

**ECE 5317/6351**  
**Microwave Engineering**  
**Fall 2019**  
**Homework #4**

Text: *Microwave Engineering* by David M. Pozar, 4th edition, Wiley, 2011.

- 1) Use the CAD formulas given in the class notes to design a microstrip line on Rogers RT/duroid 5880 ( $\epsilon_r = 2.2$ ) with a substrate thickness  $h$  of 62 [mils] (1.575 mm) at a frequency of 1 GHz so that  $Z_0 = 50 \Omega$ . In particular, find the strip width  $w$  and the effective relative permittivity  $\epsilon_r^{eff}$ . In this problem you can ignore frequency dispersion and conductor thickness. Therefore, you can use the approximate design formula in the notes that directly gives you a result for  $w/h$  (the “inverse formula”), along with the approximate formula for  $\epsilon_r^{eff}$  that ignores frequency effects.
- 2) Repeat the previous problem to find  $w$  and  $\epsilon_r^{eff}$  using the more accurate design formulas in the class notes that account for frequency dispersion and conductor thickness. Assume a frequency of 10 [GHz]. Assume “half-ounce” copper, so that the thickness of the metal  $t$  is 17.5 [ $\mu\text{m}$ ]. (Note: The design will have to be done using an iterative “trial and error” process.)
- 3) For the microstrip line in the previous problem, assume that the substrate has  $\tan\delta = 0.001$  and the copper strip and ground plane have  $\sigma = 3.0 \times 10^7$  S/m. Determine  $\alpha_c$  and  $\alpha_d$ , and the total attenuation  $\alpha$ , all in np/m, at 1 GHz. Use the CAD formulas for attenuation presented in the class notes (the ones that account for frequency and conductor thickness). Assume “half-ounce” copper, so that the thickness of the metal  $t$  is 17.5  $\mu\text{m}$ .
- 4) Repeat Prob. 2, but find the strip width and  $\epsilon_r^{eff}$  using the TXLINE program.
- 5) Repeat Prob. 3, but find the three attenuations using the TXLINE program. (Note: You can set the loss tangent of the substrate to zero or set the conductivity of the copper to be very high in order to remove either the dielectric or the conductor losses.)
- 6) Assume that the microstrip line of Prob. 1 is now modeled as a strip of width  $w^{eff}$  in an semi-infinite medium of homogeneous material having a relative permittivity of  $\epsilon_r^{eff}$ , at a height of  $h$  above a ground plane. Using your results for the known  $Z_0$  and  $\epsilon_r^{eff}$ , find the effective strip width  $w^{eff}$ . (This model of the microstrip line supports an exact TEM mode having the same characteristic impedance and phase velocity as does the actual microstrip line.) To do this,

first find the value of  $Z_0^{air}$  (the characteristic impedance that the line should have if it has a width  $w^{eff}$  but is in a semi-infinite air region over a ground plane.) Then use the approximate design formula that directly gives you a result for  $w/h$ , using air in the formula.

- 7) A rectangular waveguide carries the  $TE_{10}$  mode, which has a wave impedance of  $Z_{TE} = 500 \Omega$  at the frequency of operation. This waveguide meets a “black box” that is centered at  $z = 0$ . It is unknown what is inside the black box. The same waveguide exits and continues from the black box. From measurements, it is known that fields inside the waveguide far away from the black box are described as follows:

$$E_y(x, y, z) = \sin\left(\frac{\pi x}{a}\right) \left( e^{-k_z z} + \Gamma e^{+k_z z} \right), \quad z \ll 0$$

$$E_y(x, y, z) = \sin\left(\frac{\pi x}{a}\right) \left( T e^{-k_z z} \right), \quad z \gg 0.$$

It is found from measurements that the reflection coefficient is  $\Gamma = -0.027027 + j(0.16216)$  and that the transmission coefficient is  $T = 1 + \Gamma$ . From this information, determine what the equivalent circuit for the black box is in the TEN model, and draw a picture of the corresponding TEN circuit showing the transmission line and the circuit representation of the black box, with all impedance values labeled. Assume that the characteristic impedance of the transmission lines in the TEN is the same as the wave impedance of the  $TE_{10}$  mode. Hint: What kind of element (series, shunt, etc.) would have the property that  $T = 1 + \Gamma$ ?

- 8) An X-band rectangular waveguide has dimensions  $a = 2.0$  cm,  $b = 1.0$  cm. An air-filled section of the waveguide meets an identical section that is filled with lossless Teflon having a relative permittivity of 2.2. A  $TE_{10}$  mode at 10.7 GHz ( $a / \lambda_0 = 0.714$ ) is incident on the boundary from the air-filled section. It is desired to put a quarter-wave transformer section of waveguide between the two waveguides (air-filled and Teflon-filled) in order to get an impedance match. Determine the length  $d$  of the transformer waveguide (in cm) and also the relative permittivity  $\epsilon_r$  of the filling material that should be placed inside the transformer waveguide section. Assume that the characteristic impedance of the transmission lines in the TEN is the same as the wave impedance of the  $TE_{10}$  mode.
- 9) An X-band rectangular waveguide has dimensions  $a = 2.0$  cm,  $b = 1.0$  cm. An air-filled section of the waveguide meets an identical section that is filled with lossless Teflon having a relative permittivity of 2.2. A  $TE_{10}$  mode at 10.7 GHz ( $a / \lambda_0 = 0.714$ ) is incident on the boundary from the air-filled section. It is desired to put an inductive iris in the air-filled portion of the waveguide at a distance  $d$  from the boundary, in order to have a perfect match seen by the incident waveguide mode. Determine the distance  $d$  and also the value of the iris reactance  $X_{iris}$  in the TEN model that are necessary for a match. Use the smallest value of  $d$  possible. Assume that the characteristic impedance of the transmission lines in the TEN is the same as the wave impedance of the  $TE_{10}$  mode. It is recommended that you use the Smith chart to solve this problem.
- 10) Assume that we have a rectangular waveguide that is to be modeled using a TEN. Assume that voltage on the TEN represents the voltage drop across the  $y$  dimension of the waveguide

along the centerline ( $x = a/2$ ), along a path that goes from the bottom wall to the top wall. Assume that we also want the power flowing in the waveguide to be the same as the power flowing on the TEN. Find formulas for the coefficients  $C_1$  and  $C_2$ , and the characteristic impedance  $Z_0$  of the TEN transmission line. Use the same definitions of the normalized modal fields as in the class notes (i.e., the definition of the functions  $\underline{e}_t(x,y)$  and  $\underline{h}_t(x,y)$ ). Is the characteristic impedance of the transmission line in the TEN the same as the wave impedance of the waveguide mode?