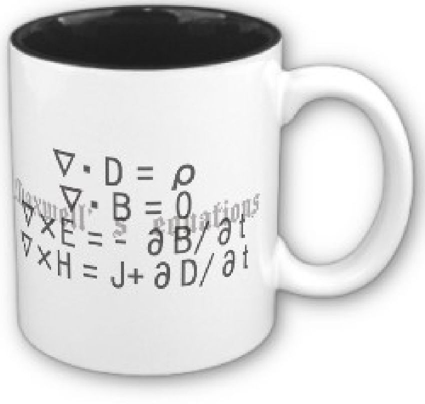
ECE 6340
Intermediate EM Waves

Fall 2016

Prof. David R. Jackson
Dept. of ECE



Notes 2

Constitutive Relations

Free Space:

$$\underline{\mathscr{D}} = \varepsilon_0 \underline{\mathscr{E}}$$

$$\underline{\mathscr{B}} = \mu_0 \underline{\mathscr{H}}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Since 1983,

$$c \equiv 2.99792458 \times 10^8 \quad [\text{m/s}]$$ (exact value)

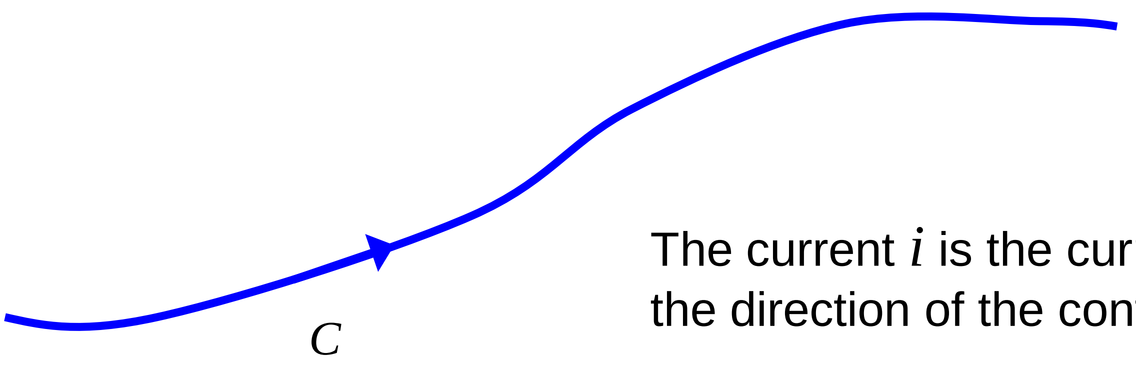
Also, $$\mu_0 = 4\pi \times 10^{-7} \quad [\text{H/m}]$$ (exact value)

Constitutive Relations

Lorentz Force law (review):

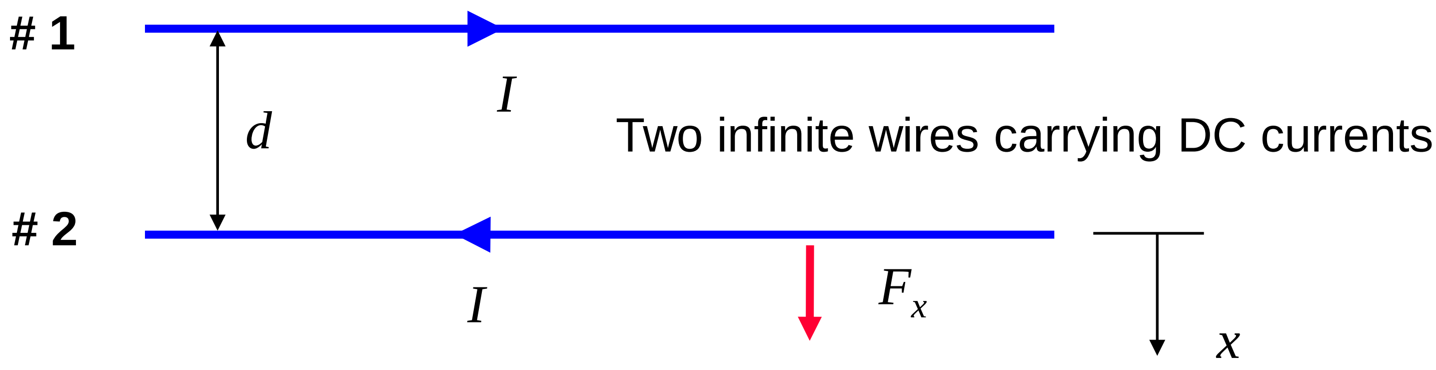
$$\underline{\mathscr{F}} = q\left(\underline{\mathscr{E}} + \underline{v} \times \underline{\mathscr{B}}\right) \quad \text{Particle}$$

$$\underline{\mathscr{F}} = i\int_C d\,\underline{l} \times \underline{\mathscr{B}} \quad \text{Wire}$$



The current i is the current flowing on the wire in the direction of the contour C.

Constitutive Relations (cont.)



$$F_{x2} = \frac{\mu_0 I^2}{2\pi d} \ [\mathrm{N/m}] \quad \Rightarrow \quad \mu_0 = \frac{2\pi d F_{x2}}{I^2}$$

Definition of I =1 Amp: $F_{x2} = 2 \times 10^{-7} \ [\mathrm{N/m}]$ when $d = 1 \ [\mathrm{m}]$

Hence $\mu_0 = 4\pi \times 10^{-7} \quad [\mathrm{H/m}]$

Constitutive Relations (cont.)

$$\varepsilon_0 = \frac{1}{\mu_0 c^2}$$

$$\varepsilon_0 \doteq 8.8541878176 2039 \times 10^{-12} \quad [\mathrm{F/m}]$$

Phasor Domain:

$$\underline{D} = \varepsilon_0 \underline{E}$$

$$\underline{B} = \mu_0 \underline{H}$$

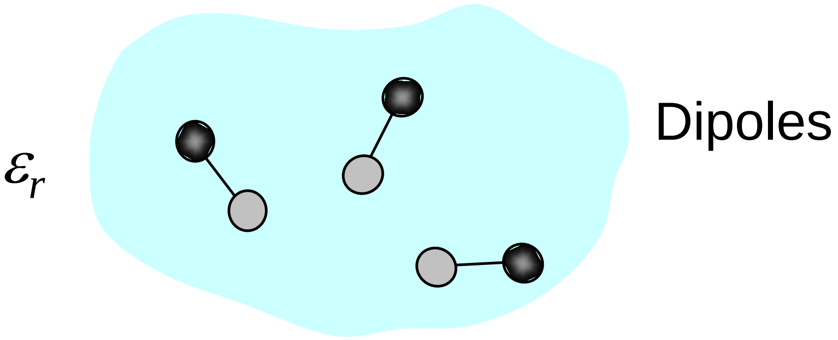
Simple Linear Media

There is a simple linear relationship between the fields in the time domain, and there is thus <u>no loss</u> due to molecular or atomic friction.

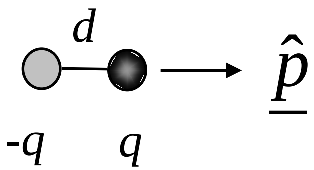
$$\underline{\mathcal{D}} = \varepsilon\,\underline{\mathcal{E}} = \varepsilon_0 \varepsilon_r\,\underline{\mathcal{E}}$$

$$\underline{\mathcal{B}} = \mu\,\underline{\mathcal{H}} = \mu_0\,\mu_r\,\underline{\mathcal{H}}$$

Atomic picture for permittivity:



Dipole moment of single molecule:



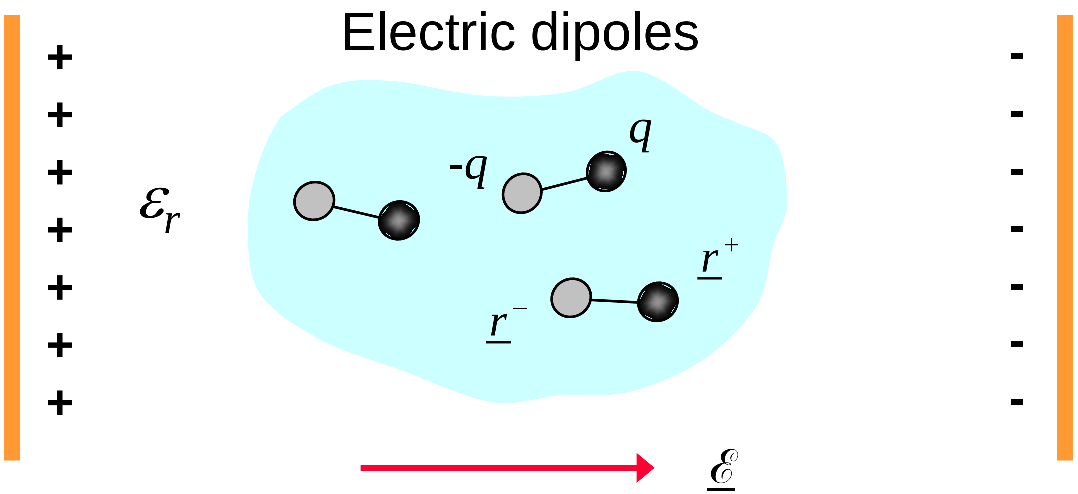
$$\underline{\mathscr{p}} = (q\,d)\,\underline{\hat{p}}$$

Simple Linear Media (cont.)

Torque on dipole: $\underline{T}_E = \underline{p} \times \underline{\mathcal{E}}$

$$\underline{T}_E = \left(\underline{r}^+ \times \underline{\mathcal{F}}^+ + \underline{r}^- \times \underline{\mathcal{F}}^-\right) = \left(q\underline{r}^+ - q\underline{r}^-\right) \times \underline{\mathcal{E}} = q\left(\underline{\hat{p}}\, d\right) \times \underline{\mathcal{E}} = \underline{p} \times \underline{\mathcal{E}}$$

Applied electric field:



Dipole moment per unit volume:

$$\underline{\mathcal{P}} \equiv \frac{1}{\Delta V} \sum_{\Delta V} \underline{p}_i$$

Simple Linear Media (cont.)

Dipole moment per unit volume: $\underline{\mathscr{P}} \equiv \frac{1}{\Delta V} \sum_{\Delta V} \underline{p}_i$

Definition of $\underline{\mathscr{D}}$ vector: $\underline{\mathscr{D}} \equiv \varepsilon_0 \underline{\mathscr{E}} + \underline{\mathscr{P}}$

Note: $\underline{\mathscr{E}}$ is the average electric field inside the material (what we would use to calculate macroscopic voltage drop).

Simple linear media: $\underline{\mathscr{P}} = \varepsilon_0 \chi_e \underline{\mathscr{E}}$

so $\underline{\mathscr{D}} = \varepsilon_0 \left(1 + \chi_e\right) \underline{\mathscr{E}}$

Define:

$\varepsilon_r = \left(1 + \chi_e\right)$

Then $\underline{\mathscr{D}} = \varepsilon_0 \varepsilon_r \underline{\mathscr{E}}$

Note that usually $\chi_e > 0$

$\left(\varepsilon_r > 1\right)$

Simple Linear Media (cont.)

Some Common Materials

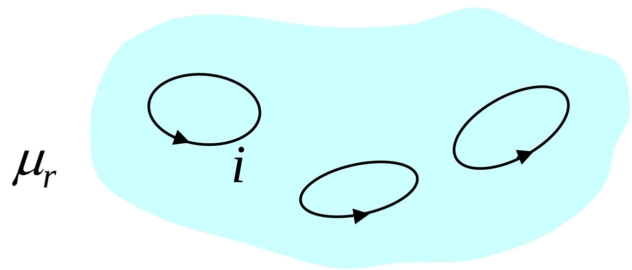
(low frequency, less than 1 GHz)

Material	Relative Permittivity ε_r
Vacuum	1
Air	1.00058986
Styrofoam	1.03
Teflon	2.1
Polyethylene	2.25
Soil	$2 < \varepsilon_r < 4$
Quartz	4
Water	81
Barium Strontium Titanate	500

Note: Water has a very polar molecule that is also fairly free to rotate (since it is a liquid).

Simple Linear Media (cont.)

Magnetic media:

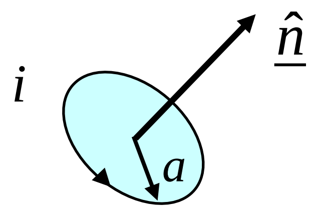


Magnetic dipoles

(from electron spin)

Torque on dipole:

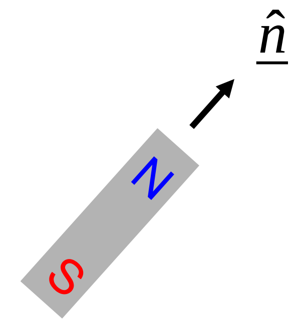
$$\underline{T}_B = \underline{m} \times \underline{\mathcal{B}}$$



$$\underline{m} = \underline{\hat{n}}\left(i\,A\right), \quad A = \pi\, a^2$$

Magnetic moment per unit volume:

$$\underline{\mathcal{M}} \equiv \frac{1}{\Delta V} \sum_{\Delta V} \underline{m}_i$$



Each magnetic dipole acts like a small bar magnet.

Definition of $\underline{\mathcal{H}}$ vector:

$$\underline{\mathcal{H}} \equiv \frac{1}{\mu_0} \underline{\mathcal{B}} - \underline{\mathcal{M}}$$

so $$\underline{\mathcal{B}} = \mu_0\, \underline{\mathcal{H}} + \mu_0\, \underline{\mathcal{M}}$$

Simple Linear Media (cont.)

Simple linear media: $\underline{\mathscr{M}} = \chi_m \underline{\mathscr{H}}$

Note:
$\underline{\mathscr{H}}$ is the *average* magnetic field inside the material.

so

$$\underline{\mathscr{B}} = \mu_0 \underline{\mathscr{H}} + \mu_0 \underline{\mathscr{M}}$$
$$= \mu_0 \underline{\mathscr{H}} + \mu_0 \chi_m \underline{\mathscr{H}}$$
$$= \mu_0 \left(1 + \chi_m\right) \underline{\mathscr{H}}$$

Define:

$$\mu_r = \left(1 + \chi_m\right)$$

Then

$$\underline{\mathscr{B}} = \mu_0 \mu_r \underline{\mathscr{H}}$$

Simple Linear Media (cont.)

Some Common Materials

Material	Relative Permeability μ_r
Vacuum	1
Air	1.0000004
Water	0.999992
Copper	0.999994
Aluminum	1.00002
Silver	0.99998
Nickel	600
Iron	5000
Carbon Steel	100
Transformer Steel	2000
Mumetal	50,000
Supermalloy	1,000,000

Note: Values can often vary depending on purity and processing.

http://en.wikipedia.org/wiki/Permeability_(electromagnetism)

Summary

Simple Linear Media (lossless)

$$\underline{\mathscr{D}} = \varepsilon \underline{\mathscr{E}} = \varepsilon_0 \varepsilon_r \underline{\mathscr{E}}$$

$$\underline{\mathscr{B}} = \mu \underline{\mathscr{H}} = \mu_0 \mu_r \underline{\mathscr{H}}$$

Phasor Domain:

$$\underline{D} = \varepsilon \underline{E} = \varepsilon_0 \varepsilon_r \underline{E}$$

$$\underline{B} = \mu \underline{H} = \mu_0 \mu_r \underline{H}$$

Note:
For simple linear media, the relative permittivity and permeability are real.

Generalized Linear Media

This accounts for molecular or atomic friction, which results in material loss.

$$\underline{\mathscr{D}} = a_0\,\underline{\mathscr{E}} + a_1 \frac{\partial \underline{\mathscr{E}}}{\partial t} + a_2 \frac{\partial^2 \underline{\mathscr{E}}}{\partial t^2} + \ldots$$

friction term

Phasor Domain:

$$\underline{D} = a_0\,\underline{E} + j\omega a_1\,\underline{E} - \omega^2 a_2\,\underline{E} + \ldots$$

$$= \left(a_0 + j\omega a_1 - \omega^2 a_2 + \ldots\right)\underline{E}$$

Define: $$\varepsilon = \left(a_0 - a_2\omega^2 + \ldots\right) + j\left(a_1\omega + \ldots\right)$$ (complex)

We still have a linear relationship in the *phasor domain*.

Then: $$\underline{D} = \varepsilon\,\underline{E}$$

Similarly: $$\underline{B} = \mu\,\underline{H}$$

$$\varepsilon = \varepsilon' - j\varepsilon''$$

$$\mu = \mu' - j\mu''$$

Anisotropic Media

$$\underline{D} = \underline{\underline{\varepsilon}} \cdot \underline{E}$$

$$\underline{B} = \underline{\underline{\mu}} \cdot \underline{H}$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

Anisotropic Media (cont.)

Isotropic:

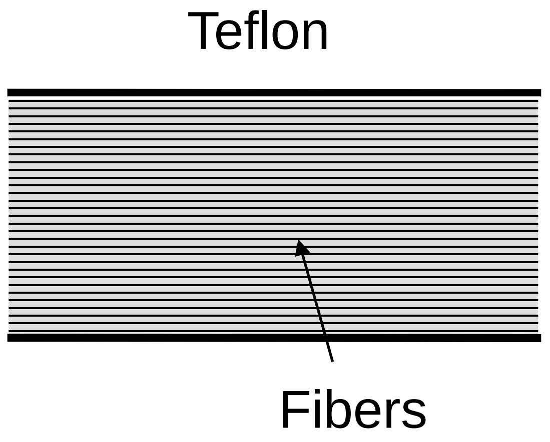
$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon$$

$$\varepsilon_{ij} = 0, \qquad i \neq j$$

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}$$

Uniaxial:

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_h & 0 & 0 \\ 0 & \varepsilon_h & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$



Example: a microwave substrate board

Anisotropic Media (cont.)

Biaxial:

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

Biased Ferrite:

$$\underline{\underline{\mu}} = \mu_0 \begin{bmatrix} \alpha & j\gamma & 0 \\ -j\gamma & \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\underline{\underline{\mu}}$ is <u>not</u> symmetric!

Summary of Possible Media

Linear, isotropic, homogeneous (simple or generalized):

$$\underline{D} = \varepsilon \, \underline{E}$$

- Simple: The permittivity is real (lossless).
- Generalized: The permittivity is complex (lossy).

Inhomogeneous: $\underline{D} = \varepsilon\left(\underline{r}\right)\underline{E}$

Anisotropic:

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

Nonlinear: $\underline{D} = \varepsilon\left(\underline{E}\right)\underline{E}$