ECE 5317/6351

Microwave Engineering

Fall 2019

Homework #5

Text: *Microwave Engineering* by David M. Pozar, 4th edition, Wiley, 2011.

1) A 75 [Ω] coaxial line used for TV has an outer radius of b = 0.25 [cm] and an inner radius of a = 0.039 [cm]. The coax is filled with Teflon (ε_r = 2.2) that has a loss tangent of 0.001. The conductors are made of copper, which is nonmagnetic. Assume that the conductivity of copper is 3.0×10^7 [S/m]. Make a table that shows the following parameters:

   - The phase velocity
   - The group velocity

   Your table should include the following frequencies: 1 [kHz], 10 [kHz], 100 [kHz], 1 [MHz], 10 [MHz], 100 [MHz], 1 [GHz].

   For the group velocity, calculate the derivative $d\beta/d\omega$ numerically using the central difference formula for approximating a derivative:

$$f'(x) \approx \frac{f\left(x+\frac{\Delta x}{2}\right)-f\left(x-\frac{\Delta x}{2}\right)}{\Delta x}.$$

   Try experimenting with different values for $\Delta\omega$ to make sure that your results are accurate to at least four significant figures.

   Comment on the agreement between the two velocities. Is there much dispersion? That is, does the phase velocity change much with frequency?

2) Assume that a waveguiding system has the property that the group velocity is less than the phase velocity (this is called "normal dispersion"). From this, prove that

$$\frac{d\overline{\beta}}{d\omega} > 0 ,$$

where

$$\overline{\beta} \equiv \beta / k_0 .$$

This means that the normalized phase constant must be an increasing function of frequency. This is the situation for many common waveguiding systems such as rectangular waveguide and microstrip line.

Hint: Start from the above equation for the normalized phase constant. Then take the derivative of both sides with respect to ω using the chain rule, and note that $\beta / c = k_0 / v_p$.

3) Assume that we have a microstrip line on Rogers RT/duroid 5880 ($\varepsilon_r = 2.2$) with a substrate thickness h of 62 mils (1.575 mm) at a frequency of 1 GHz. The strip width is 5.0 mm. Find the phase and group velocity at 1.0 GHz, 10.0, and 50 GHz. Use the effective relative permittivity ε_r^{eff} from the CAD formulas in the class notes to determine the phase constant. (The CAD formulas are the more accurate ones that account for frequency dispersion.) For the group velocity, use a numerical derivative as discussed above in Prob. 1. You can ignore the dielectric and conductor losses, and also ignore the thickness of the metal strip. (The thickness of the metal strip is important for loss calculations, but not for the effective permittivity.)

4) Consider a lossless grounded dielectric slab having a relative permittivity of 2.2 and a thickness of 50 mils (1.534 mm). Use a numerical search to find the normalized phase constant β / k_0 of the TM_0 surface-wave mode at the following frequencies: 1 GHz, 10 GHz, 100 GHz. Compare with the approximate formula given in the notes (which is accurate when the slab is thin compared to a wavelength).

5) For the above problem, determine the cutoff frequency of the next two modes that can propagate on the grounded slab. Clearly identify which modes these are.

6) A parallel RLC circuit has R = 1.0 MΩ, L = 3.0 mH, and C = 20.0 pF. Determine the complex resonance frequency ω_0 and the Q of the resonant circuit.

7) Assume that the RLC resonator in the problem above initially has a certain amount of stored energy at $t = 0$. The fields of the resonator are then allowed to oscillate and decay naturally. How long will it take before the stored energy is 10% of the initial value?

8) Find the resonance frequencies of the first three modes (i.e., the modes having the lowest cutoff frequency) of a rectangular dielectric-filled resonator, having dimensions a = 1 cm, b = 0.5 cm, and h = 2.0 cm, assuming that the resonator is filled with lossless Teflon having ε_r = 2.1, and having a PEC boundary on all six sides. Clearly indicate for each mode if it is TE_z or TM_z, and what the indices (m,n,p) are.

9) Use the transverse resonance method to formulate a transcendental equation for the wavenumber k_z of a TE_x surface wave on a lossless grounded slab. The slab has relative permittivity ε_r and relative permeability μ_r, and thickness h.

10) Use the transverse resonance method to formulate a transcendental equation for the wavenumber k_z of a TM_x surface wave on the structure shown below. Assume that there is no y variation of the mode.

Because of the symmetry, two types of TM_x modes can exist: even and odd. An even mode is one for which the horizontal component of the electric field is an even function of x (i.e., even with respect to the center of the structure). An odd mode is one for which the horizontal component of the electric field is an odd function of x (i.e., odd with respect to the center of the structure). Derive transcendental equations for both types of TM_x modes. Place the reference plane at $x = d/2$. You should therefore have a final transcendental equation for two cases: TM_x odd and TM_x even.

Hint: Think about what type of boundary condition (PEC or PMC) would exist at $x = 0$ for even and odd modes. Justify your reasoning.