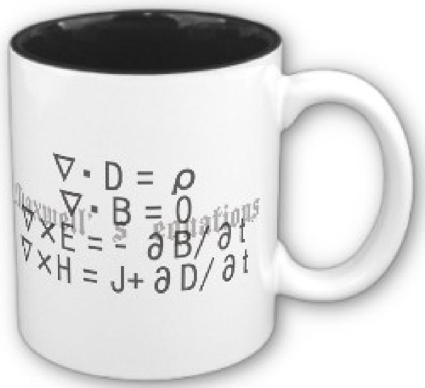
ECE 6340
Intermediate EM Waves

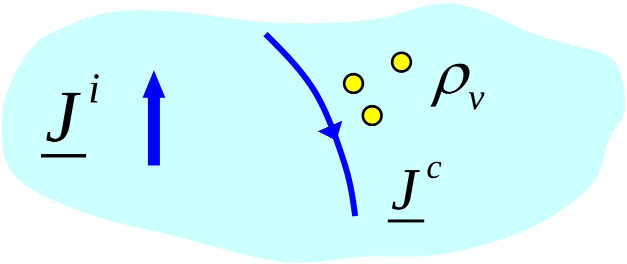
Fall 2016

Prof. David R. Jackson
Dept. of ECE



Notes 3

Types of Current



Note: The free-charge density ρ_v refers to those charge carriers (either positive or negative) that are free to move (usually electrons or ions). It is zero for perfect insulators.

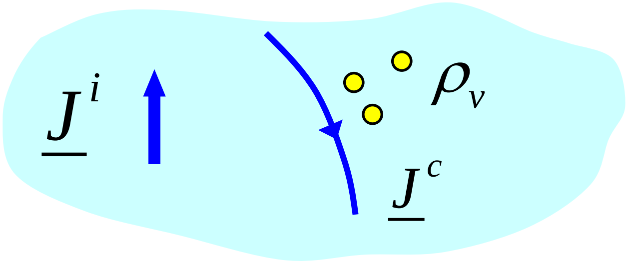
$\underline{J}^i$ impressed current (source)

$\underline{J}^c$ conduction (ohmic) current $\quad \underline{J}^c = \rho_v \, \underline{v}$

Linear medium: $\quad \underline{J}^c = \sigma \underline{E} \quad$ (Ohm's law)

Note: The electric field is set up in response to the impressed current source.

Types of Current (cont.)



Ampere's law:

$$\nabla \times \underline{H} = \underline{J} + j\omega\varepsilon\underline{E}$$

$$\nabla \times \underline{H} = \underline{J}^i + \sigma\underline{E} + j\omega\varepsilon\underline{E}$$

Source Conduction Displacement

Effective Permittivity

$$\nabla \times \underline{H} = \underline{J} + j\omega\varepsilon\underline{E}$$

$$= \underline{J}^i + \sigma\underline{E} + j\omega\varepsilon\underline{E}$$

$$= \underline{J}^i + (\sigma + j\omega\varepsilon)\underline{E}$$

$$= \underline{J}^i + j\omega\left(\varepsilon + \frac{\sigma}{j\omega}\right)\underline{E}$$

$$= \underline{J}^i + j\omega\left(\varepsilon - j\frac{\sigma}{\omega}\right)\underline{E}$$

Effective Permittivity (cont.)

Define:

$$\varepsilon_c \equiv \varepsilon - j\frac{\sigma}{\omega}$$

This "effective" permittivity accounts for the conductivity.

Note: If there is polarization loss (molecular or atomic friction), than ε will be complex in addition to ε_c.

Ampere's law becomes:

$$\nabla \times \underline{H} = \underline{J}^i + j\omega\varepsilon_c\,\underline{E}$$

Ampere's law thus becomes in the same form as for free space:

$$\nabla \times \underline{H} = \underline{J}^i + j\omega\varepsilon_0\,\underline{E}$$

Effective Permittivity (cont.)

Note: ε_c is often called ε for simplicity in most books.

However, be careful!

$$\underline{D} = \varepsilon \underline{E}$$

$$\underline{D} \neq \varepsilon_c \underline{E}$$

Even though the effective permittivity appears in Ampere's law, it is the actual permittivity that relates the flux density to the electric field.

Effective Permittivity Principle

This principle allows us to solve problems involving a homogeneous (lossy) material, as long as we know how to solve the corresponding free-space problems.

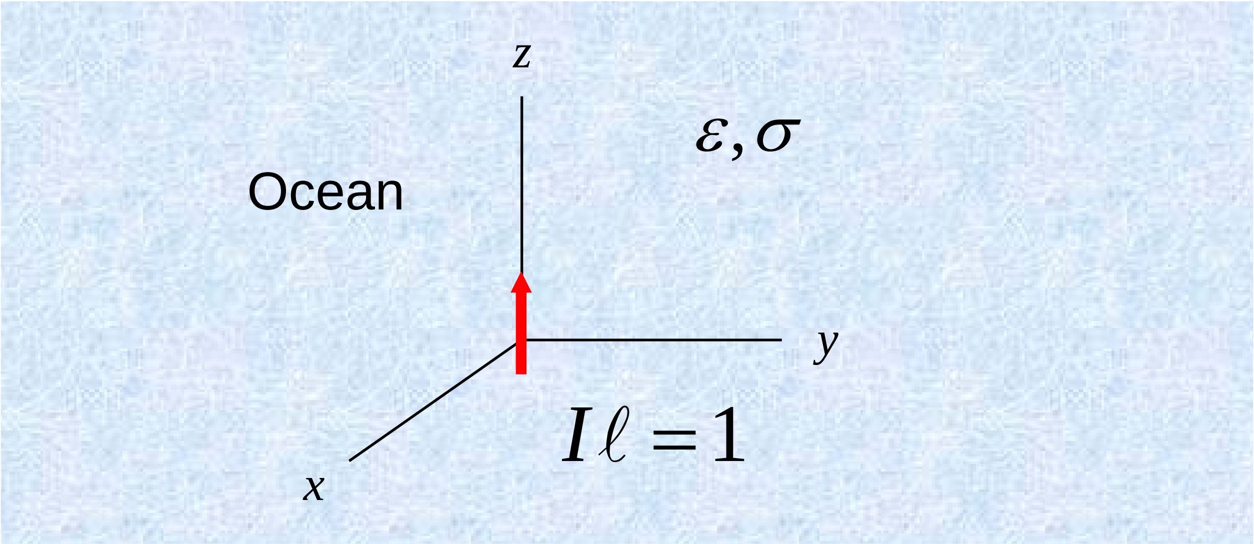
$$\nabla \times \underline{H} = \underline{J}^{i} + j\omega\varepsilon_0 \underline{E}$$ (Free-space problem)

$$\downarrow \qquad \varepsilon_0 \rightarrow \varepsilon_c$$

$$\nabla \times \underline{H} = \underline{J}^{i} + j\omega\varepsilon_c \underline{E}$$ (Material problem)

The formulas for the fields remains the same: we simply make this simple substitution.

Example

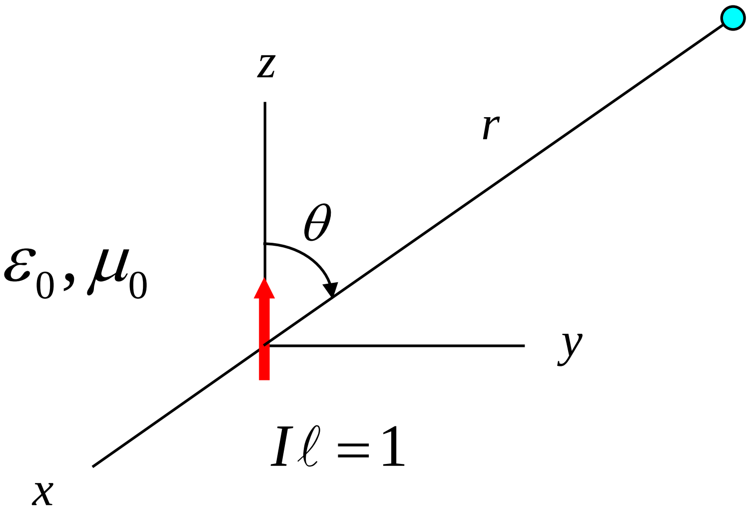


A dipole is embedded in an infinite medium of ocean water. What is the far-field of the dipole?

First examine problem in *free space* (next slide).

Example (cont.)

Dipole in free space:

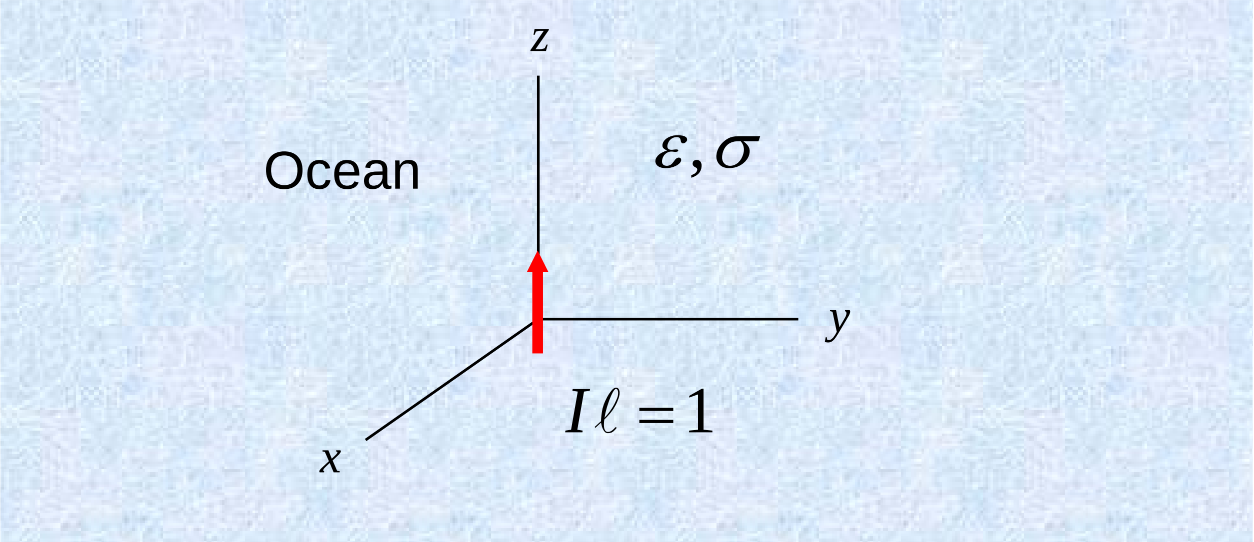


As $r \to \infty$

$$E_\theta = \frac{j\omega\mu_0}{4\pi r} \sin\theta \, e^{-jk_0 r}, \quad k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$$

Example (cont.)

Dipole in ocean:



As $r \to \infty$

$$E_\theta = \frac{j\omega\mu_0}{4\pi r} \sin\theta \, e^{-jk_1 r}, \quad k_1 = \omega\sqrt{\mu_0 \varepsilon_c} = k_1' - jk_1''$$

$$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega} \qquad \varepsilon = \varepsilon_0 \varepsilon_r = \varepsilon_0 \left(\varepsilon_r' - j\varepsilon_r''\right)$$

Loss Tangent

$$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}$$

Write this as:

$$\varepsilon_c = \varepsilon_c' - j\varepsilon_c''$$

The loss tangent is defined as:

$$\tan\delta \equiv \frac{\varepsilon_c''}{\varepsilon_c'}$$

Note: The loss tangent combines losses from atomic and molecular friction together with loss from conductivity.

$$\varepsilon_c' = \mathrm{Re}(\varepsilon)$$

$$\varepsilon_c'' = -\mathrm{Im}(\varepsilon) + \frac{\sigma}{\omega}$$

Note: In most books, the symbol ε is used to denote ε_c in the time-harmonic steady state.

Loss Tangent

Some Common Materials

f = 3 GHz

Material	$\tan\delta$
Water (pure)	0.156
FR4	0.018
Duroid board (typical)	0.001
Polyethelene	0.00031
Teflon	0.00014
Quartz	0.000061
Sapphire	0.00002

Polarization Current

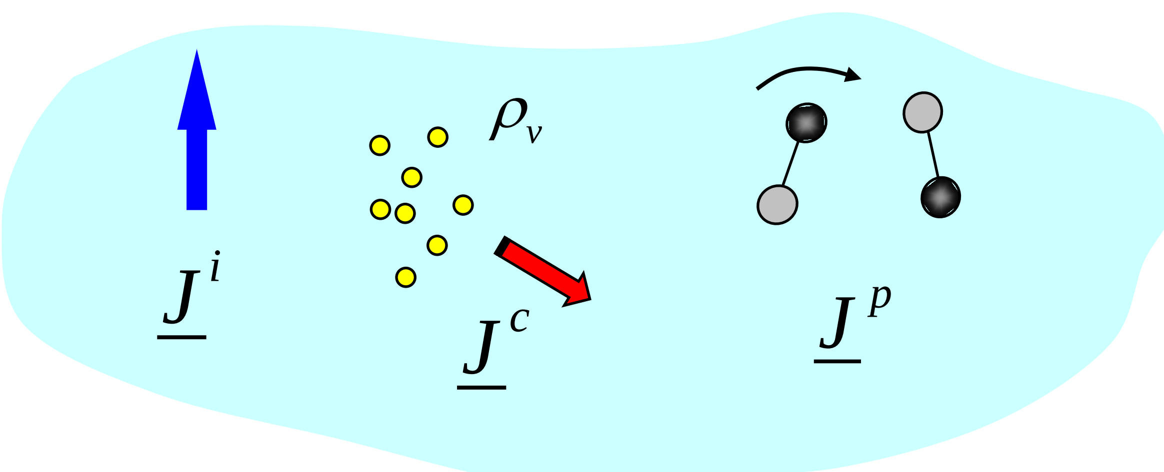
$$\nabla \times \underline{H} = \underline{J}^{i} + j\omega\varepsilon_c \underline{E}$$

$$= \underline{J}^{i} + \sigma \underline{E} + j\omega\varepsilon \underline{E}$$

$$= \underline{J}^{i} + \sigma \underline{E} + j\omega\varepsilon_0 \underline{E} + j\omega\left(\varepsilon - \varepsilon_0\right)\underline{E}$$

Source	Conduction $\underline{J}^{c}$	Free-space displacement	Polarization $\underline{J}^{p}$

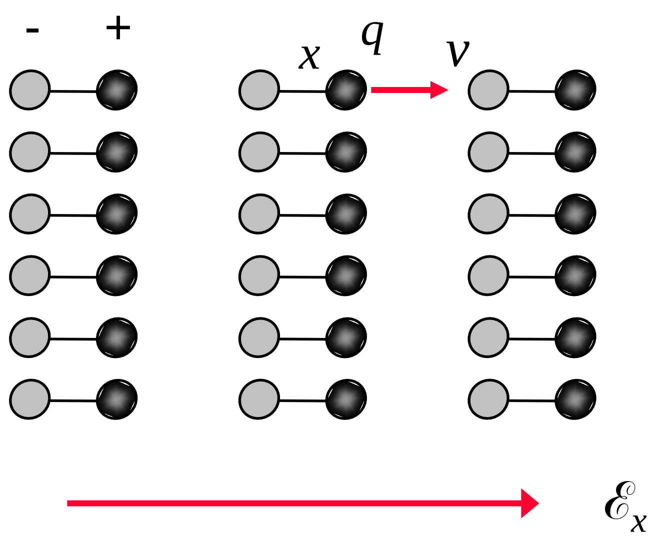
Four types of current density (nonmagnetic medium)



Note: The free-space displacement current is not an actual current flow.

Polarization Current (cont.)

Model of polarization current:



N_d dipoles per unit volume

The dipoles stretch rather than rotate.

As the electric field changes, we imagine that the position x of the positive charge changes, with the negative charge being stationary.

$$\mathscr{P}_x = (qx)N_d$$

$$\Rightarrow \frac{d\mathscr{P}_x}{dt} = (N_d q)\frac{dx}{dt} = (N_d q)v$$

From the charge-current equation:

$$\mathscr{J}_x^{\,p} = \rho_v^q v = (qN_d)v$$

Hence $\mathscr{J}_x^{\,p} = \dfrac{d\mathscr{P}_x}{dt}$

Polarization Current (cont.)

In general,

$$\underline{\mathcal{J}}^{p} = \frac{d\underline{\mathcal{P}}}{dt}$$

Time-harmonic steady state:

$$\underline{J}^{p} = j\omega\underline{P} = j\omega\varepsilon_0\chi_e\underline{E} = j\omega\varepsilon_0\left(\varepsilon_r - 1\right)\underline{E} = j\omega\left(\varepsilon - \varepsilon_0\right)\underline{E}$$

$$\underline{J}^{p} = j\omega\left(\varepsilon - \varepsilon_0\right)\underline{E}$$

This agrees with the conclusion from Amperes' law.

Polarization Current (cont.)

If magnetic material is present: $\mathscr{H} \equiv \frac{1}{\mu_0}\underline{\mathscr{B}} - \underline{\mathscr{M}}$ ⇨ $H = \frac{1}{\mu_0}\underline{B} - \underline{M}$

$$\nabla \times \left(\frac{1}{\mu_0}\underline{B} - \underline{M} \right) = \underline{J}^i + j\omega\varepsilon_c\,\underline{E}$$

$$= \underline{J}^i + \sigma\underline{E} + j\omega\varepsilon\underline{E}$$

$$= \underline{J}^i + \sigma\underline{E} + j\omega\varepsilon_0\underline{E} + j\omega\left(\varepsilon - \varepsilon_0\right)\underline{E}$$

LHS is that of Ampere's law in free space.

$$\nabla \times \left(\frac{1}{\mu_0}\underline{B} \right) = \underline{J}^i + \sigma\underline{E} + j\omega\varepsilon_0\underline{E} + j\omega\left(\varepsilon - \varepsilon_0\right)\underline{E} + \nabla \times \underline{M}$$

Polarization current from dielectric properties

Polarization current from magnetic properties

Polarization Current (cont.)

$$\nabla \times \left(\frac{1}{\mu_0} \underline{B} \right) = \underline{J}^{i} + \sigma \underline{E} + j\omega\varepsilon_0 \underline{E} + j\omega(\varepsilon - \varepsilon_0)\underline{E} + \nabla \times \underline{M}$$

Source

Conduction $\underline{J}^{c}$

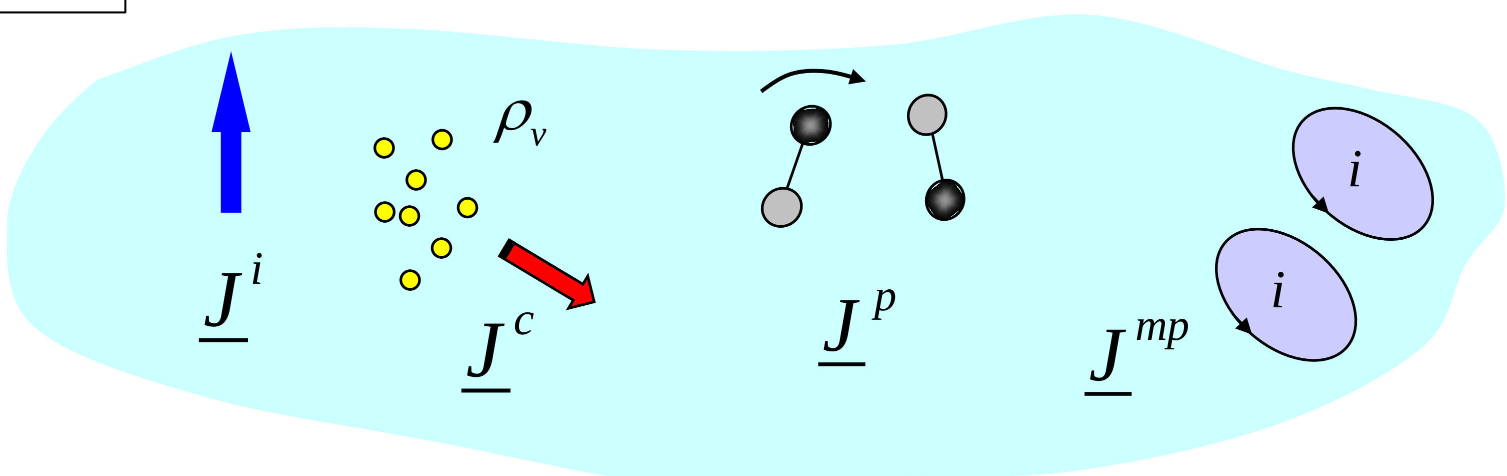
Free-space displacement

Polarization $\underline{J}^{p}$

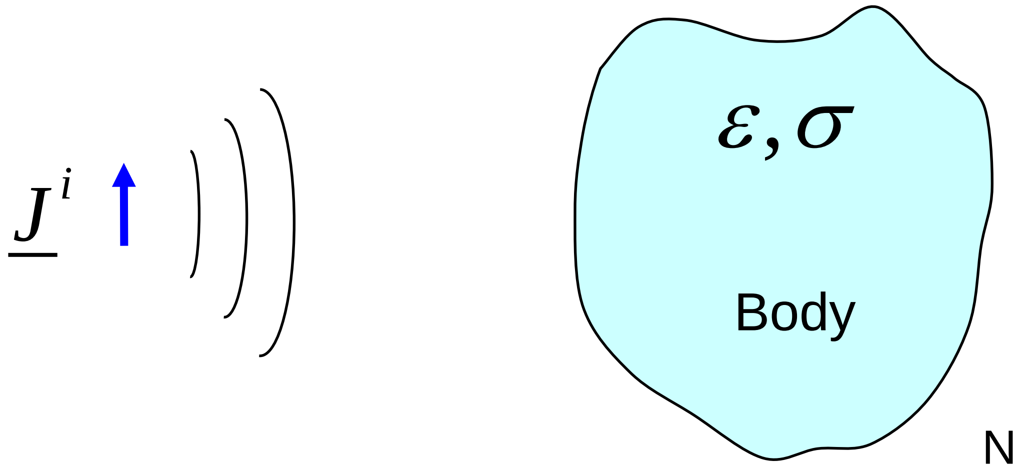
Magnetic polarization $\underline{J}^{mp}$

Note: The free-space displacement current is not an actual current flow.

Five types of current density



Equivalent Current



$$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}$$

Nonmagnetic body

Inside the body,

$$\nabla \times \underline{H} = j\omega\varepsilon_c\,\underline{E}$$
$$= j\omega\varepsilon_0\underline{E} + j\omega\left(\varepsilon_c - \varepsilon_0\right)\underline{E}$$

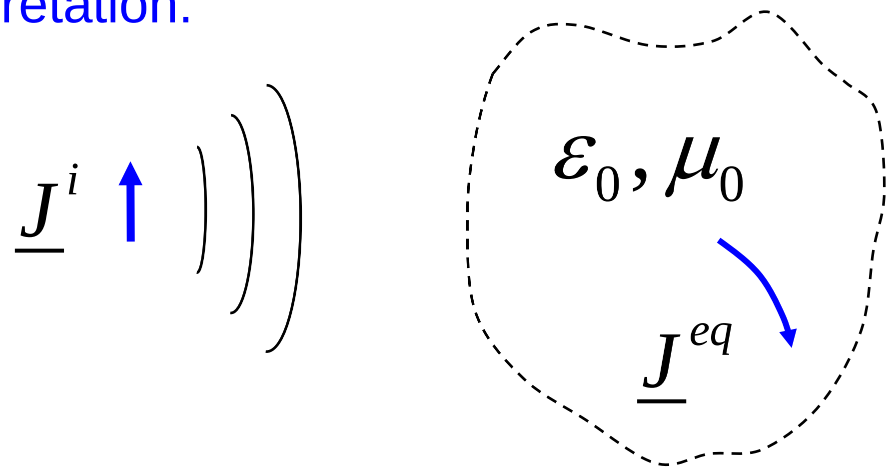
Define:

$$\underline{J}^{eq} \equiv j\omega\left(\varepsilon_c - \varepsilon_0\right)\underline{E}$$

Equivalent Current (cont.)

$$\nabla \times \underline{H} = j\omega\varepsilon_0 \underline{E} + \underline{J}^{eq}$$

Interpretation:



The body is replaced by its equivalent current in free space.

Note: The equivalent current is unknown, since the electric field inside the body is unknown.

Equivalent Current (cont.)

The equivalent current combines the conduction current and the polarization current.

$$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}$$

$$\underline{J}^{eq} \equiv j\omega\left(\varepsilon_c - \varepsilon_0\right)\underline{E}$$

$$= j\omega\left(\varepsilon - \varepsilon_0\right)\underline{E} + \sigma\underline{E}$$

so

$$\underline{J}^{eq} = j\omega\left(\varepsilon - \varepsilon_0\right)\underline{E} + \sigma\underline{E}$$

Polarization current — $j\omega\left(\varepsilon - \varepsilon_0\right)\underline{E}$

Conduction current — $\sigma\underline{E}$