ECE 6340
Intermediate EM Waves

Fall 2016

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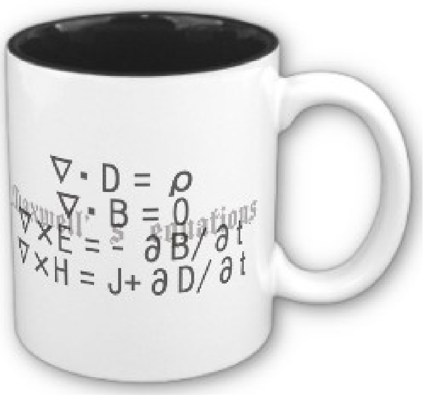
Dept. of ECE



Notes 5

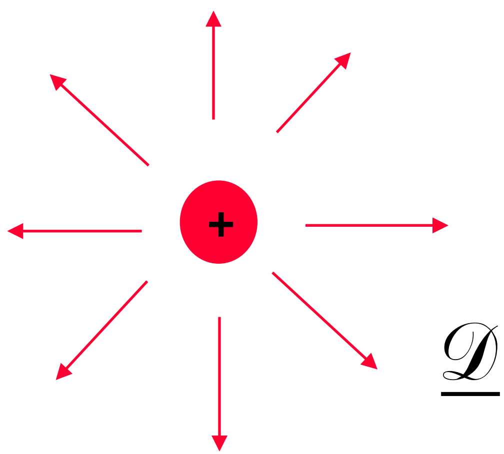
Magnetic Current

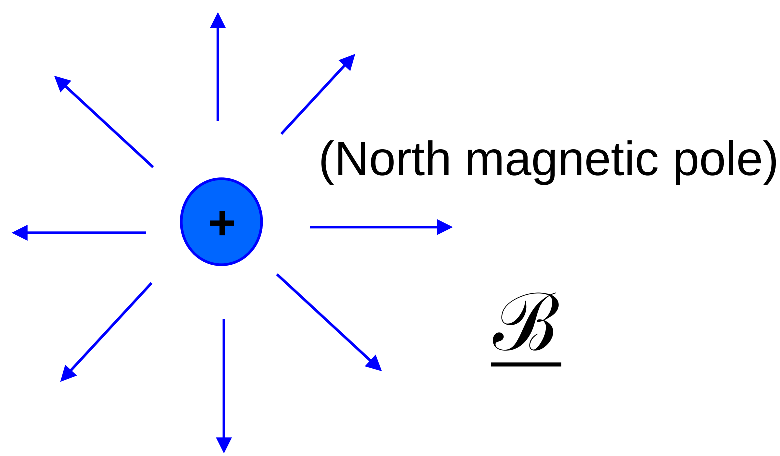
Maxwell's Equations:

$$\nabla \cdot \underline{\mathscr{D}} = \rho_v \qquad \nabla \times \underline{\mathscr{H}} = \underline{\mathscr{J}} + \frac{\partial \underline{\mathscr{D}}}{\partial t}$$

$$\nabla \cdot \underline{\mathscr{B}} = 0 \qquad \nabla \times \underline{\mathscr{E}} = \underline{0} - \frac{\partial \underline{\mathscr{B}}}{\partial t}$$

Missing terms correspond to ρ_v^m , $\underline{\mathscr{M}}$





Magnetic Current (cont.)

Define magnetic charge so that

$$\nabla \cdot \underline{\mathscr{B}} = \rho_v^m$$

(A positive magnetic charge corresponds to a north magnetic pole.)

Assume that a continuity equation holds:

$$\nabla \cdot \underline{\mathscr{M}} = -\frac{\partial \rho_v^m}{\partial t}$$

From this, we can show that $\underline{\mathscr{M}}$ belongs in Faraday's Law as:

$$\nabla \times \underline{\mathscr{E}} = -\underline{\mathscr{M}} - \frac{\partial \underline{\mathscr{B}}}{\partial t}$$

Magnetic Current (cont.)

Proof:

Take the divergence of both sides:

$$\nabla \cdot \left(\nabla \times \underline{\mathcal{E}} \right) = -\nabla \cdot \underline{\mathcal{M}} - \frac{\partial}{\partial t} \left(\nabla \cdot \underline{\mathcal{B}} \right)$$

so $$\nabla \cdot \underline{\mathcal{M}} = -\frac{\partial \rho_v^m}{\partial t}$$

Magnetic Current

Maxwell's Equations:

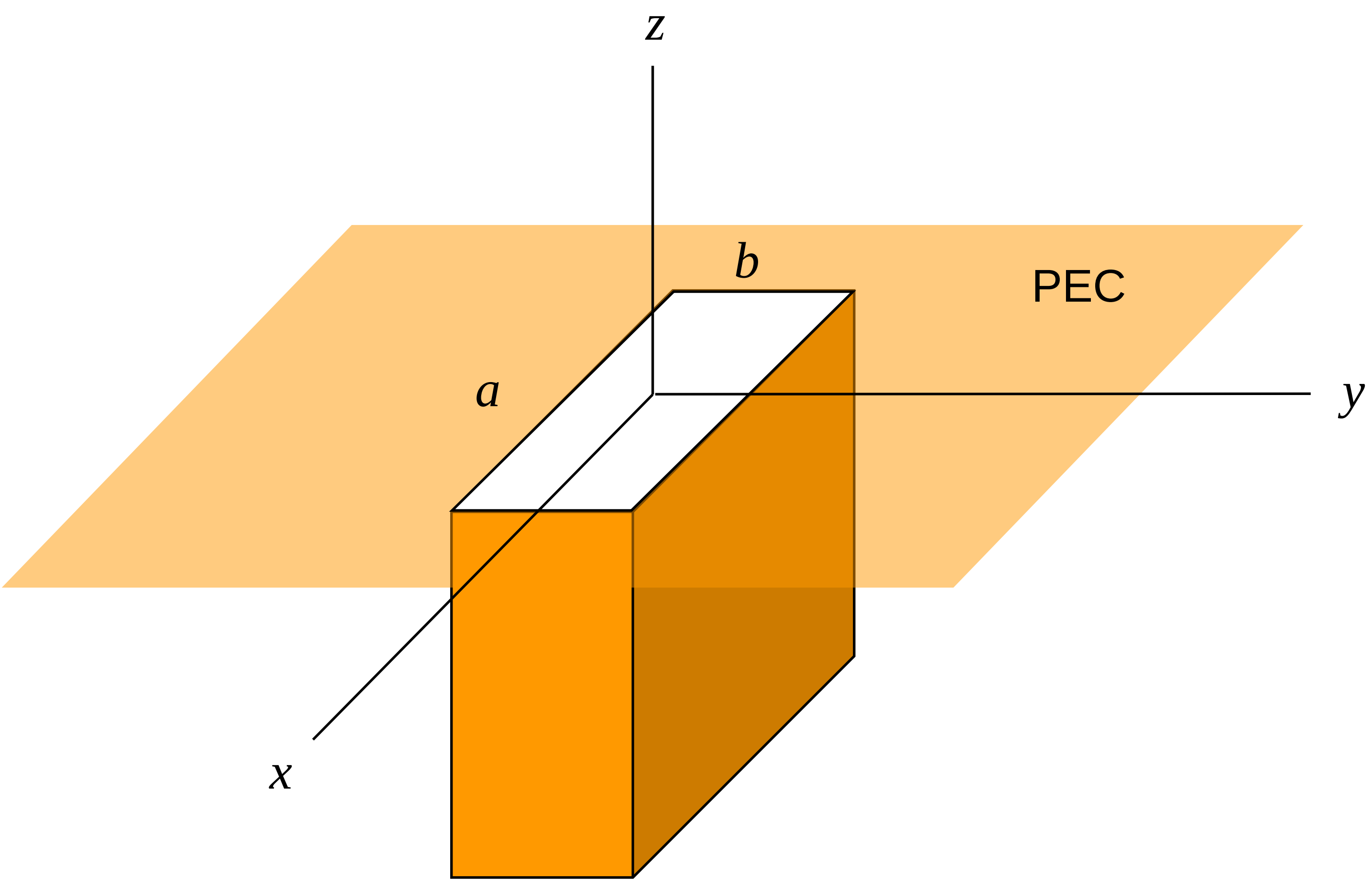
$$\nabla \times \underline{\mathscr{H}} = \underline{\mathscr{J}} + \frac{\partial \underline{\mathscr{D}}}{\partial t} \qquad \nabla \cdot \underline{\mathscr{D}} = \rho_v$$

$$\nabla \times \underline{\mathscr{E}} = -\underline{\mathscr{M}} - \frac{\partial \underline{\mathscr{B}}}{\partial t} \qquad \nabla \cdot \underline{\mathscr{B}} = \rho_v^m$$

Note: Maxwell's equations are now <u>symmetric</u>. If we know how an electric current radiates, it will be easy to figure out how a magnetic current radiates (this is called duality).

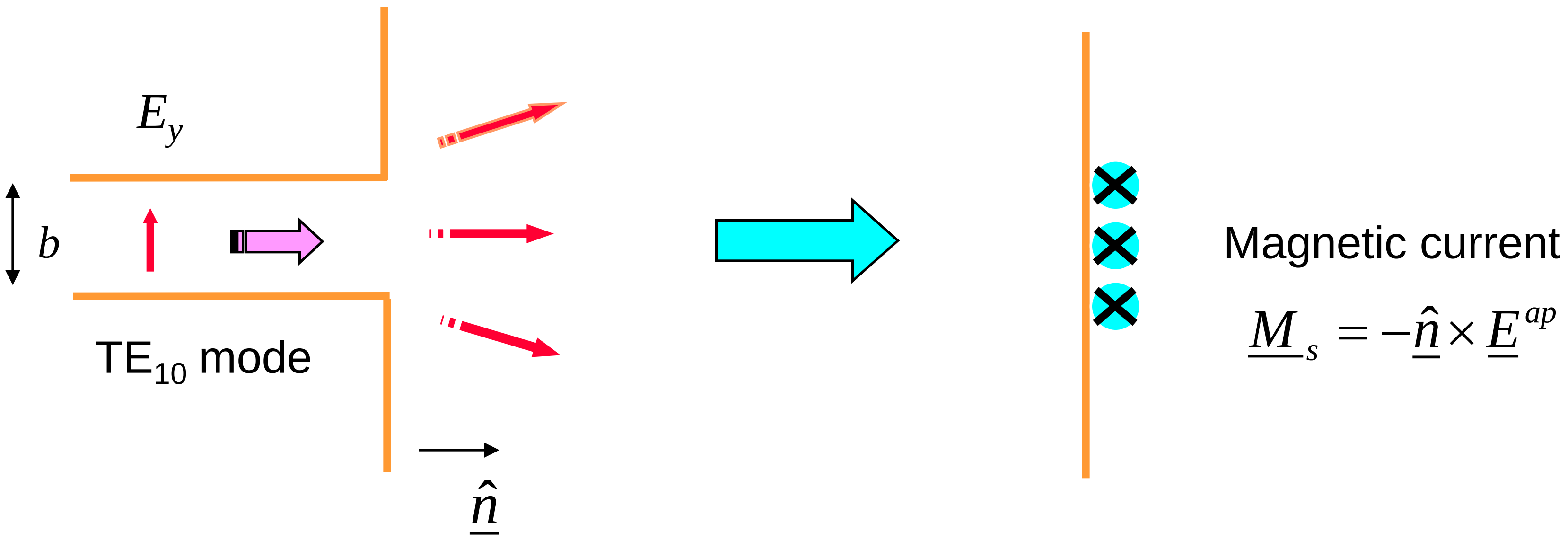
Magnetic Current (cont.)

Example: Radiation from a waveguide-fed aperture



Magnetic Current (cont.)

Usefulness of magnetic current concept: radiation from a waveguide



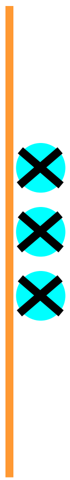
Waveguide with infinite baffle

Equivalence Principle

(discussed later in the semester)

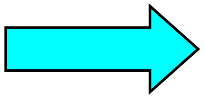
Magnetic Current (cont.)

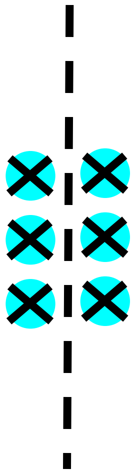
We can now remove the ground plane:



Magnetic current

$$\underline{M}_s = -\underline{\hat{n}} \times \underline{E}^{ap}$$





Magnetic current

$$\underline{M}_s = -2\underline{\hat{n}} \times \underline{E}^{ap}$$

Image Theory

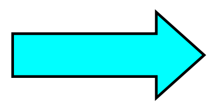
(discussed later in the semester)

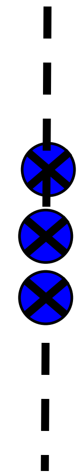
Magnetic Current (cont.)

The radiation from the magnetic current is related to radiation from a corresponding electric current:

Magnetic current

$$\underline{M}_s = -2\underline{\hat{n}} \times \underline{E}^{ap}$$





Electric current

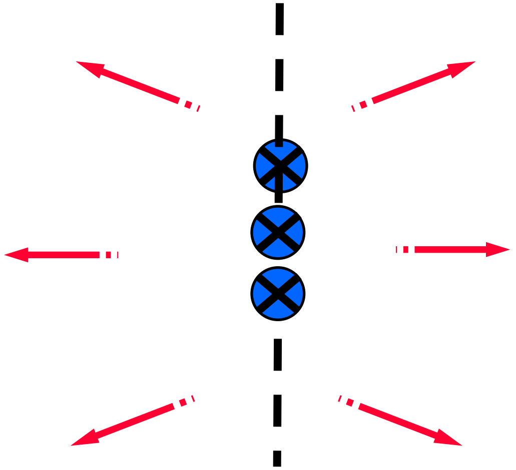
$$\underline{J}_s = -2\underline{\hat{n}} \times \underline{E}^{ap}$$

Duality

(discussed later in the semester)

Magnetic Current (cont.)

Summary of final radiation picture



Electric current

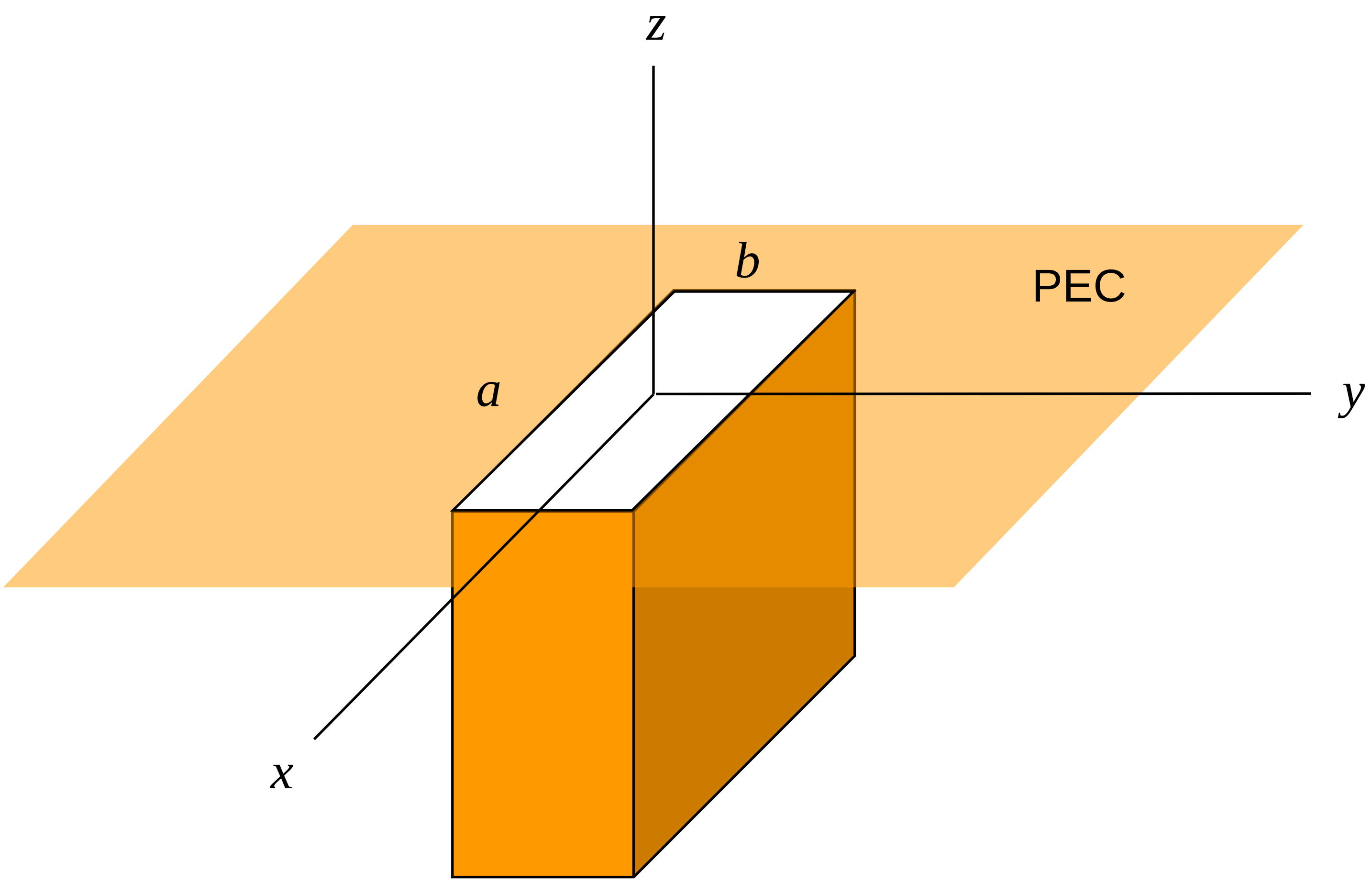
$$\underline{J}_s = 2\left(-\underline{\hat{n}} \times \underline{E}^{ap}\right)$$

Radiation from electric current in free space

(discussed later in the semester)

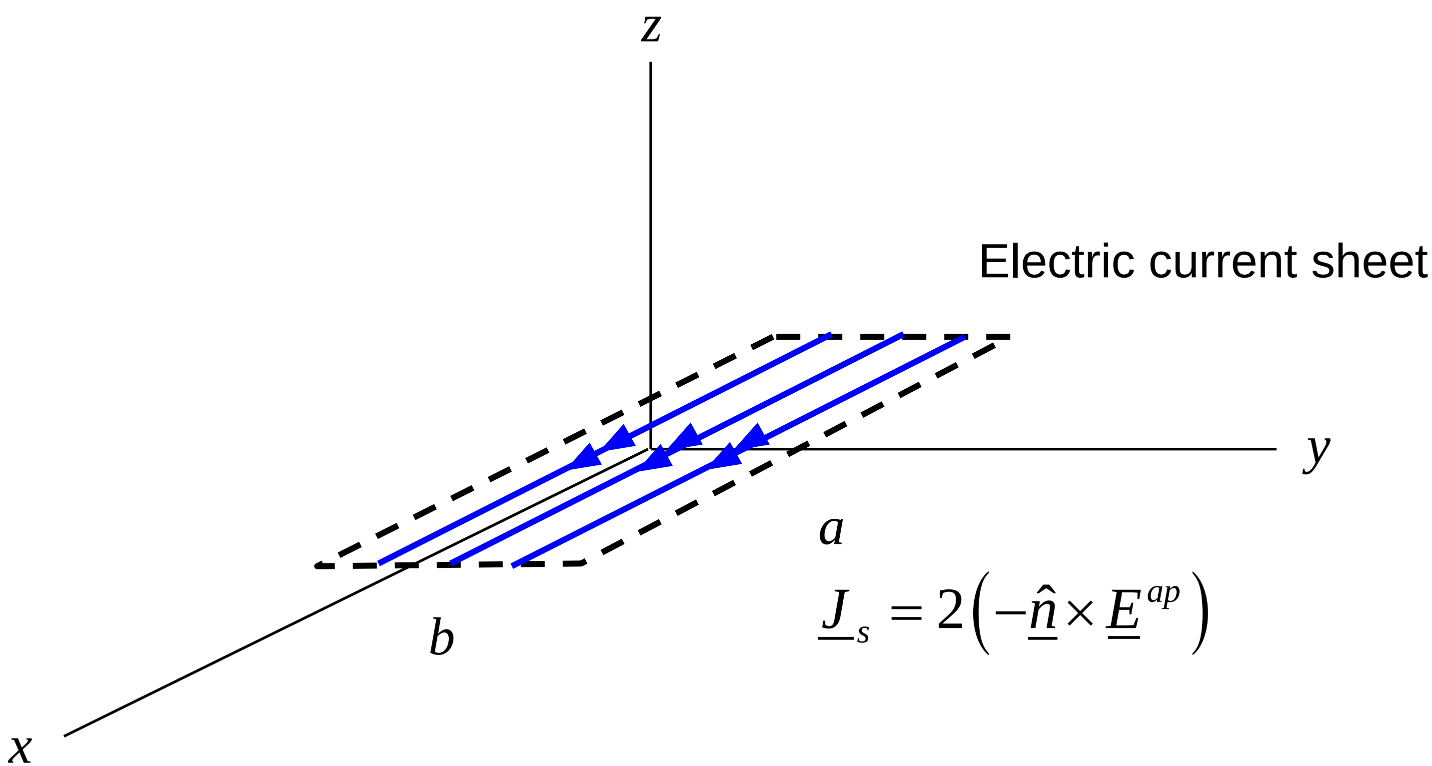
Magnetic Current (cont.)

3D view of original problem



Magnetic Current (cont.)

3D view of final radiation model



$$\underline{J}_s = 2\left(-\underline{\hat{n}} \times \underline{E}^{ap}\right)$$