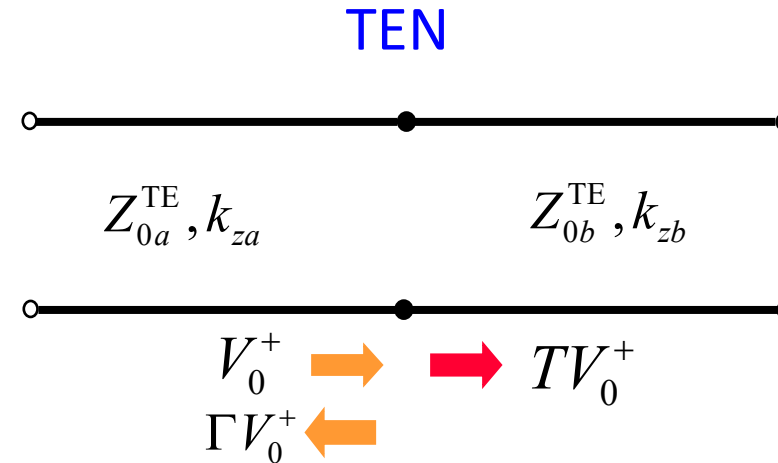


$$V_0^+ = C_1 A^+ = C_1 = \sqrt{\frac{ab}{2}}$$

Example (cont.)

$$\begin{aligned}
 V_a(z) &= V_0^+ \left(e^{-jk_{za}z} + \Gamma e^{+jk_{za}z} \right) \\
 V_b(z) &= V_0^+ T e^{-jk_{zb}z} \\
 I_a(z) &= \frac{V_0^+}{Z_{0a}} \left(e^{-jk_{za}z} - \Gamma e^{+jk_{za}z} \right) \\
 I_b(z) &= \frac{V_0^+ T}{Z_{0b}} e^{-jk_{zb}z}
 \end{aligned}$$

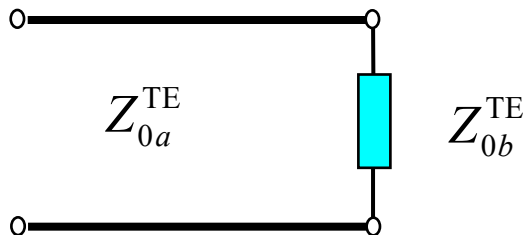
$$V_0^+ = \sqrt{\frac{ab}{2}}$$



$$\Gamma = \frac{Z_{0b} - Z_{0a}}{Z_{0b} + Z_{0a}} = -0.316$$

$$T = 1 + \Gamma = 0.684$$

Equivalent reflection problem:



Note:

The above TEN enforces the continuity of voltage and current at the junction, and hence the tangential electric and magnetic fields are automatically continuous in the WG problem.

Example (cont.)

Hence, we have:

$$V_a(z) = \sqrt{\frac{ab}{2}} \left(e^{-jk_{za}z} + (-0.316)e^{+jk_{za}z} \right)$$

$$V_b(z) = \sqrt{\frac{ab}{2}} (0.684)e^{-jk_{zb}z}$$

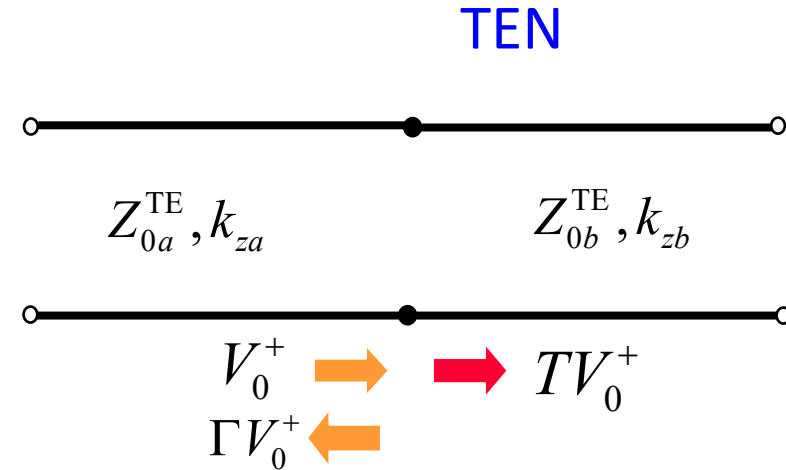
$$I_a(z) = \sqrt{\frac{ab}{2}} \frac{1}{Z_{0a}} \left(e^{-jk_{za}z} - (-0.316)e^{+jk_{za}z} \right)$$

$$I_b(z) = \sqrt{\frac{ab}{2}} \frac{(0.684)}{Z_{0b}} e^{-jk_{zb}z}$$

Recall that for the TE₁₀ mode:

$$\underline{E}_t(x, y, z) = \frac{1}{C_1} \underline{e}_t(x, y) \mathbf{V}(z)$$

$$\underline{H}_t(x, y, z) = \frac{1}{C_2} \underline{h}_t(x, y) \mathbf{I}(z)$$



$$k_{za} = 158.0 \text{ [rad / m]}$$

$$k_{zb} = 304.1 \text{ [rad / m]}$$

$$Z_{0a} = 499.7 \text{ [}\Omega\text{]}$$

$$Z_{0b} = 259.6 \text{ [}\Omega\text{]}$$

$$\underline{e}_t = \underline{\hat{y}} \sin\left(\frac{\pi}{a}x\right)$$

$$\underline{h}_t = -\underline{\hat{x}} \frac{1}{Z_{\text{TE}}} \sin\left(\frac{\pi}{a}x\right)$$

$$Z_0 = Z_{\text{TE}} = \frac{\omega\mu}{k_z^{10}}$$

$$k_z^{10} = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}$$

$$C_1 = \sqrt{\frac{ab}{2}}$$

$$C_2 = \frac{1}{Z_{\text{TE}}} \sqrt{\frac{ab}{2}}$$

Example (cont.)

Hence, for the waveguide problem we have the fields as:

Air region

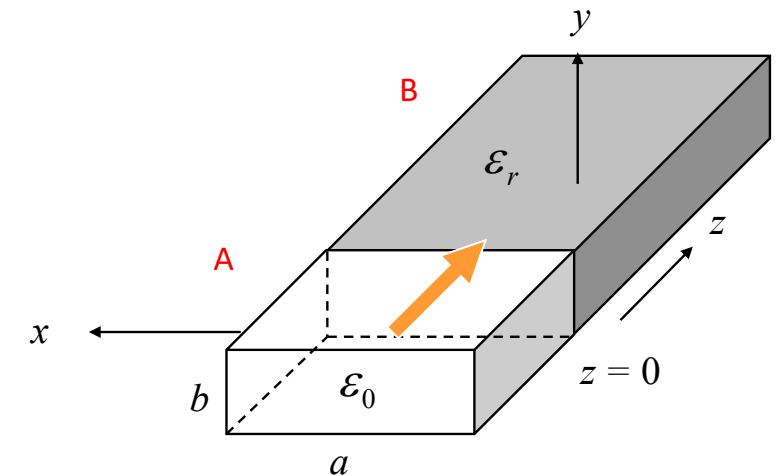
$$\underline{E}_{ta}(x, y, z) = \frac{1}{C_1} \underline{e}_t(x, y) \sqrt{\frac{ab}{2}} \left(e^{-jk_{za}z} + (-0.316) e^{+jk_{za}z} \right)$$

$$\underline{H}_{ta}(x, y, z) = \frac{1}{C_2} \underline{h}_t(x, y) \sqrt{\frac{ab}{2}} \frac{1}{Z_{0a}} \left(e^{-jk_{za}z} - (-0.316) e^{+jk_{za}z} \right)$$

Dielectric region

$$\underline{E}_{tb}(x, y, z) = \frac{1}{C_1} \underline{e}_t(x, y) \sqrt{\frac{ab}{2}} (0.684) e^{-jk_{zb}z}$$

$$\underline{H}_{tb}(x, y, z) = \frac{1}{C_2} \underline{h}_t(x, y) \sqrt{\frac{ab}{2}} \frac{1}{Z_{0b}} (0.684) e^{-jk_{zb}z}$$

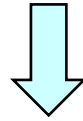


Example (cont.)

Substituting in for C_1 and $\underline{e}_t$, we have (guide A):

Air region

$$\underline{E}_{ta}(x, y, z) = \frac{1}{C_1} \underline{e}_t(x, y) \sqrt{\frac{ab}{2}} \left(e^{-jk_{za}z} + (-0.316) e^{+jk_{za}z} \right)$$



$$\underline{E}_{ta}(x, y, z) = \frac{1}{\sqrt{\frac{ab}{2}}} \left[\underline{\hat{y}} \sin\left(\frac{\pi}{a}x\right) \right] \sqrt{\frac{ab}{2}} \left(e^{-jk_{za}z} + (-0.316) e^{+jk_{za}z} \right)$$

$$C_1 = \sqrt{\frac{ab}{2}}$$
$$C_2 = \frac{1}{Z_{TE}} \sqrt{\frac{ab}{2}}$$

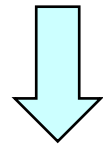
$$\underline{e}_t = \underline{\hat{y}} \sin\left(\frac{\pi}{a}x\right)$$
$$\underline{h}_t = -\underline{\hat{x}} \frac{1}{Z_{TE}} \sin\left(\frac{\pi}{a}x\right)$$
$$Z_{TE} = \frac{\omega\mu}{k_z^{10}}$$

Example (cont.)

Substituting in for C_2 and $\underline{h}_t$, we have (guide A):

Air region

$$\underline{H}_{ta}(x, y, z) = \frac{1}{C_2} \underline{h}_t(x, y) \sqrt{\frac{ab}{2}} \frac{1}{Z_{0a}} \left(e^{-jk_{za}z} - (-0.316) e^{+jk_{za}z} \right)$$


$$Z_{0a} = Z_a^{\text{TE}}$$

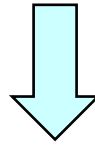
$$\underline{H}_{ta}(x, y, z) = \frac{1}{\frac{1}{Z_a^{\text{TE}}} \sqrt{\frac{ab}{2}}} \left[-\hat{x} \frac{1}{Z_a^{\text{TE}}} \sin\left(\frac{\pi}{a}x\right) \right] \sqrt{\frac{ab}{2}} \frac{1}{Z_a^{\text{TE}}} \left(e^{-jk_{za}z} - (-0.316) e^{+jk_{za}z} \right)$$

Example (cont.)

Substituting in for C_1 and $\underline{e}_t$, we have (guide B):

Dielectric region

$$\underline{E}_{tb}(x, y, z) = \frac{1}{C_1} \underline{e}_t(x, y) \sqrt{\frac{ab}{2}} (0.684) e^{-jk_{zb}z}$$



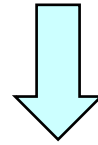
$$\underline{E}_{tb}(x, y, z) = \frac{1}{\sqrt{\frac{ab}{2}}} \left[\underline{\hat{y}} \sin\left(\frac{\pi}{a}x\right) \right] \sqrt{\frac{ab}{2}} (0.684) e^{-jk_{zb}z}$$

Example (cont.)

Substituting in for C_2 and $\underline{h}_t$, we have (guide B):

Dielectric region

$$\underline{H}_{tb}(x, y, z) = \frac{1}{C_2} \underline{h}_t(x, y) \sqrt{\frac{ab}{2}} \frac{1}{Z_{0b}} (0.684) e^{-jk_{zb}z}$$



$$Z_{0b} = Z_b^{\text{TE}}$$

$$\underline{H}_{tb}(x, y, z) = \frac{1}{\frac{1}{Z_b^{\text{TE}}} \sqrt{\frac{ab}{2}}} \left[-\hat{x} \frac{1}{Z_b^{\text{TE}}} \sin\left(\frac{\pi}{a}x\right) \right] \sqrt{\frac{ab}{2}} \frac{1}{Z_b^{\text{TE}}} (0.684) e^{-jk_{zb}z}$$

Example (cont.)

Summary of Fields

$$\underline{E}_{ta}(x, y, z) = \left[\underline{\hat{y}} \sin\left(\frac{\pi}{a}x\right) \right] \left(e^{-jk_{za}z} + (-0.316)e^{+jk_{za}z} \right)$$

$$\underline{H}_{ta}(x, y, z) = \left[-\underline{\hat{x}} \frac{1}{Z_a^{\text{TE}}} \sin\left(\frac{\pi}{a}x\right) \right] \left(e^{-jk_{za}z} - (-0.316)e^{+jk_{za}z} \right)$$

$$\underline{E}_{tb}(x, y, z) = \left[\underline{\hat{y}} \sin\left(\frac{\pi}{a}x\right) \right] (0.684)e^{-jk_{zb}z}$$

$$\underline{H}_{tb}(x, y, z) = \left[-\underline{\hat{x}} \frac{1}{Z_b^{\text{TE}}} \sin\left(\frac{\pi}{a}x\right) \right] (0.684)e^{-jk_{zb}z}$$

$$E_y^{\text{inc}} = \sin\left(\frac{\pi}{a}x\right) e^{-jk_{za}z}$$

$$Z_a^{\text{TE}} = 499.7 \text{ } [\Omega]$$

$$Z_b^{\text{TE}} = 259.6 \text{ } [\Omega]$$

$$k_{za} = 158.0 \text{ } [\text{rad/m}]$$

$$k_{zb} = 304.1 \text{ } [\text{rad/m}]$$

Example (cont.)

Power Calculations:

Note: In this example, Z_0 and Γ are real.

Recall: $V_0^+ \equiv C_1 A^+ = C_1 (1) = C_1 = \sqrt{\frac{ab}{2}}$

$$P_a^{\text{inc}} = \text{Re} \left(\frac{1}{2} V_0^+ I_0^{+*} \right) = \text{Re} \left(\frac{1}{2} |V_0^+|^2 \frac{1}{Z_{0a}^*} \right) = \frac{1}{2} \left| \sqrt{\frac{ab}{2}} \right|^2 \frac{1}{Z_a^{\text{TE}}}$$

$$P_a^{\text{ref}} = \text{Re} \left(\frac{1}{2} V_0^+ I_0^{+*} \right) |\Gamma|^2 = \frac{1}{2} \left| \sqrt{\frac{ab}{2}} \right|^2 \frac{1}{Z_a^{\text{TE}}} |\Gamma|^2$$

$$P_b^{\text{trans}} = \text{Re} \left(\frac{1}{2} V_0^+ I_0^{+*} \right) (1 - |\Gamma|^2) = \frac{1}{2} \left| \sqrt{\frac{ab}{2}} \right|^2 \frac{1}{Z_a^{\text{TE}}} (1 - |\Gamma|^2)$$

Example (cont.)

Final Results:

$$P_a^{\text{inc}} = 1.161 \text{ [mW]}$$

$$P_a^{\text{ref}} = 0.116 \text{ [mW]}$$

$$P_b^{\text{trans}} = 1.045 \text{ [mW]}$$

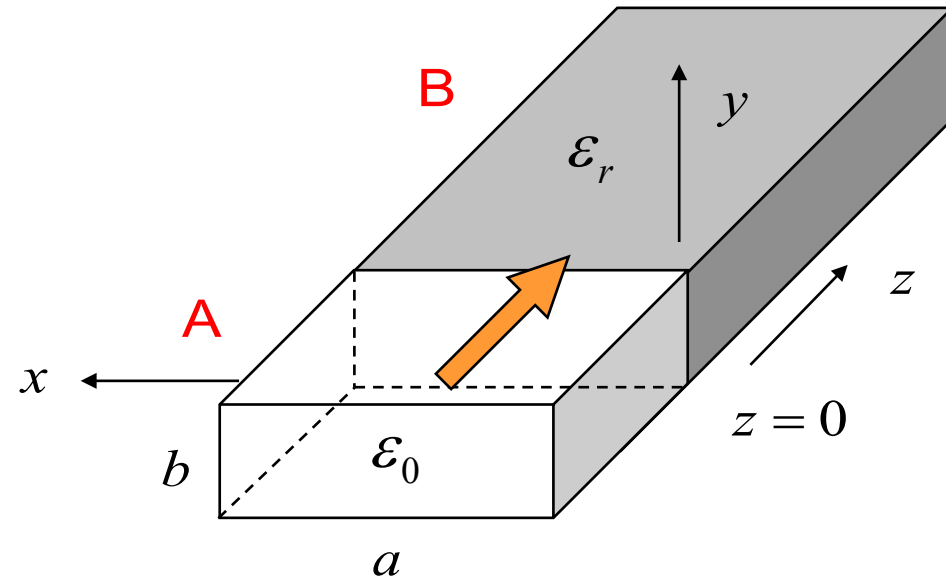
$$a = 2.2856 \text{ cm}$$

$$b = 1.016 \text{ cm}$$

$$\epsilon_r = 2.54$$

$$f = 10 \text{ GHz}$$

$$E_y^{\text{inc}} = \sin\left(\frac{\pi}{a}x\right)e^{-jk_z z}$$



Note: 90.1% of the incident power is transmitted.

Quarter-Wave Transformer in Waveguide

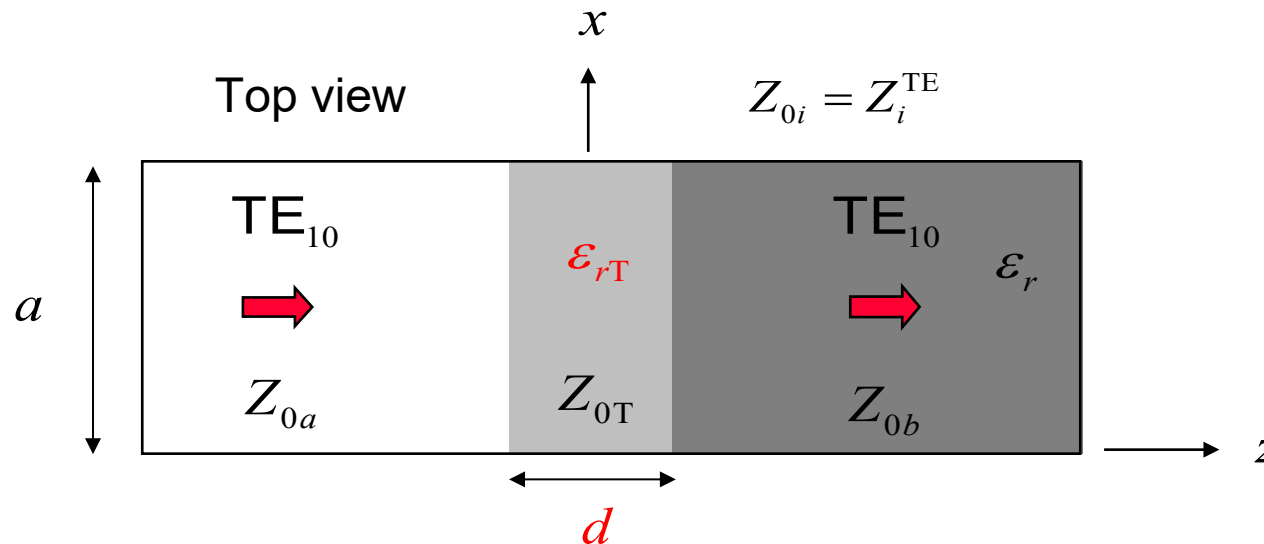
A quarter-wave transformer is shown here.

$$Z_{0T} = \sqrt{Z_{0a} Z_{0b}}$$

$$d = \lambda_{gT} / 4$$

Goal:

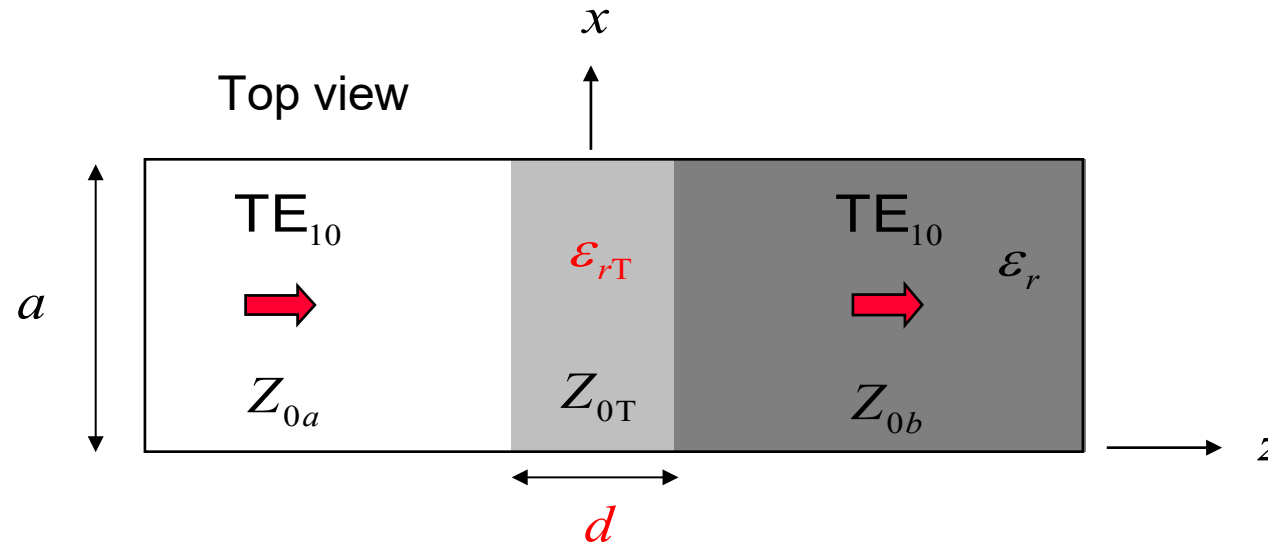
Determine: d, ϵ_T



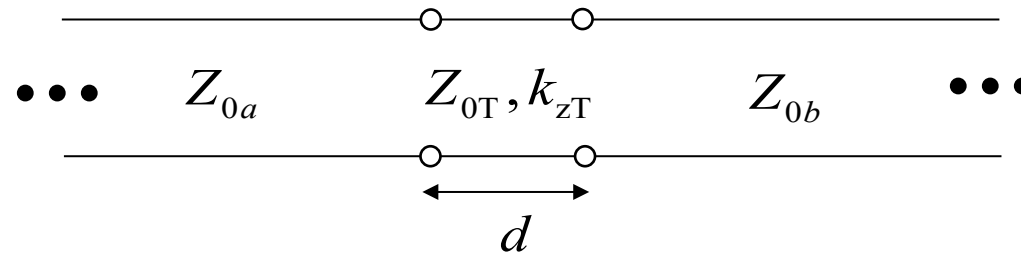
Now 100% of the incident power is transmitted.

Quarter-Wave Transformer in Waveguide (cont.)

Waveguide problem:



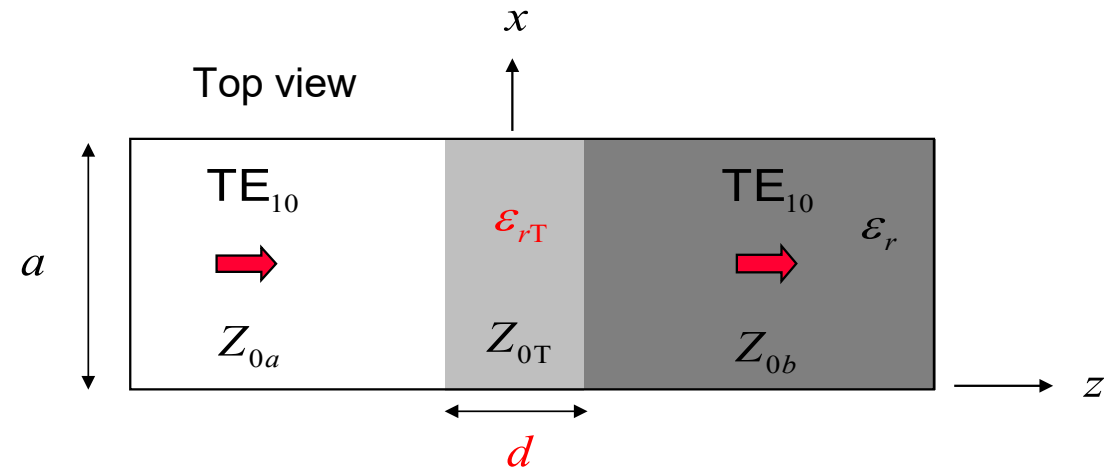
TEN:



Quarter-Wave Transformer in Waveguide (cont.)

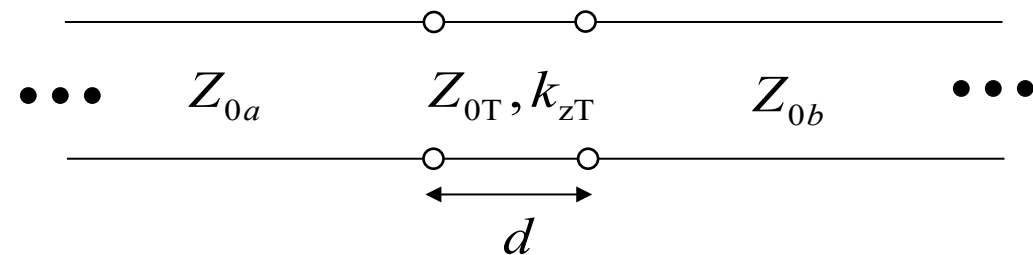
Design recipe:

- 1) Determine Z_{0T} : $Z_{0T} = \sqrt{Z_{0a}Z_{0b}}$
- 2) Determine k_{zT} : $Z_{0T} = \omega\mu_0 / k_{zT}$
- 3) Determine k_T : $k_{zT} = \sqrt{k_T^2 - \left(\frac{\pi}{a}\right)^2}$
- 4) Determine ϵ_{rT} : $k_T = k_0\sqrt{\epsilon_{rT}}$
- 5) Determine β_T : $\beta_T = k_{zT}$
- 6) Determine λ_{gT} : $\lambda_{gT} = 2\pi / \beta_T$
- 7) Determine d : $d = \lambda_{gT} / 4$



$$Z_{0T} = \sqrt{Z_{0a}Z_{0b}}$$

$$d = \lambda_{gT} / 4$$



Quarter-Wave Transformer in Waveguide (cont.)

Example:

$$a = 2.2856 \text{ cm}$$

$$b = 1.016 \text{ cm}$$

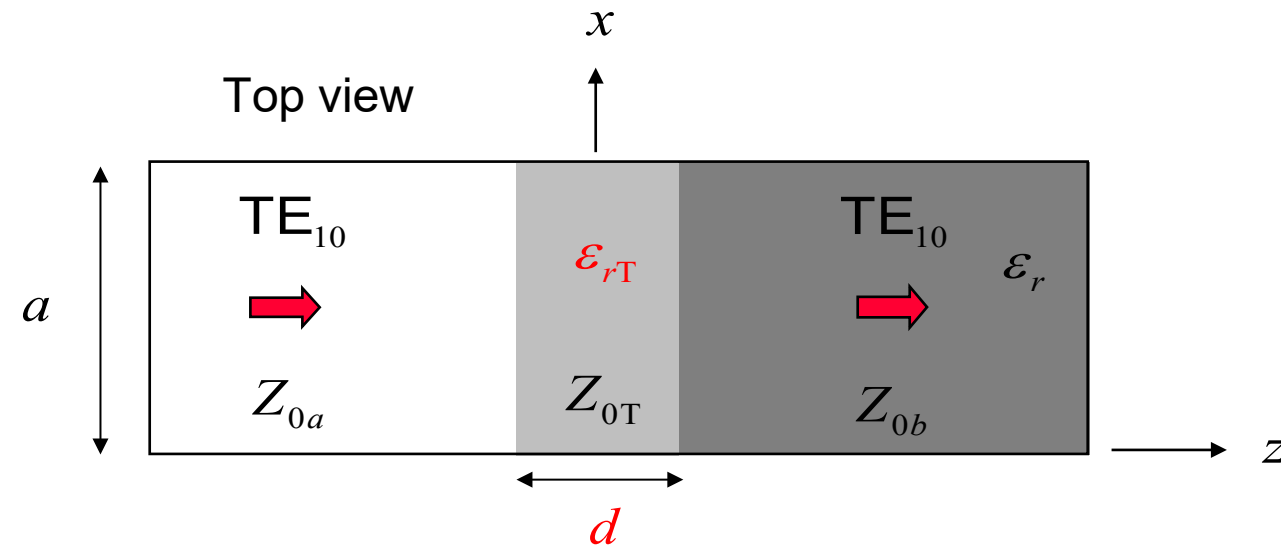
$$\epsilon_r = 2.54$$

$$f = 10 \text{ GHz}$$

Results:

$$\epsilon_{rT} = 1.53$$

$$d = 0.716 \text{ cm}$$



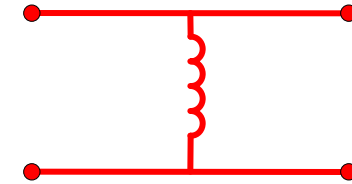
Matching Elements in Waveguide

Rectangular Waveguide
(end view)

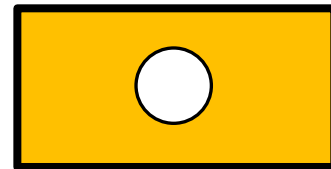
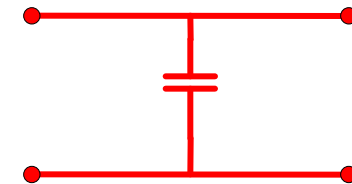
Note:
Planar discontinuities are modeled as purely shunt elements.



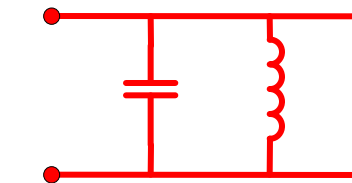
Inductive iris



Capacitive iris



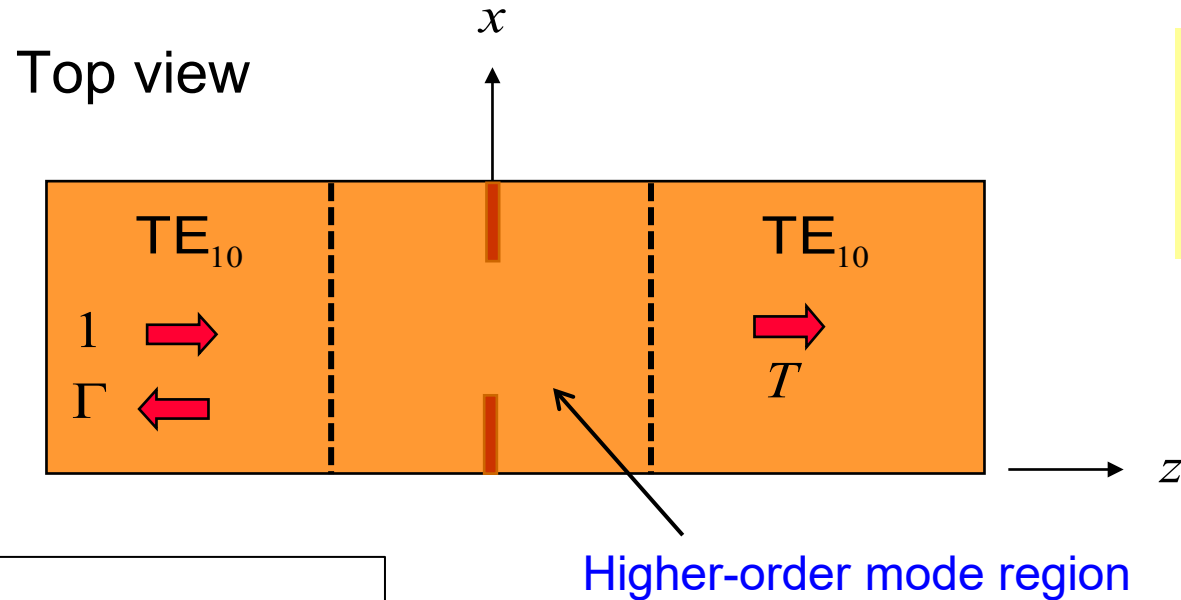
Resonant iris



The equivalent circuit gives us the correct reflection and transmission of the TE_{10} mode.

Matching Elements in Waveguide (cont.)

Inductive iris in air-filled waveguide



$$Z_0 = Z_{\text{TE}}^{10} = \frac{\omega\mu_0}{k_z^{10}} = \frac{\eta_0}{\sqrt{1 - \left(\frac{\pi}{k_0 a}\right)^2}}$$

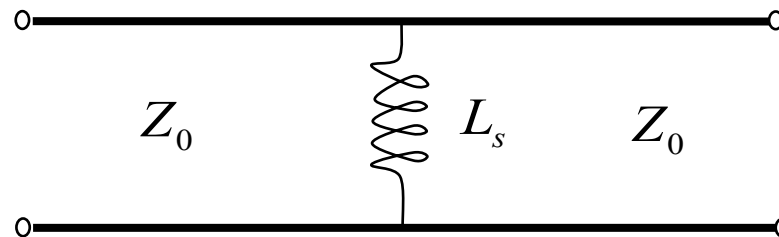
Note:

The shunt inductor models the effects of the iris and gives the amplitudes of the TE_{10} mode correctly everywhere, but the TEN model does not tell us how strong the higher-order modes are.

Because the element is a shunt discontinuity, we have:

$$T = 1 + \Gamma$$

TEN Model



Matching Elements in Waveguide (cont.)

Much more information can be found in the following reference:

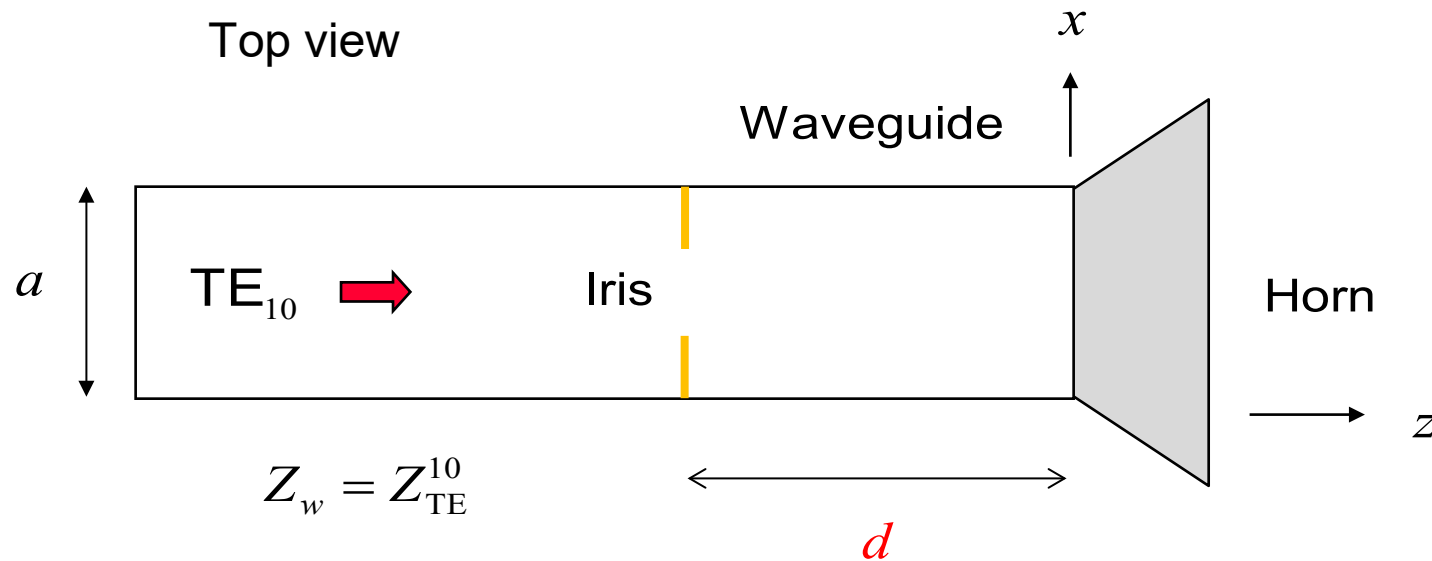
N. Marcuvitz, *Waveguide Handbook*, Peter Perigrinus, Ltd. (on behalf of the Institute of Electrical Engineers), 1986.

- Equivalent circuits for many types of discontinuities
- Accurate CAD formulas for many of the discontinuities
- Graphical results for many of the cases
- Sometimes, measured results

Using Irises for Matching

A “single-stub” matching scheme is shown here.

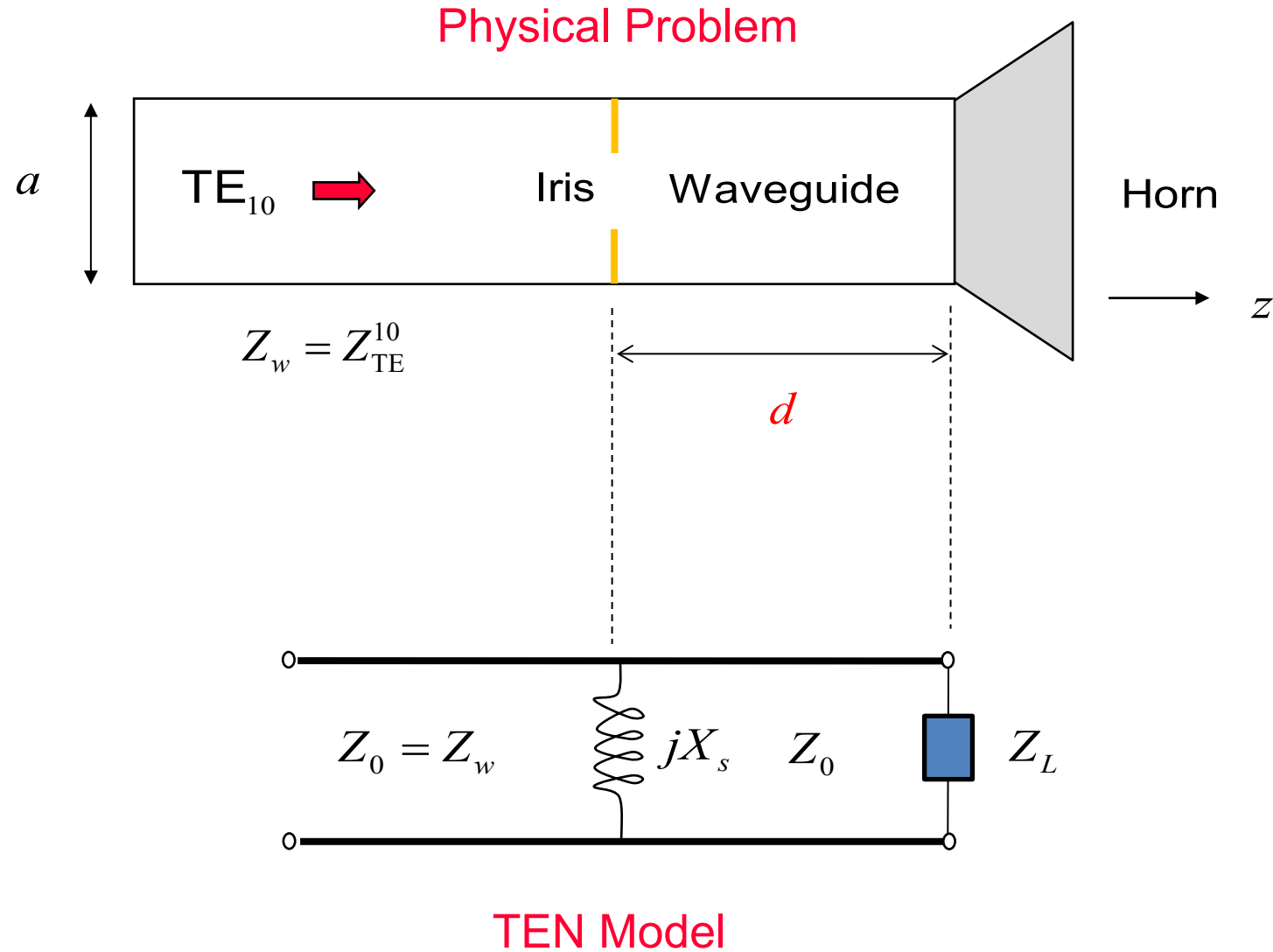
(The procedure is illustrated for a horn antenna load).



An inductive iris is shown being used for matching.

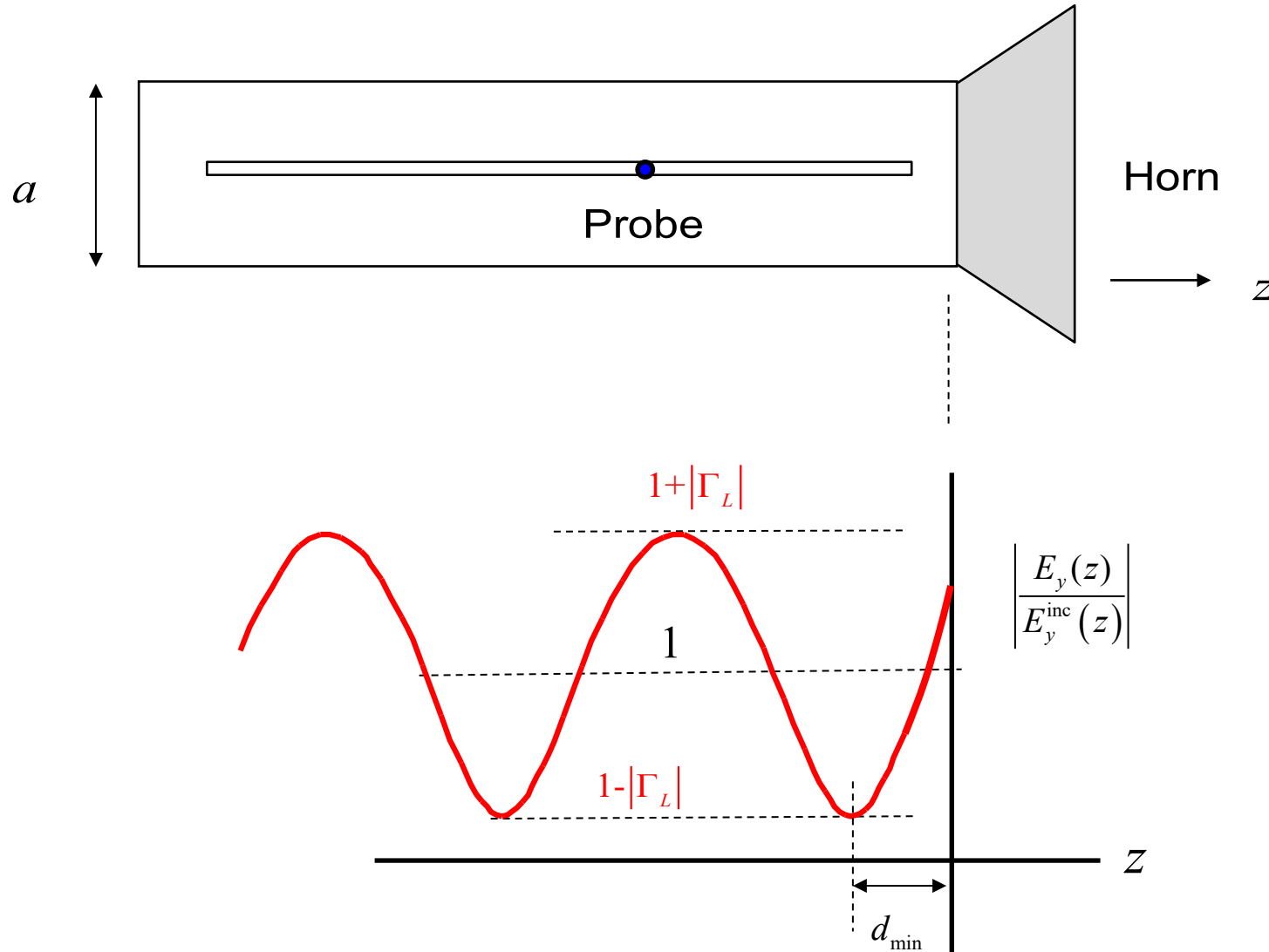
Using Irises for Matching (cont.)

The TEN is shown



Using Irises for Matching (cont.)

A field probing can be used to determine the unknown load impedance Z_L .



$$\text{SWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$$|\Gamma_L| = \frac{\text{SWR} - 1}{\text{SWR} + 1}$$

Using Irises for Matching (cont.)

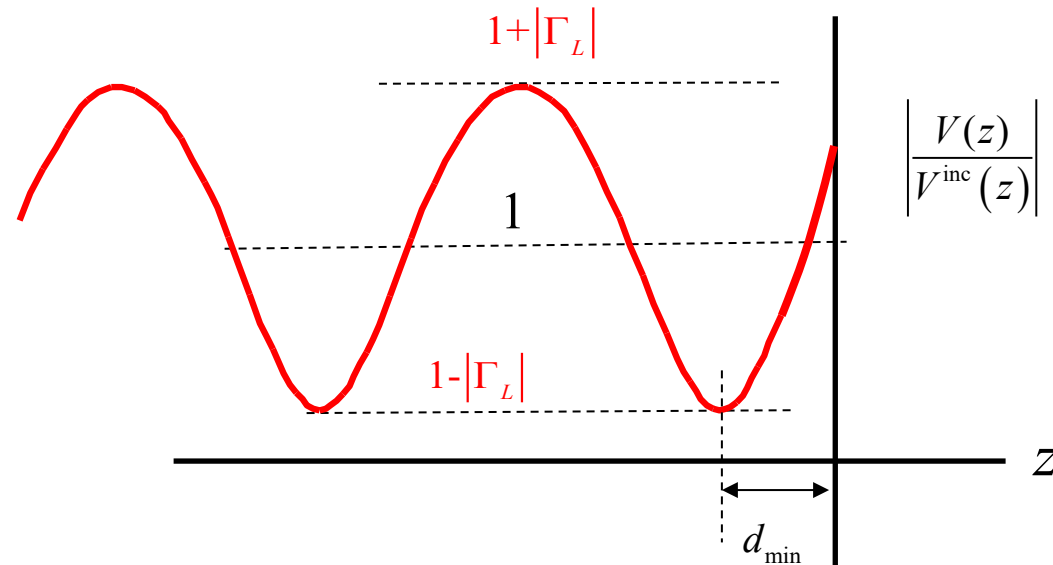
A field probing can be used to determine the unknown load impedance Z_L .



TEN Model

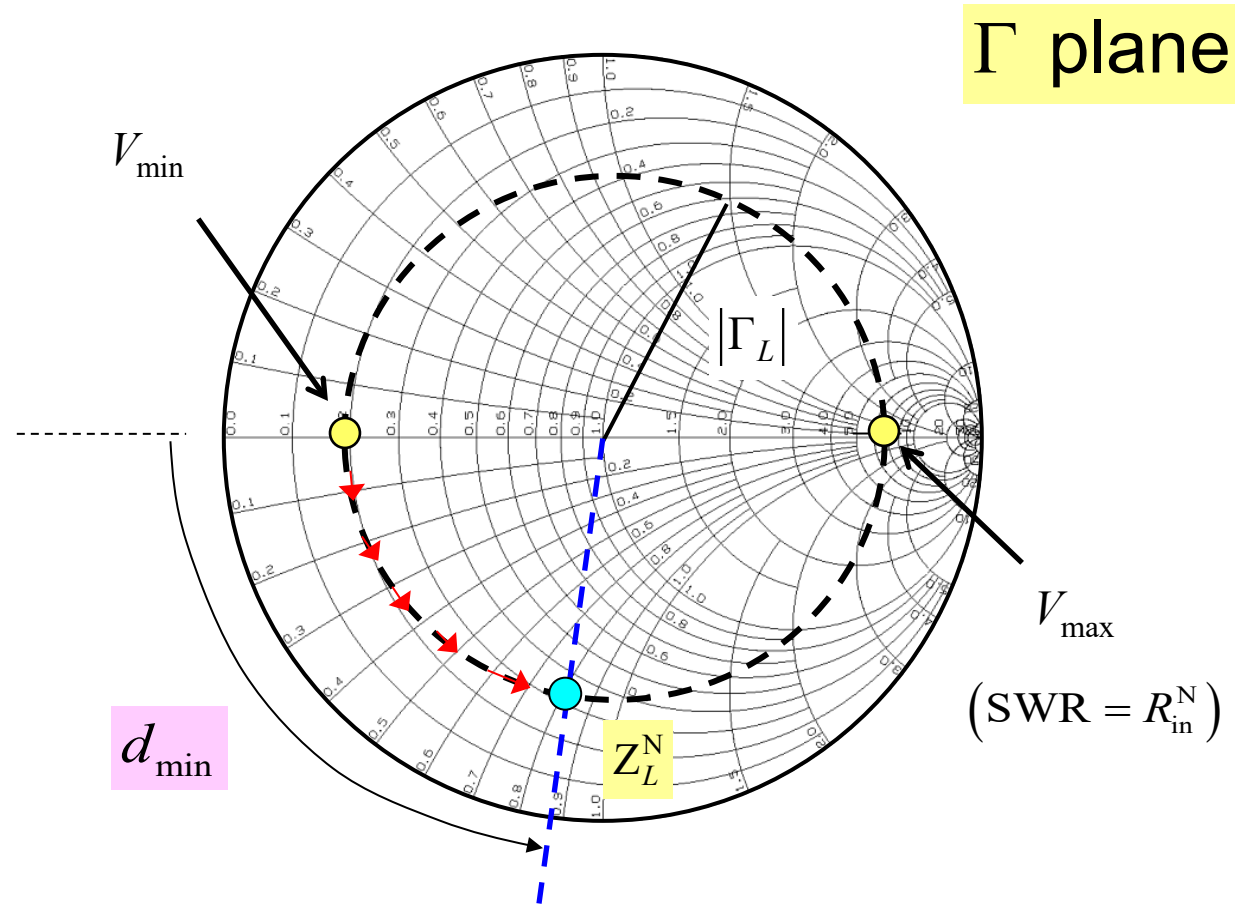
$$\text{SWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$$|\Gamma_L| = \frac{\text{SWR} - 1}{\text{SWR} + 1}$$



Using Irises for Matching (cont.)

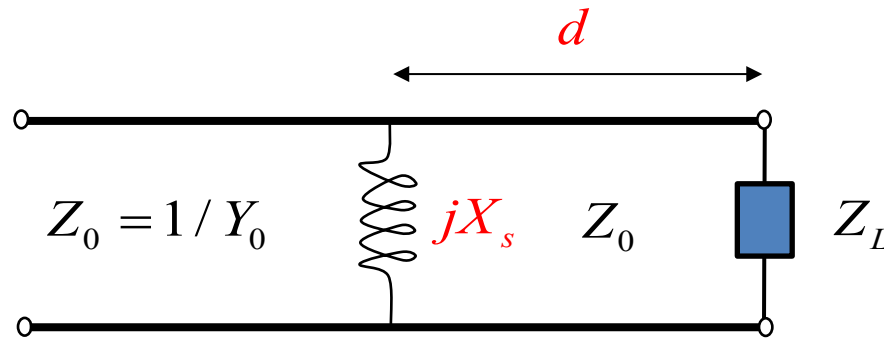
A Smith chart can then be used to find the unknown load impedance of the antenna in the TEN.



Using Irises for Matching (cont.)

A Smith chart can also be used to find the location of the shunt susceptance and its value.

“single-stub matching”



We'll assume that we are using an inductive iris in this problem as the matching element.

$$Z_s = jX_s$$

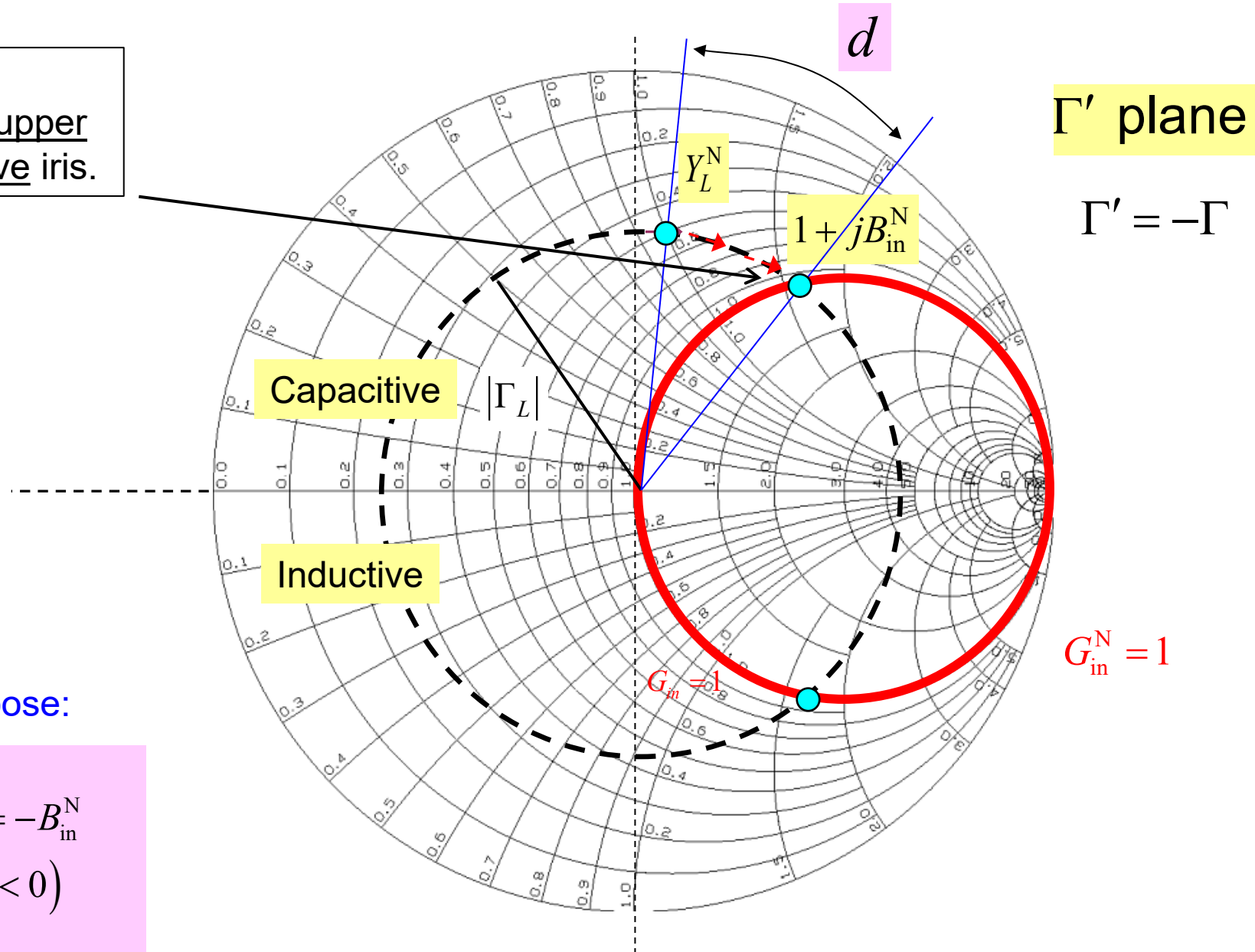
$$Y_s = jB_s$$

$$\text{Note: } B_s = -\frac{1}{X_s} \quad B_s^N \equiv \frac{B_s}{Y_0}$$

Using Irises for Matching (cont.)

Note:

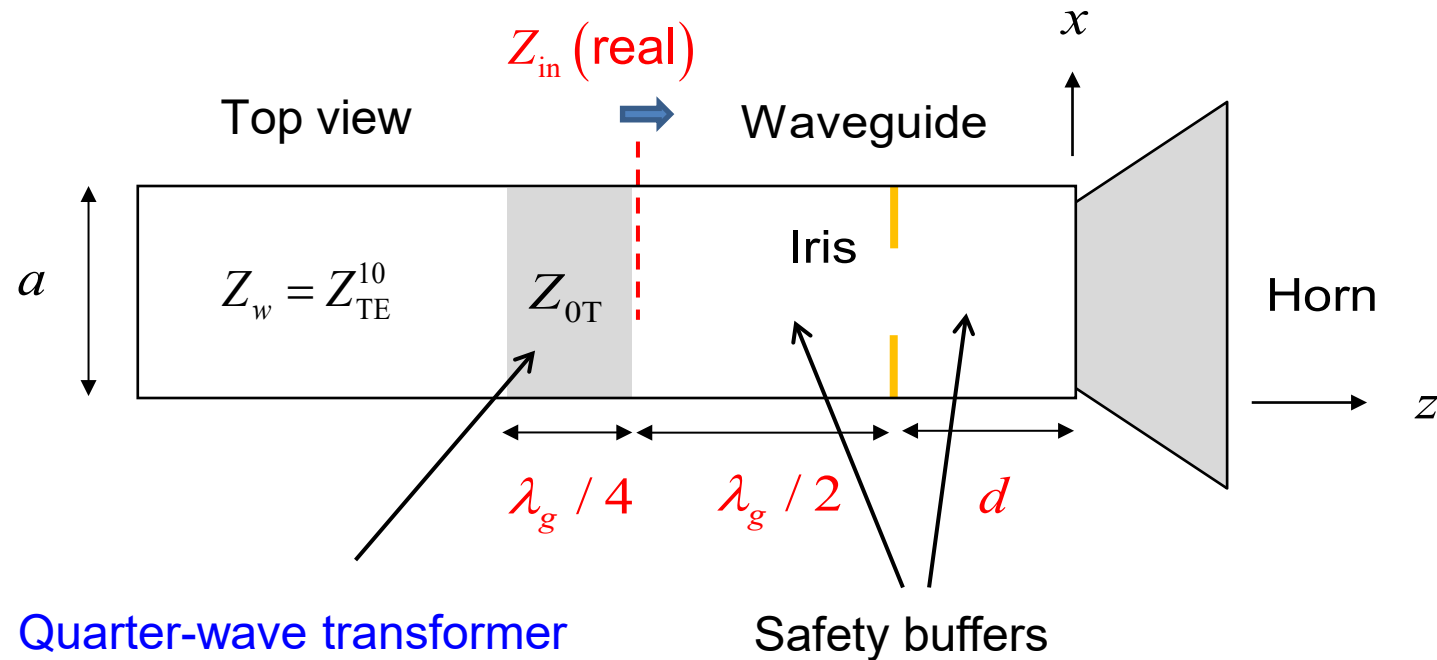
We must use this upper point for an inductive iris.



Using Irises for Matching (cont.)

A quarter-wave transformer matching scheme is shown here.

Here is the distance d is arbitrary.



Note:

The arbitrary distance d gives us a “safety buffer” from the horn discontinuity, to allow the higher-order modes to decay. The extra $\lambda_g/2$ is also a safety buffer.

Note:

Input impedance repeats after one-half of a guided wavelength.

$$Z_{0T} = \sqrt{Z_{TE}^{10} Z_{in}}$$

The iris is used to cancel the input susceptance at $z = -d$.

Using Irises for Matching (cont.)

Going a distance d (arbitrary) from the horn:

Note:

The arbitrary distance d must be chosen so that we end up in the capacitive region of the Smith chart.

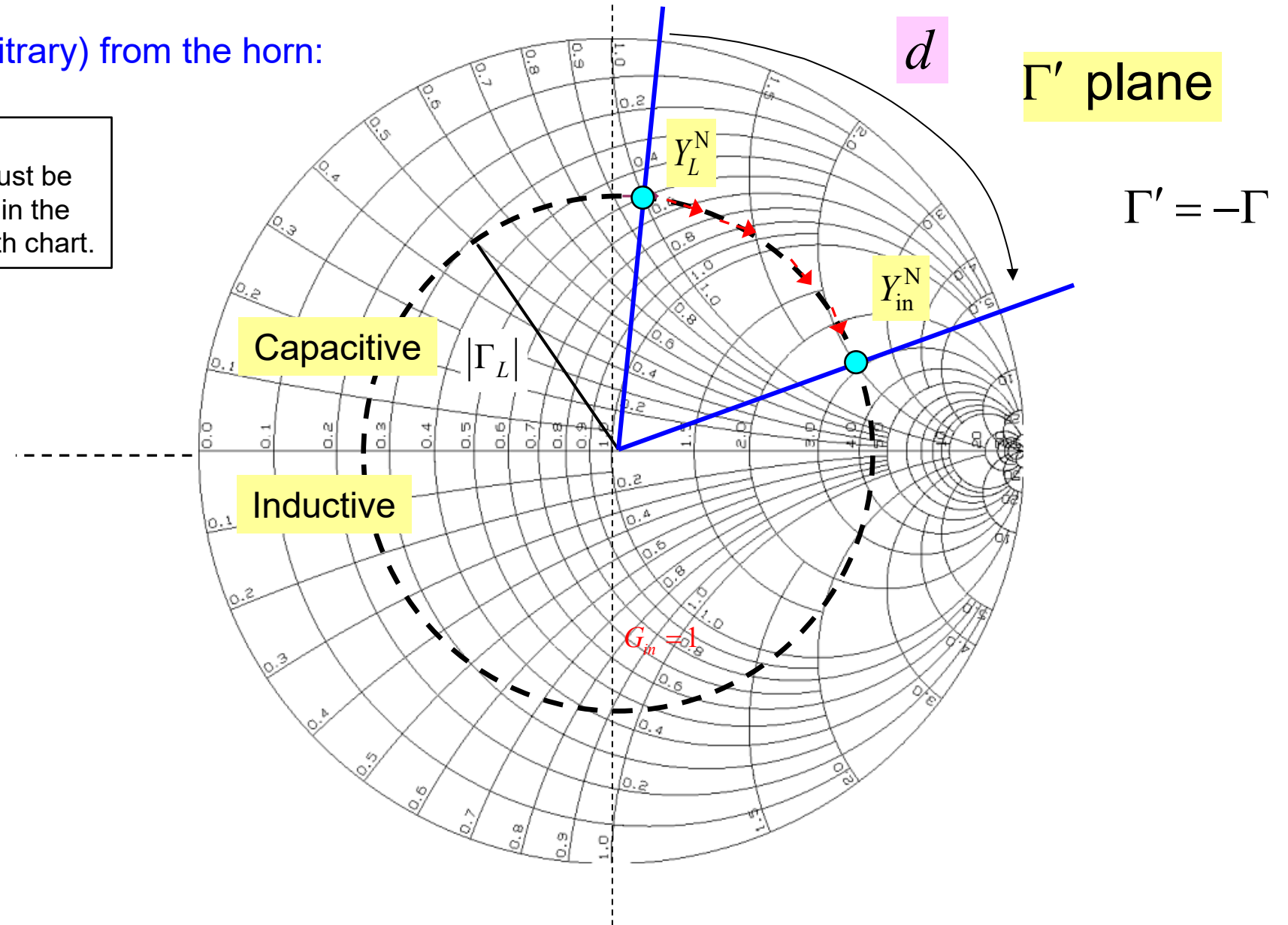
$$Z_{\text{in}}^N = \frac{1}{G_{\text{in}}^N} = \frac{1}{\text{Re}(Y_{\text{in}}^N)}$$

$$Z_{\text{in}} = Z_{\text{in}}^N Z_{\text{TE}}^{10}$$

Choose:

$$B_s^N = -B_{\text{in}}^N$$

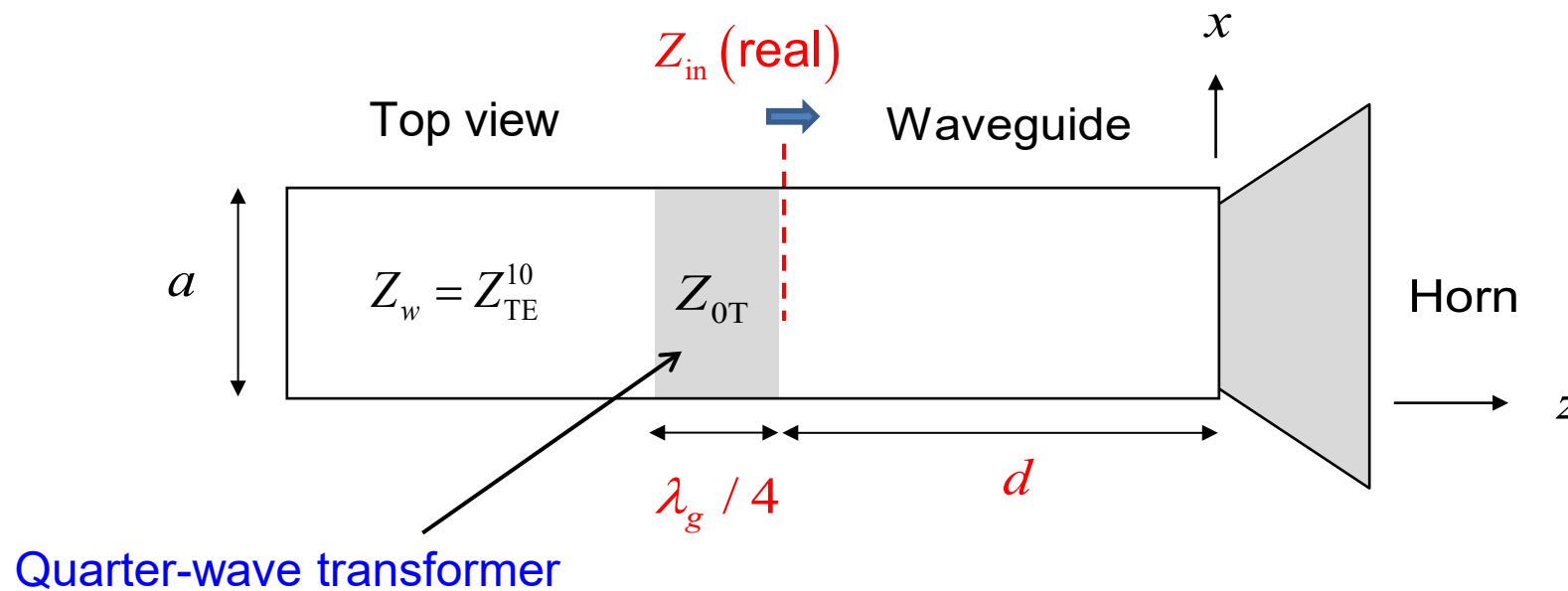
$$(B_s^N < 0)$$



Quarter-Wave Transformer

A quarter-wave transformer matching scheme is shown here without needing an iris.

Here we choose distance d so that Z_{in} is real.



Quarter-Wave Transformer (cont.)

Choosing distance d so we don't need an iris:

$$Z_{\text{in}}^N = \frac{1}{G_{\text{in}}^N} = \frac{1}{Y_{\text{in}}^N}$$

$$Z_{\text{in}} = Z_{\text{in}}^N Z_{\text{TE}}^{10}$$

Now we don't need an iris!

