ECE 6340
Intermediate EM Waves

Fall 2016

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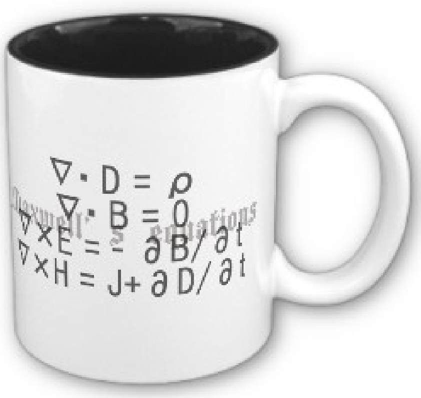
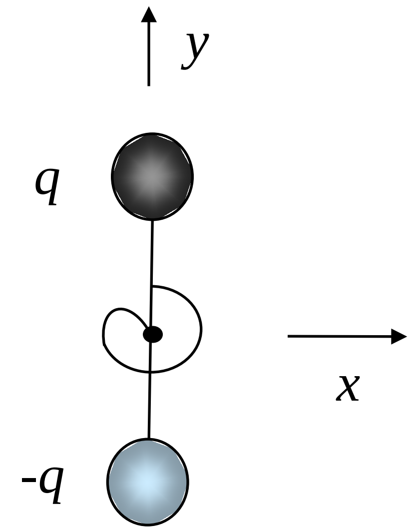


Notes 4

Debye Model

This model explains molecular effects.

Molecule:



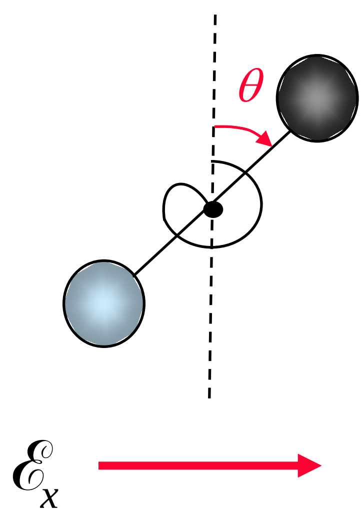
Molecule at rest

$$p_x = 0$$

We consider an electric field applied in the x direction.

The dipole moment p_x of this single molecule represents the <u>average</u> dipole moment in the x direction for all of the dipoles in a little volume.

A zero dipole moment p_x corresponds to a <u>random</u> dipole alignment in the actual material.



Molecule with applied field $\mathcal{E}_x$

$$p_x > 0$$

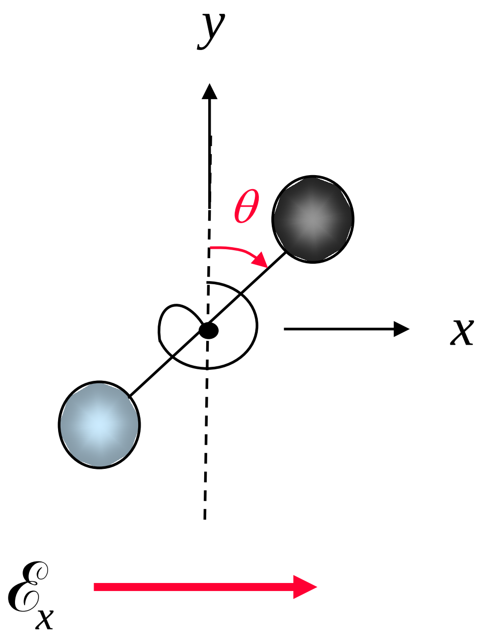
Debye Model (cont.)

Torque on dipole due to electric field:

$$\underline{T}_E = \underline{r}^+ \times \left(q\underline{\mathscr{E}}\right) + \underline{r}^- \times \left(-q\underline{\mathscr{E}}\right)$$

$$= \left(\underline{r}^+ - \underline{r}^-\right) \times q\underline{\mathscr{E}}$$

$$= \underline{p} \times \underline{\mathscr{E}}$$

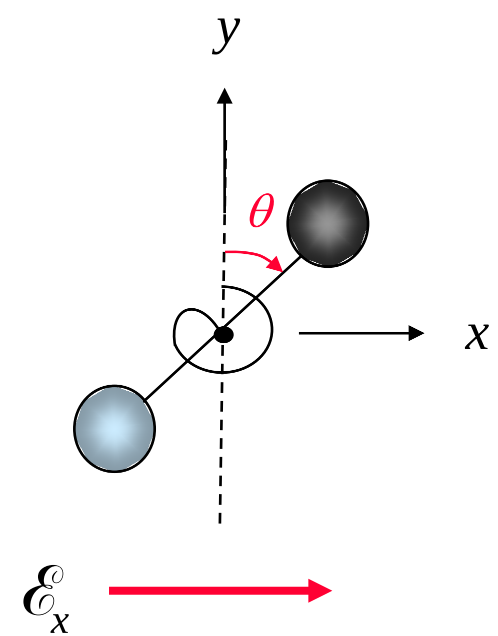


Debye Model (cont.)

$$T = I\frac{d^2\theta}{dt^2}$$

Note: $T = -T_z$

$$T = T_E + T_S + T_F$$



$$\underline{T}_E = \underline{p} \times \underline{\mathcal{E}} = -\underline{\hat{z}}\, p\, \mathcal{E}_x \sin\left(\frac{\pi}{2} - \theta\right)$$
$$= qd\,\mathcal{E}_x \cos\theta$$

$$T_E = qd\,\mathcal{E}_x \cos\theta$$

$$T_S = -s\theta$$

$$T_F = -c\frac{d\theta}{dt}$$

s = spring constant

c = friction constant

Debye Model (cont.)

Hence

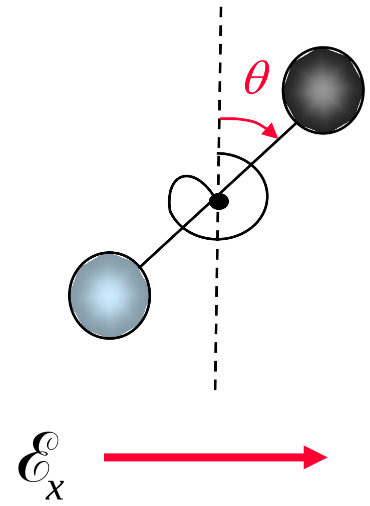
$$q\,d\,\mathscr{E}_x\cos\theta - s\theta - c\frac{d\theta}{dt} = I\frac{d^2\theta}{dt^2}$$

Assume $\theta << 1, \quad \cos\theta \approx 1$ (small fields)

$$q\,d\,\mathscr{E}_x \approx s\theta + c\frac{d\theta}{dt} + I\frac{d^2\theta}{dt^2}$$

Debye Model (cont.)

$$q\,d\,\mathcal{E}_x \approx s\theta + c\frac{d\theta}{dt} + I\frac{d^2\theta}{dt^2}$$



Note that

$$p_x = q\,d\sin\theta$$

$$\approx (q\,d)\,\theta$$

Hence:

$$\theta \approx \frac{p_x}{q\,d}$$

Insert this into the top equation.

Debye Model (cont.)

Then we have:

$$q\,d\,\mathscr{E}_x \approx s\left(\frac{\wp_x}{q\,d}\right)+c\left(\frac{1}{q\,d}\right)\frac{d\,\wp_x}{dt}+I\left(\frac{1}{q\,d}\right)\frac{d^2\wp_x}{dt^2}$$

or

$$s\,\wp_x + c\frac{d\,\wp_x}{dt}+I\,\frac{d^2\wp_x}{dt^2}=\left(q\,d\right)^2\mathscr{E}_x$$

Assume sinusoidal steady state:

$$sp_x + j\,\omega\,cp_x - \omega^2\,I\,p_x = \left(q\,d\right)^2 E_x$$

Debye Model (cont.)

Hence, we have

$$p_x = E_x \left[\frac{(q\,d)^2}{\left(s - \omega^2 I\right) + j\,\omega\,c} \right]$$

Denote $$N_m = \frac{\#\,\text{molecules}}{m^3}$$

Then we have $$P_x^M = N_m\,p_x$$

The term P denotes the total dipole moment per unit volume.

The M superscript reminds us that we are talking about molecules.

Debye Model (cont.)

Also, for a linear material, $P_x^M = \varepsilon_0 \chi_e^M E_x$

Hence $\chi_e^M = \dfrac{P_x^M}{\varepsilon_0 E_x} = \dfrac{N_m p_x}{\varepsilon_0 E_x}$

Therefore $\chi_e^M = \left(\dfrac{N_m}{\varepsilon_0}\right)(q\,d)^2 \left[\dfrac{1}{\left(s-\omega^2 I\right)+j\omega c}\right]$

Assume $\omega^2 I << s$

(The frequency is fairly low relative to molecular resonance frequencies. That is, the frequency is at millimeter wave frequency and below.)

Debye Model (cont.)

$$\chi_e^M \approx \left(\frac{N_m}{\varepsilon_0}\right)(q\,d)^2\left(\frac{1}{s}\right)\left[\frac{1}{1+j\omega\left(\frac{c}{s}\right)}\right]$$

Denote the time constant as: $\tau = \frac{c}{s}$

Denote the zero-frequency value as: $\chi_e^M(0) = \left(\frac{N_m}{\varepsilon_0}\right)(q\,d)^2\left(\frac{1}{s}\right)$

(real constant)

Then we have $\chi_e^M \approx \frac{\chi_e^M(0)}{1+j\omega\tau}$

Debye Model (cont.)

$$\chi_e^M \approx \frac{\chi_e^M(0)}{1 + j\omega\tau}$$

This would imply that

$$\varepsilon_r = 1 + \frac{\chi_e^M(0)}{1 + j\omega\tau}$$

At high frequency the molecules cannot respond to the field, so the relative permittivity due to the molecules tends to unity.

This equation gives the wrong result at high frequency, where atomic effects become important.

Debye Model (cont.)

Include BOTH molecule and atomic effects:

$$
\begin{aligned}
P_x &= P_x^M + P_x^A \\
&= \varepsilon_0 \chi_e^M E_x + \varepsilon_0 \chi_e^A E_x \\
&= \varepsilon_0 \chi_e E_x
\end{aligned}
$$

Molecule effects: $$\chi_e^M = \frac{\chi_e^M(0)}{1 + j\omega\tau}$$

Atomic effects: $$\chi_e^A = \text{constant (real)}$$

Note:
Atoms can respond much faster to the field than molecules, so the atomic susceptibility is almost constant (unless the frequency is very high, e.g., at THz frequencies and above).

Debye Model (cont.)

We then have that $\chi_e = \chi_e^M + \chi_e^A$

Hence: $$\chi_e = \frac{\chi_e^M(0)}{1 + j\omega\tau} + \chi_e^A$$

Debye Model (cont.)

Permittivity formula: $\varepsilon_r = 1 + \chi_e$

Hence, we have

$$\varepsilon_r = 1 + \chi_e^A + \frac{\chi_e^M(0)}{1 + j\omega\tau}$$

$$= a_1 + \frac{a_2}{1 + j\omega\tau}$$

where

$$a_1 = 1 + \chi_e^A$$

$$a_2 = \chi_e^M(0)$$

(a_1 and a_2 are real constants.)

Debye Model (cont.)

Note that: $$\varepsilon_r(0) = a_1 + a_2$$

$$\varepsilon_r(\infty) = a_1$$

$$\varepsilon_r = a_1 + \frac{a_2}{1 + j\omega\tau}$$

so

$$a_1 = \varepsilon_r(\infty)$$

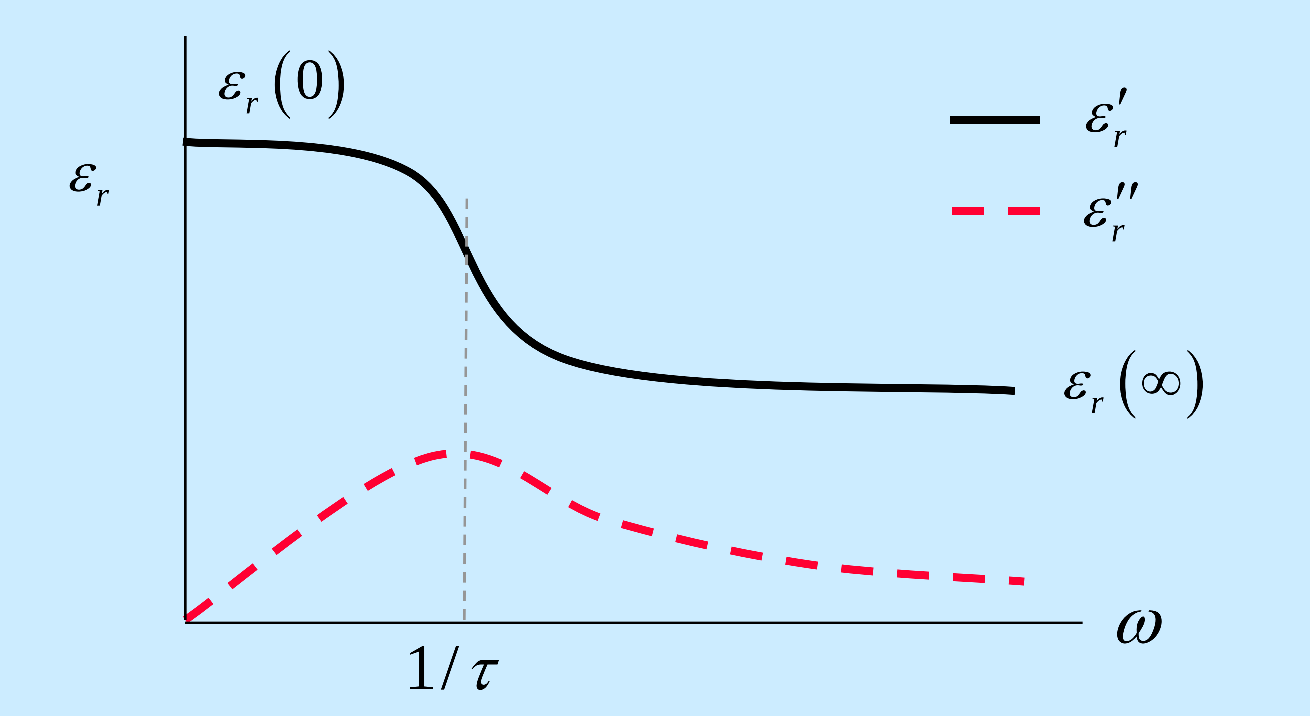
$$a_2 = \varepsilon_r(0) - \varepsilon_r(\infty)$$

Hence: $$\varepsilon_r = \varepsilon_r(\infty) + \frac{\varepsilon_r(0) - \varepsilon_r(\infty)}{1 + j\omega\tau}$$

Debye Model (cont.)

$$\varepsilon_r' = \varepsilon_r(\infty) + \frac{\varepsilon_r(0) - \varepsilon_r(\infty)}{1 + (\omega\tau)^2}$$

$$\varepsilon_r'' = \omega\tau \left[\frac{\varepsilon_r(0) - \varepsilon_r(\infty)}{1 + (\omega\tau)^2} \right]$$



Debye Model (cont.)

Frequency for maximum loss:

Let $x = \omega\tau$

$$\varepsilon_r'' = \left(\frac{x}{1+x^2}\right)\left(\varepsilon_r(0) - \varepsilon_r(\infty)\right)$$

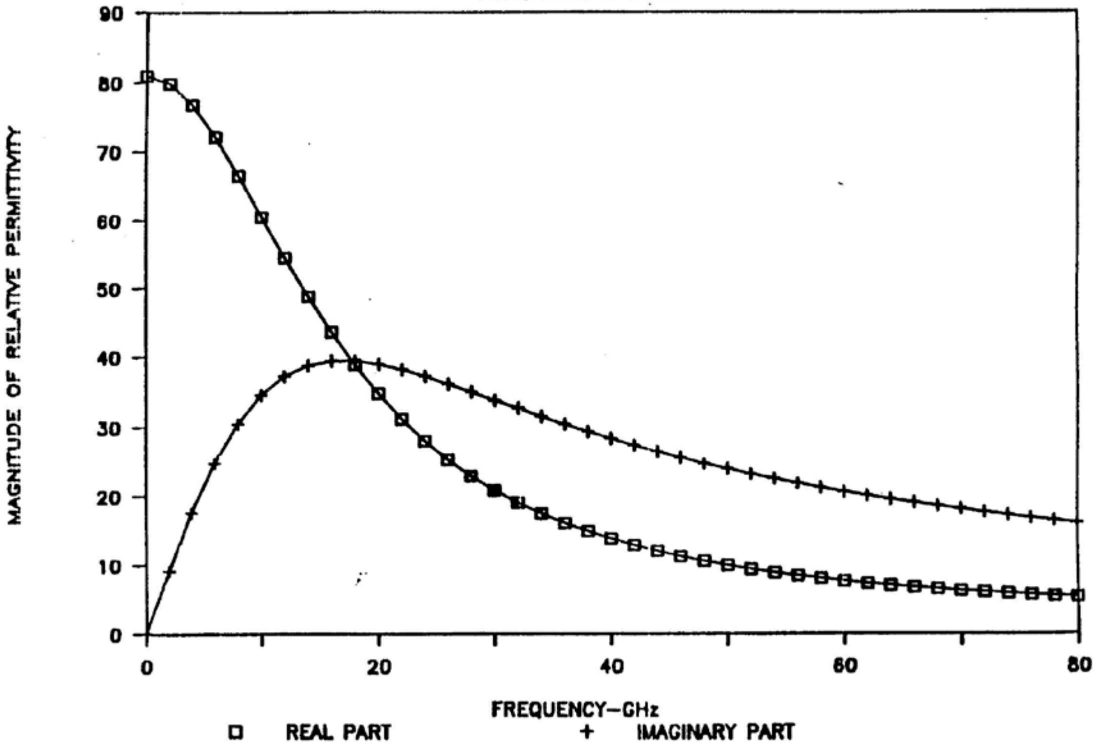
A maximum occurs at $x = 1$

or $\omega = \dfrac{1}{\tau}$

Debye Model (cont.)

Complex relative permittivity for pure (distilled) water

Water obeys the Debye model quite well.



Example

Calculate the complex relative permittivity ε_{rc} for ocean water at 10.0 GHz.

Ocean water: $\sigma = 4$ [S/m]

$$\varepsilon_{rc} = \frac{\varepsilon_c}{\varepsilon_0} = \frac{1}{\varepsilon_0}\left(\varepsilon - j\frac{\sigma}{\omega}\right) = \varepsilon_r - j\frac{\sigma}{\omega\varepsilon_0}$$

$$\varepsilon_{rc} = (60 - j35) - j\frac{4}{2\pi\left(10.0\times10^{9}\right)\left(8.854\times10^{-12}\right)}$$

from previous plot for distilled water

Hence

$$\varepsilon_{rc} = (60 - j35) - j(7.19)$$ or $$\varepsilon_{rc} = 60 - j42.19$$

Cole-Cole Model

This is a modification of the Debye model.

$$\varepsilon_r = \varepsilon_r(\infty) + \frac{\varepsilon_r(0) - \varepsilon_r(\infty)}{1 + (j\omega\tau)^{1-\alpha}}$$

When $\alpha = 0$, the model reduces to the Debye model.

This model has often been used to describe the permittivity of some polymers, as well as biological tissues.

Cole-Cole Model (Cont.)

Parameters for Some Biological Tissues

TABLE I

FITTING PARAMETERS FOR THE COMPLEX PERMITTVITY VALUES OF MUSCLE AND FAT FROM [2] FITTED TO A SINGLE-TERM COLE–COLE RELATION, FROM 0.5 TO 30 GHz

Tissue	τ_0 (ps)	ε_∞	ε_s	α	σ (s/m)	E_r	Max Error (%)	
							$\Delta\varepsilon'/\varepsilon'$(@f, GHz)	$\Delta\varepsilon''/\varepsilon''$(@f, GHz)
Muscle	7.23	2.93	55.32	0.125	0.777	0.0049	-2.7(0.5)	-1.7(1.4)
Fat-Inf	7.98	2.41	11.62	0.212	0.073	0.0016	-0.9(0.5)	-0.8(1.1)

2588 IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 57, NO. 10, OCTOBER 2009

FDTD Modeling of Biological Tissues Cole–Cole Dispersion for 0.5–30 GHz Using Relaxation Time Distribution Samples—Novel and Improved Implementations

Mohammad-Reza Tofighi, *Senior Member, IEEE*

Havriliak–Negami Model

This is another modification of the Debye model.

$$\varepsilon_r = \varepsilon_r(\infty) + \frac{\varepsilon_r(0) - \varepsilon_r(\infty)}{\left(1 + (j\omega\tau)^{\alpha}\right)^{\beta}}$$

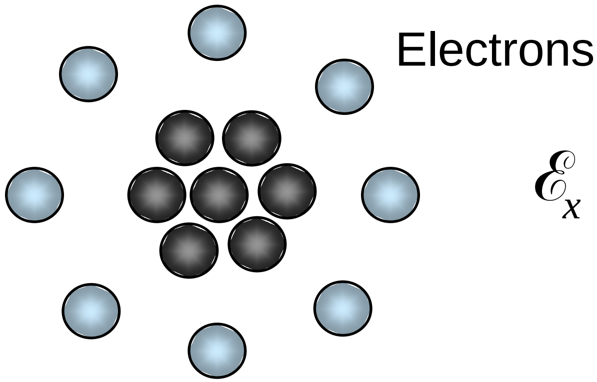
When $\alpha = 1$ and $\beta = 1$, the model reduces to the Debye model.

This has been used to describe the permittivity of some polymers.

Lorentz Model

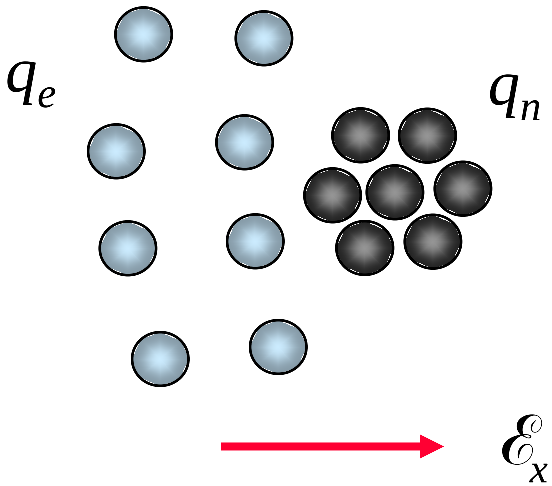
Explains atom and electron resonance effects (usually observed at high frequencies, such as THz frequencies and optical frequencies, respectively).

Atom:



$$\mathcal{E}_x = 0$$

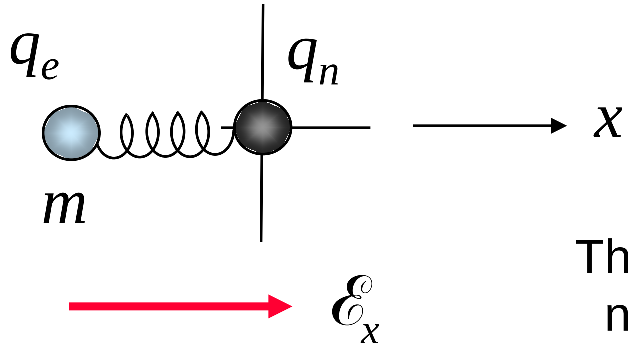
Dipole effect:



$$q_n = - q_e$$

Lorentz Model (cont.)

Model:



The heavy positive nucleus is fixed.

Equation of motion for electrons in atom:

$$\mathcal{F}_x = m\frac{d^2x}{dt^2}$$

$$\mathcal{F}_x = \mathcal{F}_x^E + \mathcal{F}_x^S + \mathcal{F}_x^F$$

$$= q_e\mathcal{E}_x - sx - c\frac{dx}{dt}$$

Lorentz Model (cont.)

so $$q_e \mathscr{E}_x - sx - c\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

For a single atom, $\wp_x = q_n(-x) = q_e x$ or $x = \wp_x / q_e$

Hence $$q_e^2\,\mathscr{E}_x = s\,\wp_x + c\frac{d\,\wp_x}{dt} + m\frac{d^2\,\wp_x}{dt^2}$$

Sinusoidal Steady State:

$$q_e^2\,E_x = s\,p_x + j\,\omega\,c\,p_x - \omega^2 m\,p_x$$

Lorentz Model (cont.)

$$p_x = E_x \left[\frac{q_e^2}{\left(s - \omega^2 m\right) + j\omega c} \right]$$

Denote $N_a = \dfrac{\#\,\text{atoms}}{m^3}$

$$P_x^A = N_a p_x$$
$$= \varepsilon_0 \chi_e^A E_x$$

The term P denotes the total dipole moment per unit volume.

The A superscript reminds us that we are talking about atoms.

Therefore, $\chi_e^A = \dfrac{N_a}{\varepsilon_0}\left(\dfrac{p_x}{E_x}\right)$

Lorentz Model (cont.)

Hence

$$\chi_e^A = \left(\frac{N_a}{\varepsilon_0}\right)\left[\frac{q_e^2}{\left(s-\omega^2 m\right)+j\,\omega\,c}\right]$$

$$= \left(\frac{N_a\,q_e^2}{\varepsilon_0 m}\right)\left[\frac{1}{\left(\frac{s}{m}\right)-\omega^2+j\,\omega\left(\frac{c}{m}\right)}\right]$$

Denote:

$$A = \frac{N_a\,q_e^2}{\varepsilon_0 m} \qquad \omega_0^2 = \frac{s}{m} \qquad c_f = \frac{c}{m}$$

(real constants)

Lorentz Model (cont.)

We then have

$$\chi_e^A = \frac{A}{\left(\omega_0^2 - \omega^2\right) + j\,\omega\,c_f}$$

Permittivity formula: $\varepsilon_r^A = 1 + \chi_e^A$

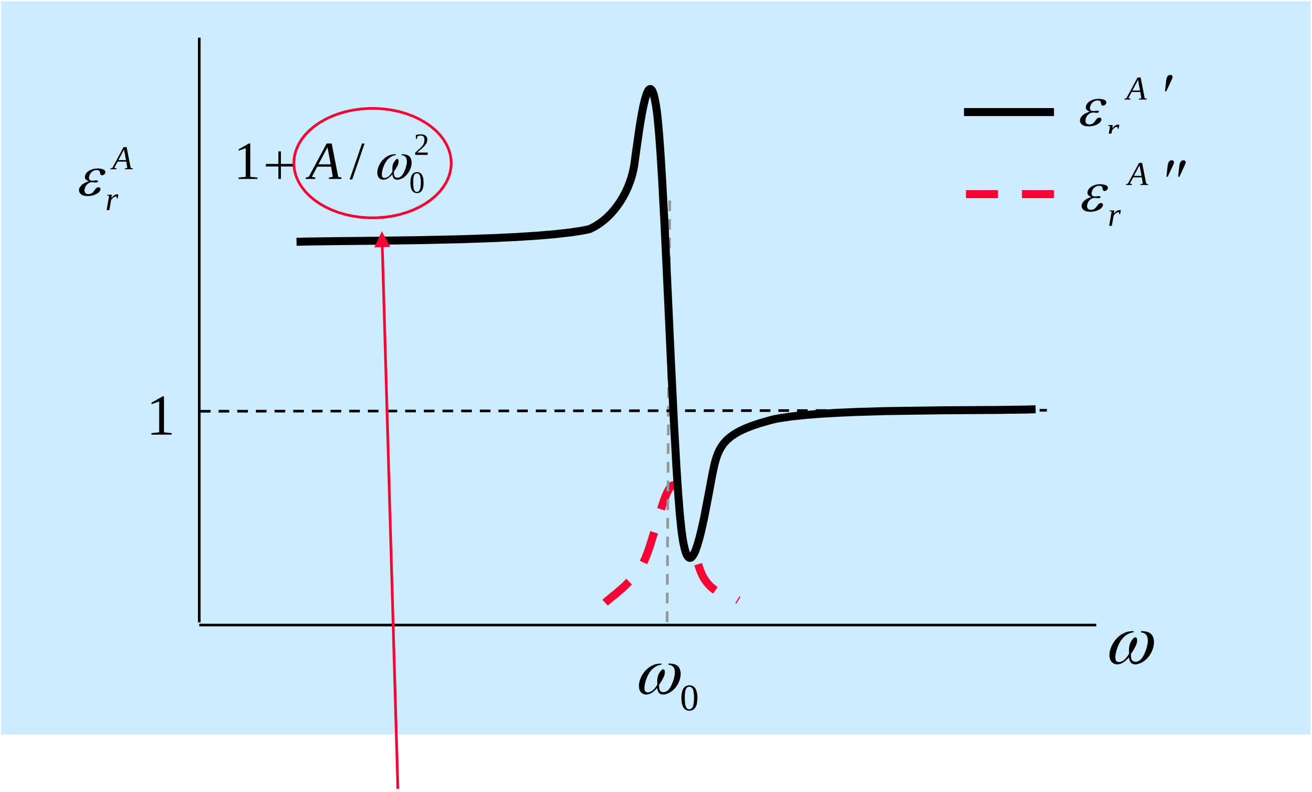
Hence

$$\varepsilon_r^A = 1 + \frac{A}{\left(\omega_0^2 - \omega^2\right) + j\,\omega\,c_f}$$

Real and imaginary parts:

$$\varepsilon_r^{A}{}' = 1 + \frac{\left(\omega_0^2 - \omega^2\right)A}{\left(\omega_0^2 - \omega^2\right)^2 + \left(\omega\,c_f\right)^2} \qquad \varepsilon_r^{A}{}'' = \frac{\omega\,c_f\,A}{\left(\omega_0^2 - \omega^2\right)^2 + \left(\omega\,c_f\right)^2}$$

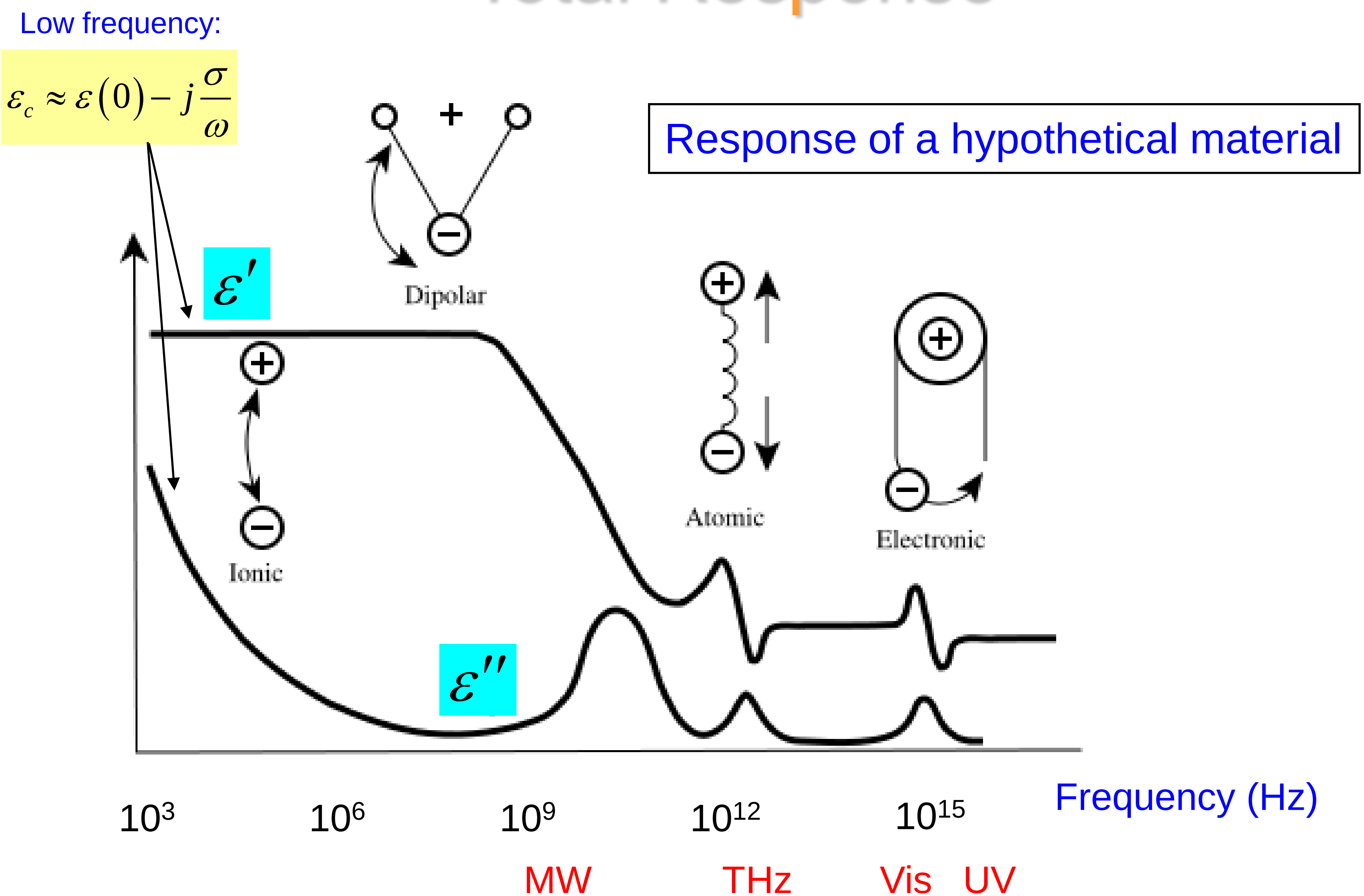
Lorentz Model (cont.)



Low frequency value for χ_e^A (This is still "high" frequency in the Debye model.)

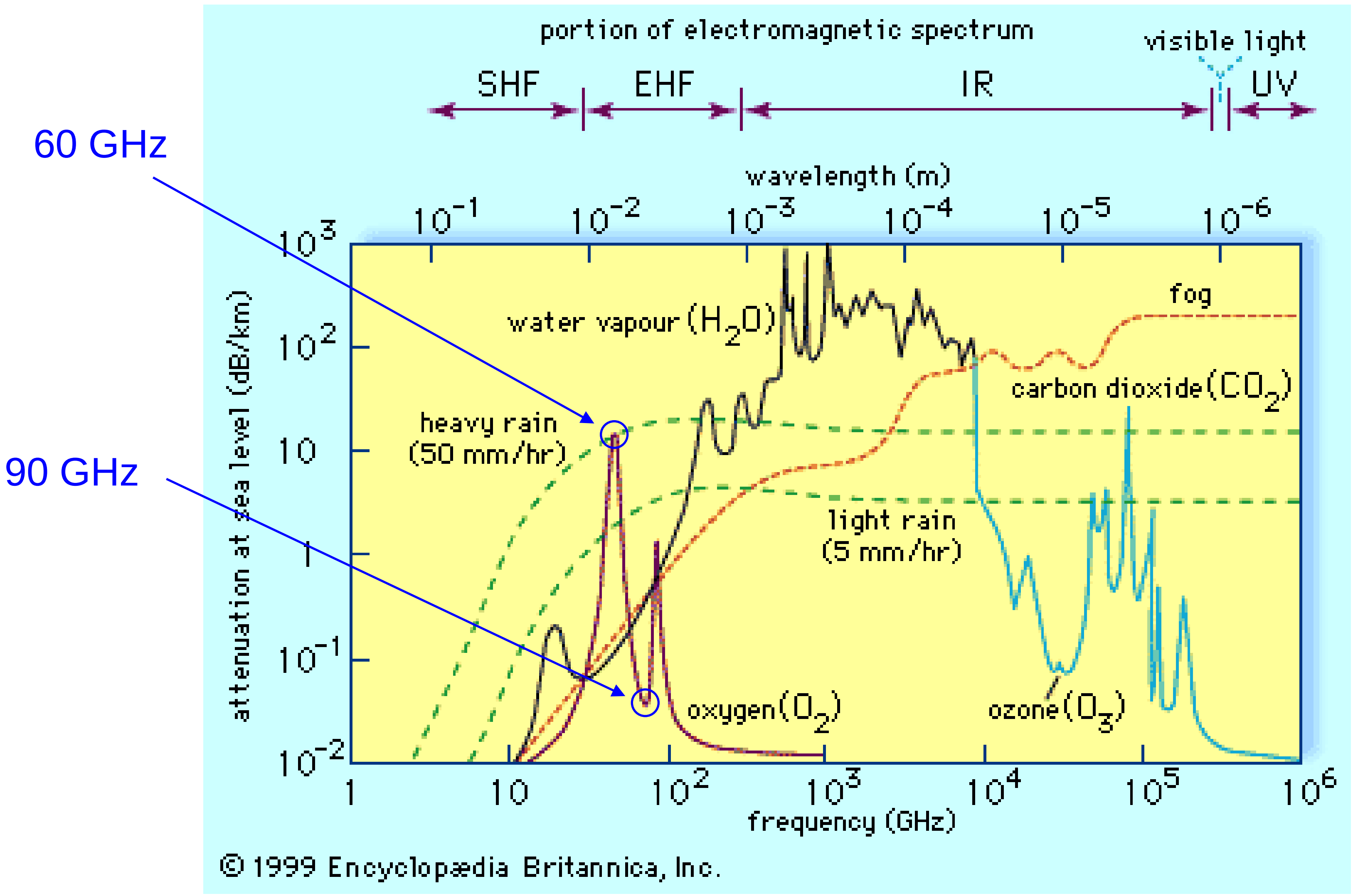
A sharp resonance occurs at $\omega = \omega_0$

Total Response



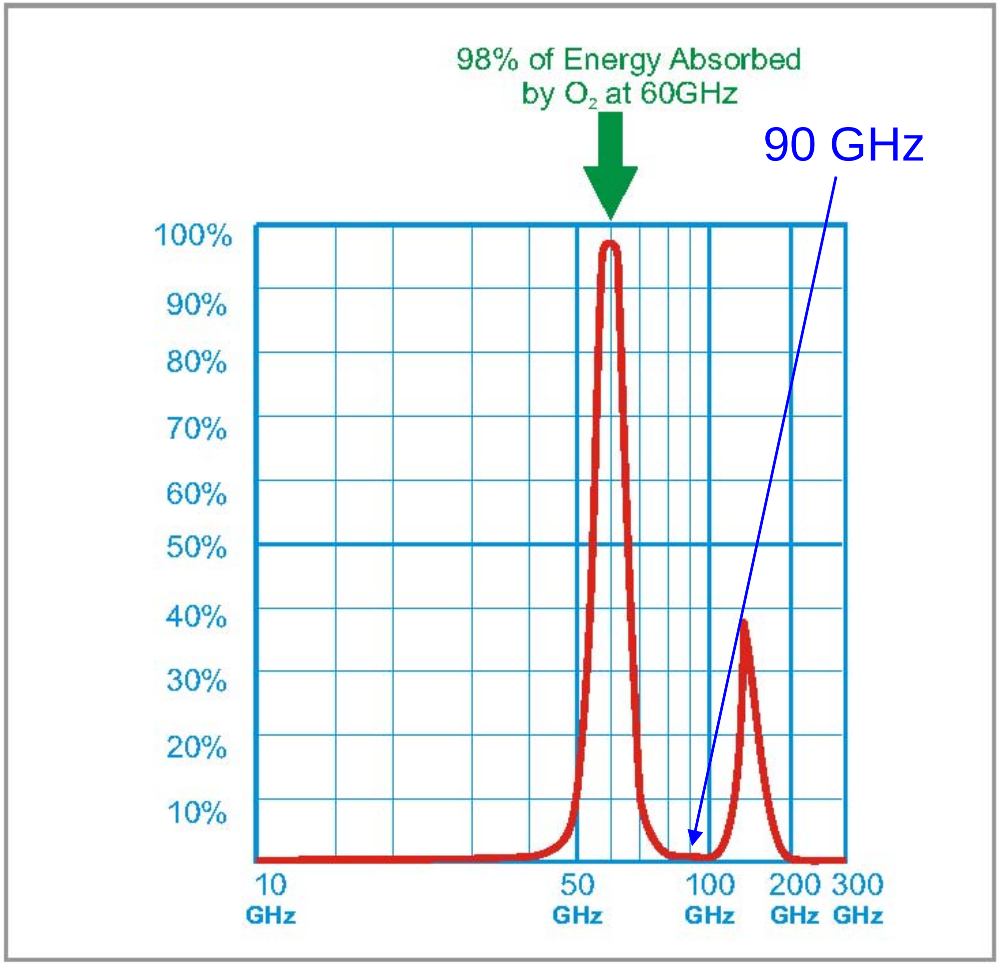
From the dielectric spectroscopy page of the research group of Dr. Kenneth A. Mauritz.

Atmospheric Attenuation



Atmospheric Attenuation (cont.)

Atmospheric absorption (% power absorbed) for millimeter-wave frequencies over a 1-km path



% power absorbed:

$$P_{abs}^{\%} = 100\left(1 - e^{-2\alpha(1000)}\right)$$

Attenuation in dB/km:

$$\text{dB / km} = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

$$= 10\log_{10}\left(e^{-2\alpha(1000)}\right)$$

$$= 10\log_{10}\left(1 - \frac{P_{abs}^{\%}}{100}\right)$$

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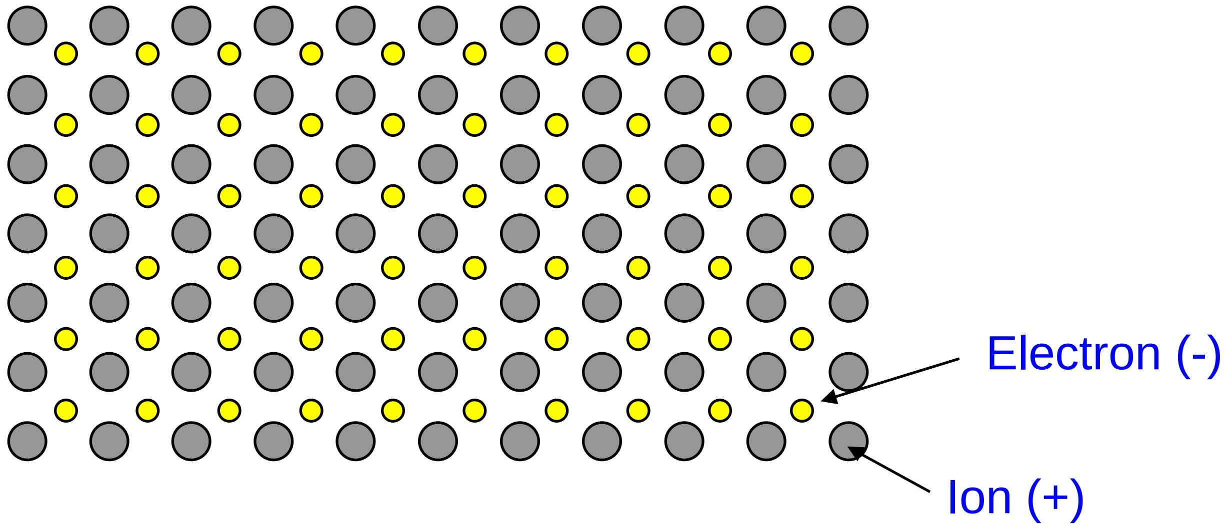
Plasma

Electrically neutral plasma medium (positive ions and electrons):



We assume that only the electrons are free to move when an electric field is applied. This causes a current to flow.

Plasma (cont.)

Equation of motion for average electron:

$$\underline{\mathscr{F}} = m\frac{d\underline{v}}{dt} = (-e)\underline{\mathscr{E}} - m\underline{v}(\upsilon)$$

There is no "spring" force now.

Force due to electric field

Force due to collisions with ions (loss of momentum)

υ = collision frequency (rate of collisions per second of average electron)

Notes: (1) The last term assumes perfect inelastic collisions (loss mechanism).
(2) We neglect the force due to the magnetic field.

Plasma (cont.)

Sinusoidal steady state:

$$m\left(j\omega\underline{v}\right)=\left(-e\right)\underline{E}-m\underline{v}\,\upsilon$$

$$\Rightarrow \qquad \underline{v}=\underline{E}\,\frac{\left(-e\right)}{m\left(j\omega+\upsilon\right)}$$

Current: $$\underline{J}=\rho_{ve}\underline{v}=\underline{E}\left[\frac{\rho_{ve}\left(-e\right)}{m\left(j\omega+\upsilon\right)}\right]$$

Plasma (cont.)

$$\underline{J} = \rho_{ve}\underline{v} = \underline{E}\left(\frac{\rho_{ve}(-e)}{m(j\omega+\upsilon)}\right)$$

Amperes' law:

We assume that there is no polarization current – only conduction current. Hence we use ε_0.

$$\begin{aligned}
\nabla\times\underline{H} &= \underline{J} + j\omega\varepsilon_0\underline{E} \\
&= \underline{E}\left(\frac{\rho_{ve}(-e)}{m(j\omega+\upsilon)}\right) + j\omega\varepsilon_0\underline{E} \\
&= \left(j\omega\varepsilon_0 + \left(\frac{\rho_{ve}(-e)}{m(j\omega+\upsilon)}\right)\right)\underline{E} \\
&= j\omega\varepsilon_c\underline{E}
\end{aligned}$$

Plasma (cont.)

Hence:

$$j\omega\varepsilon_c = j\omega\varepsilon_0 + \left(\frac{\rho_{ve}(-e)}{m(j\omega+\upsilon)}\right)$$

so

$$\varepsilon_c = \varepsilon_0 + \frac{1}{j\omega}\left(\frac{\rho_{ve}(-e)}{m(j\omega+\upsilon)}\right)$$

or

$$\varepsilon_c = \varepsilon_0 - \frac{\rho_{ve}(-e)}{m}\frac{1}{\omega(\omega - j\upsilon)}$$

Plasma (cont.)

$$\varepsilon_c = \varepsilon_0 - \frac{\rho_{ve}(-e)}{m}\frac{1}{\omega(\omega - j\upsilon)}$$

Define:

$$\varepsilon_0 \omega_p^2 \equiv \frac{\rho_{ve}(-e)}{m}$$ ($\omega_p \equiv$ plasma frequency)

We then have

$$\varepsilon_c = \varepsilon_0 \left[1 - \frac{\omega_p^2}{\omega(\omega - j\upsilon)}\right]$$

Plasma (cont.)

Lossless plasma: $\upsilon = 0$

$$\varepsilon_c = \varepsilon_0 \left[1 - \left(\frac{\omega_p}{\omega} \right)^2 \right]$$

(Drude equation)

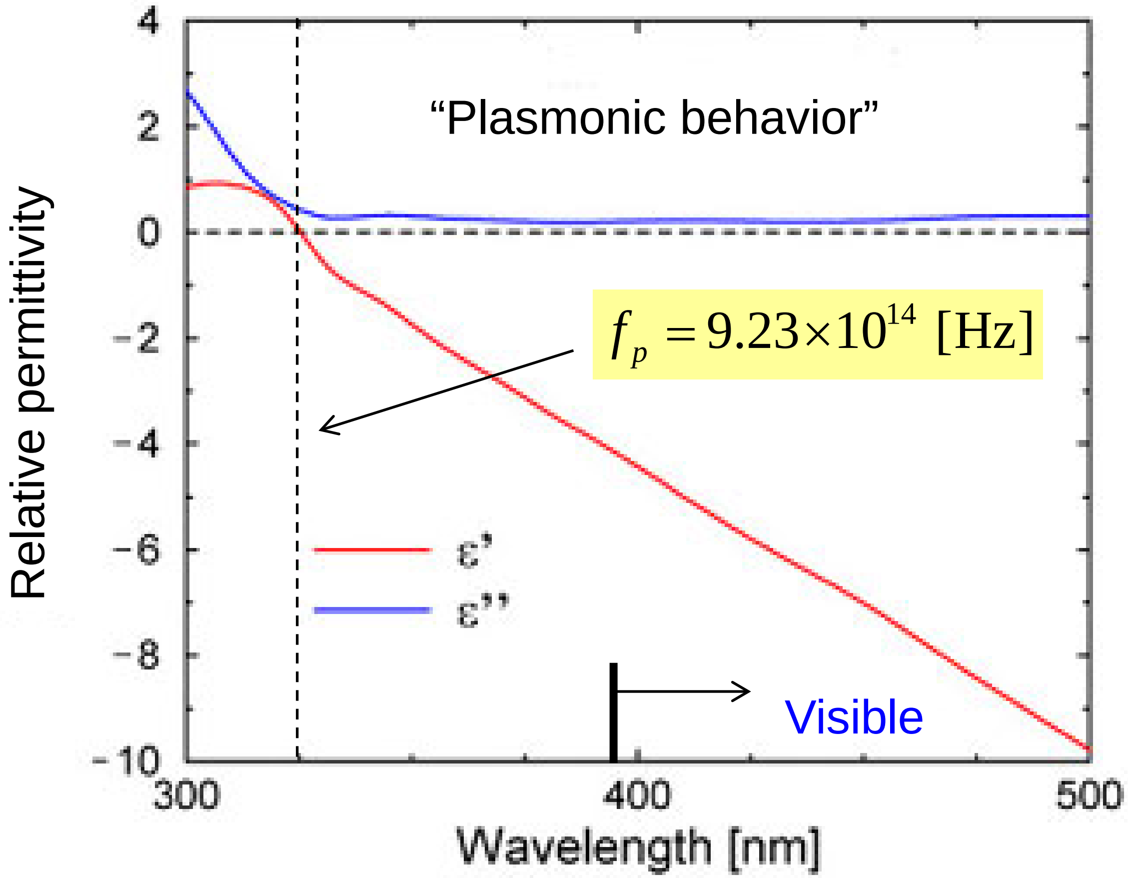
Plane wave in lossless plasma:

$$\omega > \omega_p : \; \varepsilon_c = |\varepsilon_c| > 0, \quad k = \omega\sqrt{\mu_0 \varepsilon_c} = \beta \quad \text{(propagation)}$$

$$\omega < \omega_p : \; \varepsilon_c = -|\varepsilon_c| < 0, \quad k = \omega\sqrt{\mu_0 \varepsilon_c} = -j\alpha \quad \text{(attenuation)}$$

Plasma (cont.)

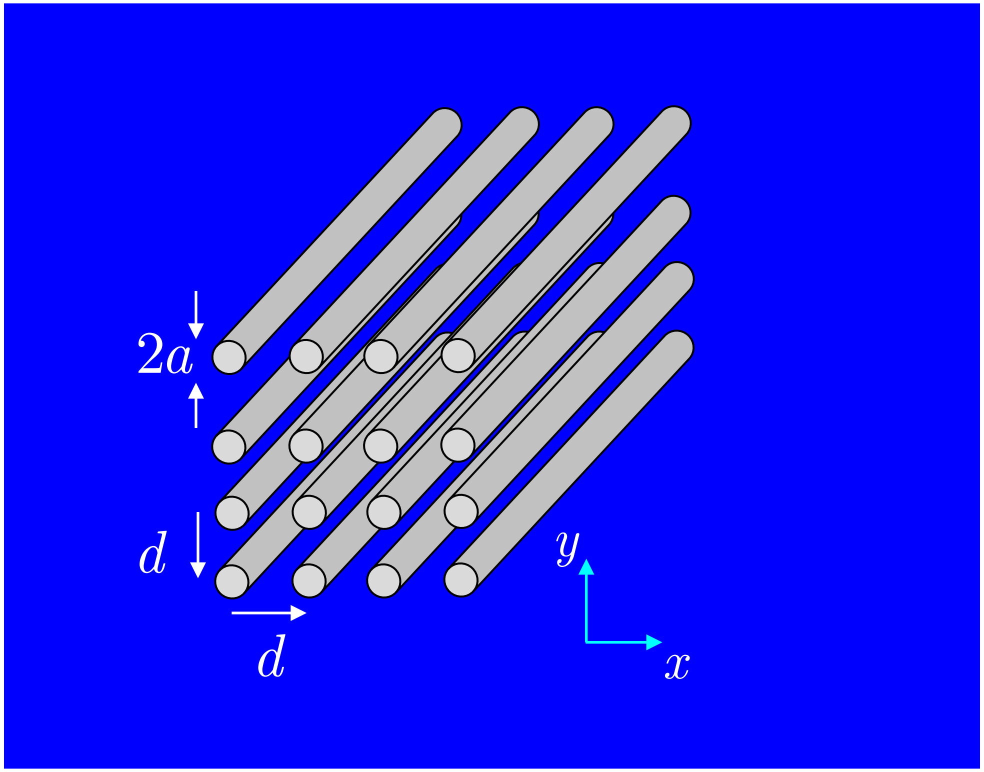
The Drude model is an approximate model for how metals behave at optical frequencies.



Measured complex relative permittivity of silver at optical frequencies

Plasma (cont.)

At microwave frequencies, a plasma-like medium can be simulated by using a wire medium.



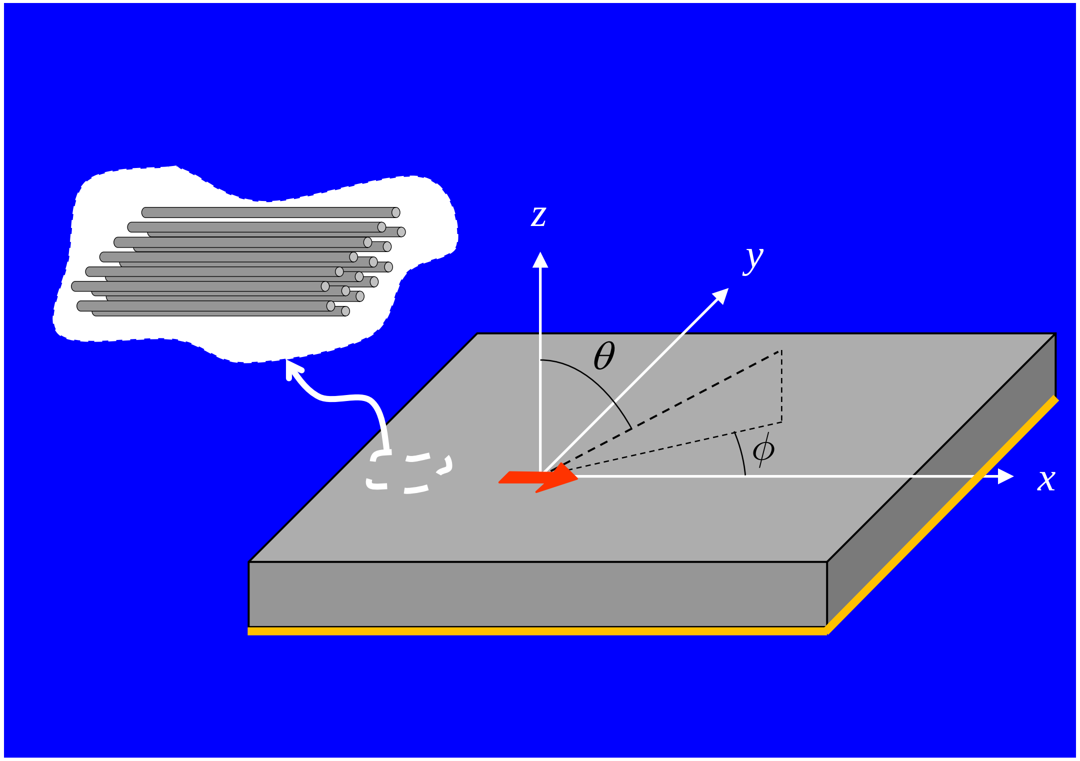
$$\omega_p = \frac{c}{d}\frac{1}{\sqrt{2\pi\left(\ln\frac{d}{2\pi a}+0.5275\right)}}$$

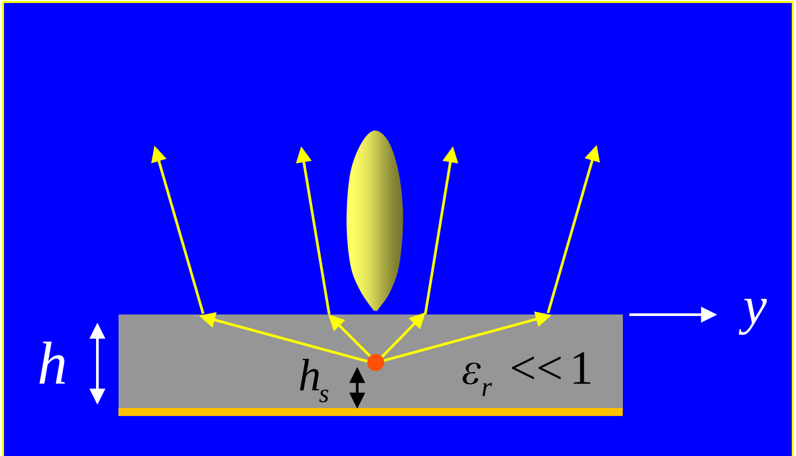
$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Plasma (cont.)

An example of a directive antenna using a wire medium:

$$\omega > \omega_p, \;\; \omega \approx \omega_p \quad \rightarrow \;\; 0 < \varepsilon_r << 1$$





Geometrical optics (GO):
Refraction towards broadside