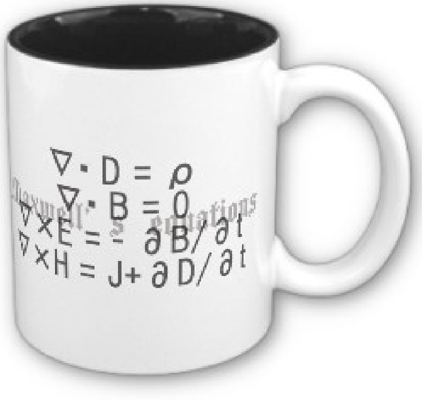
ECE 6340
Intermediate EM Waves

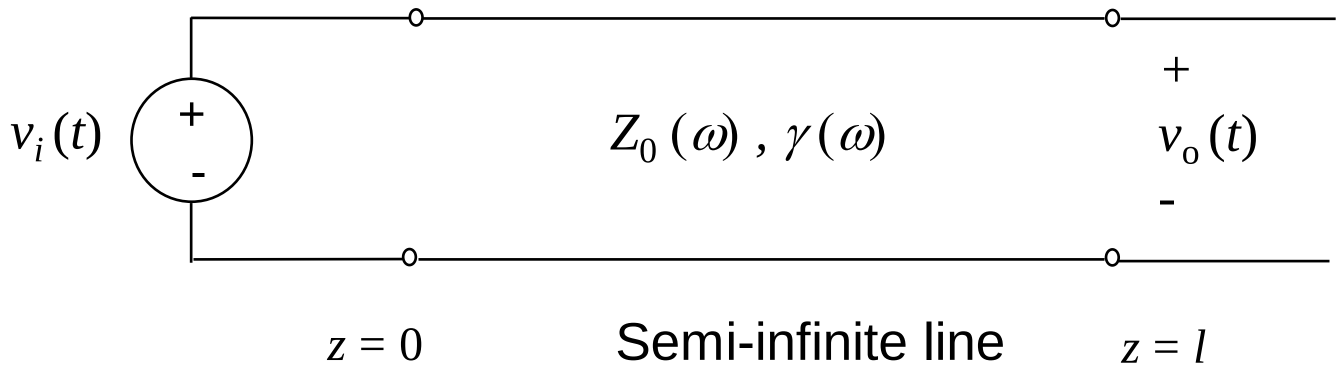
Fall 2016

Prof. David R. Jackson
Dept. of ECE



Notes 8

Signal Propagation on Line



Introduce Fourier transform:

$$\tilde{v}_i(\omega) = \int_{-\infty}^{\infty} v_i(t)\, e^{-j\omega t} dt$$

$$v_i(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{v}_i(\omega)\; e^{j\omega t} d\omega$$

Signal Propagation on Line (cont.)

$$v_i(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{v}_i(\omega)\, e^{j\omega t}\, d\omega$$

Goal: To get this into a form that looks like

$$v_i(t) = \sum_n \mathrm{Re}\left(A_n\, e^{j\omega_n t} \right), \quad \omega_n > 0$$

The transform variable ω can then be interpreted as (radian) frequency.

Signal Propagation on Line (cont.)

We start by considering a useful property of the transform.

Transform definition: $$\tilde{v}_i(\omega) = \int_{-\infty}^{\infty} v_i(t)\, e^{-j\omega t} dt$$

Since $v_i(t) = \mathrm{Real}(t)$

it follows that $$\tilde{v}_i(-\omega) = \tilde{v}_i^*(\omega)$$ (assuming that ω is real)

Next, use

$$v_i(t) = \frac{1}{2\pi}\int_{0}^{\infty} \tilde{v}_i(\omega)\, e^{j\omega t} d\omega + \frac{1}{2\pi}\int_{-\infty}^{0} \tilde{v}_i(\omega)\, e^{j\omega t} d\omega$$

Signal Propagation on Line (cont.)

$$v_i(t) = \frac{1}{2\pi}\int_0^{\infty} \tilde{v}_i(\omega) e^{j\omega t} d\omega + \frac{1}{2\pi}\int_{-\infty}^{0} \tilde{v}_i(\omega) e^{j\omega t} d\omega$$

Use $\omega' = -\omega$

$$\frac{1}{2\pi}\int_{-\infty}^{0} \tilde{v}_i(\omega) e^{j\omega t} d\omega = \frac{-1}{2\pi}\int_{\infty}^{0} \tilde{v}_i(-\omega') e^{-j\omega' t} d\omega'$$

$$= \frac{1}{2\pi}\int_{0}^{\infty} \tilde{v}_i(-\omega') e^{-j\omega' t} d\omega'$$

$$= \frac{1}{2\pi}\int_{0}^{\infty} \tilde{v}_i^*(\omega') e^{-j\omega' t} d\omega'$$

$$= \frac{1}{2\pi}\int_{0}^{\infty} \tilde{v}_i^*(\omega) e^{-j\omega t} d\omega$$

Signal Propagation on Line (cont.)

Hence

$$v_i(t) = \frac{1}{2\pi}\int_0^{\infty}\left(\tilde{v}_i(\omega)e^{j\omega t} + \tilde{v}_i^*(\omega)\, e^{-j\omega t}\right) d\omega$$

$$= \frac{1}{2\pi}\int_0^{\infty}\mathrm{Re}\left(\tilde{v}_i(\omega)\, e^{j\omega t} + \left(\tilde{v}_i(\omega)\, e^{j\omega t}\right)^*\right) d\omega$$

$$= \frac{1}{\pi}\int_0^{\infty}\mathrm{Re}\left(\tilde{v}_i(\omega)\, e^{j\omega t}\right) d\omega$$

The transform variable is now interpreted as radian frequency.

or

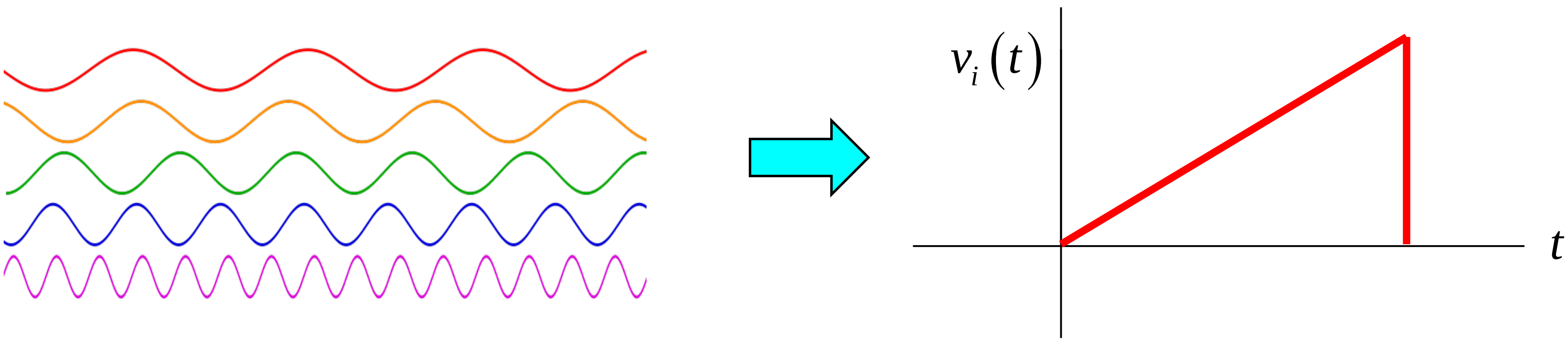
$$\omega = 2\pi f$$

$$v_i(t) = \int_0^{\infty}\mathrm{Re}\left[\left(\frac{1}{\pi}\tilde{v}_i(\omega)\, d\omega\right) e^{j\omega t}\right]$$

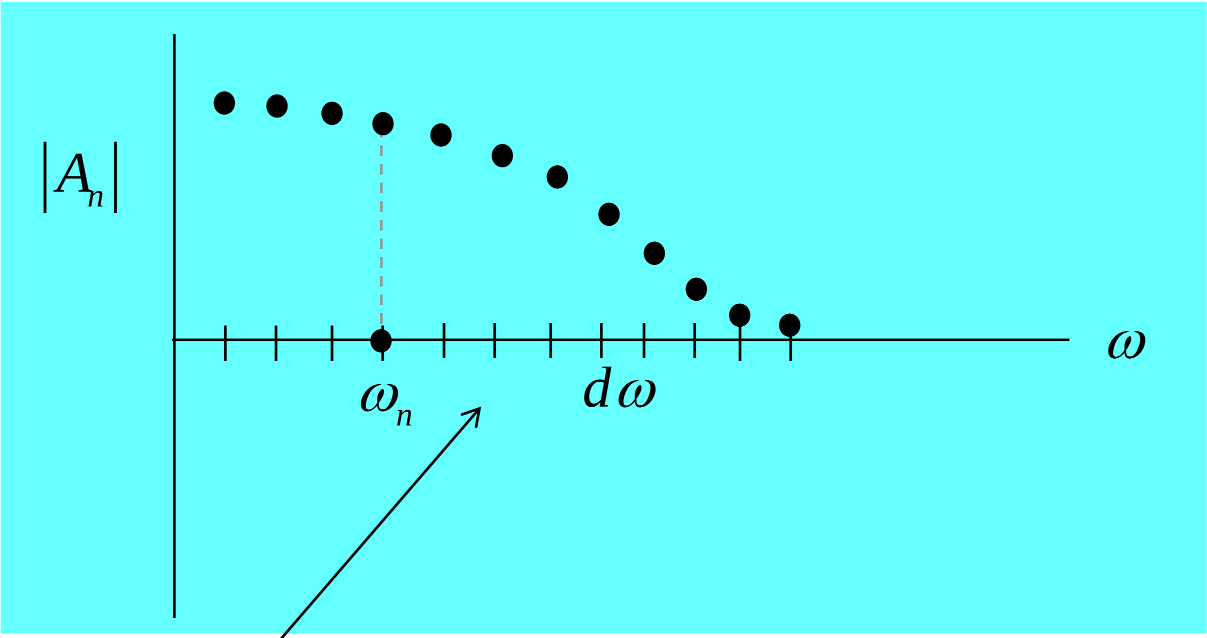
Signal Propagation on Line (cont.)

$$v_i(t) = \int_0^{\infty} \mathrm{Re}\left[\left(\frac{1}{\pi} \tilde{v}_i(\omega)\, d\omega \right) e^{j\omega t} \right]$$

Any signal can be expressed as a collection of infinite sinusoidal signals.



Signal Propagation on Line (cont.)



The integral is approximated as a sum.

$$v_i(t) = \int_0^\infty \mathrm{Re}\left[\left(\frac{1}{\pi}\tilde{v}_i(\omega)\, d\omega\right) e^{j\omega t}\right]$$

Define $A_n \equiv \frac{1}{\pi}\tilde{v}_i(\omega_n)\, d\omega$

Then $v_i(t) \approx \sum_n \mathrm{Re}\left[A_n e^{j\omega_n t}\right]$

Signal Propagation on Line (cont.)

$$v_i(t) = \sum_n \mathrm{Re}\left[A_n e^{j\omega_n t} \right]$$

In the phasor domain: Input $A_n \rightarrow A_n \, e^{-\gamma_n l}$ Output

$$\gamma_n = \gamma(\omega_n)$$

Therefore,

$$v_o(t) = \sum_n \mathrm{Re}\left[A_n e^{-\gamma_n l} e^{j\omega_n t} \right]$$

$$= \sum_n \mathrm{Re}\left[\left(\frac{1}{\pi} \tilde{v}_i(\omega_n) \, d\omega \right) e^{-\gamma_n l} e^{j\omega_n t} \right]$$

Taking the limit,

$$v_o(t) = \int_0^\infty \mathrm{Re}\left(\frac{1}{\pi} \tilde{v}_i(\omega) e^{-\gamma l} e^{j\omega t} d\omega \right)$$

where $\gamma = \gamma(\omega)$

Signal Propagation on Line (cont.)

Final result:

$$v_o(t) = \frac{1}{\pi} \mathrm{Re} \int_0^{\infty} \tilde{v}_i(\omega) e^{-\gamma l} e^{j\omega t} d\omega$$

where $\gamma = \gamma(\omega)$

$$\tilde{v}_i(\omega) = \int_{-\infty}^{\infty} v_i(t) e^{-j\omega t} dt$$

Signal Propagation on Line (cont.)

Compare:

$$v_o(t) = \frac{1}{\pi} \operatorname{Re} \int_0^{\infty} \tilde{v}_i(\omega) e^{-\gamma l} e^{j\omega t} d\omega$$

Output voltage expression

$$v_o(t) = \frac{1}{\pi} \operatorname{Re} \int_0^{\infty} \tilde{v}_o(\omega) e^{j\omega t} d\omega$$

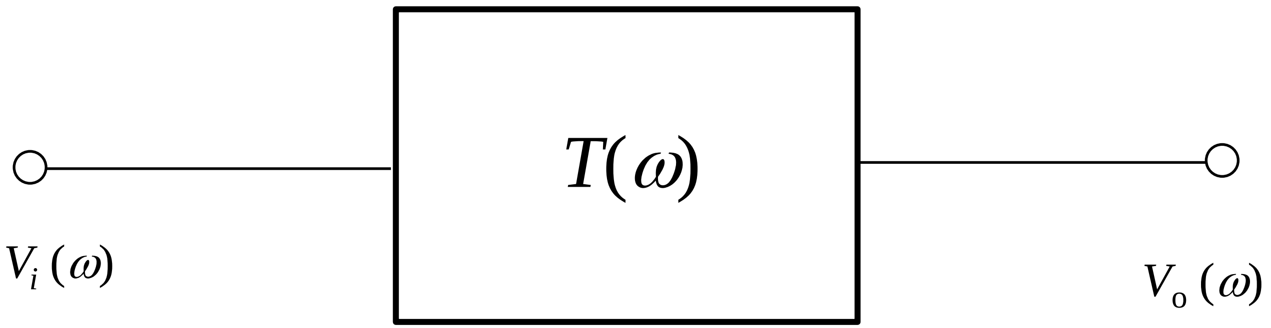
Output voltage expressed in inverse transform form

Conclusion:

$$\tilde{v}_o(\omega) = \tilde{v}_i(\omega) e^{-\gamma \ell}$$

Signal Propagation on Line (cont.)

The most general scenario:



$T(\omega)$ is the *frequency-domain transfer function*.

$$T(\omega) \equiv \frac{V_o(\omega)}{V_i(\omega)}$$ (the ratio of the phasor amplitudes)

Then we have

$$v_o(t) = \frac{1}{\pi}\,\mathrm{Re}\int_0^{\infty} \tilde{v}_i(\omega)\,T(\omega)\,e^{j\omega t}\,d\omega$$

$$\tilde{v}_o(\omega) = \tilde{v}_i(\omega)\,T(\omega)$$

Lossless Line

$$v_o(t) = \frac{1}{\pi} \mathrm{Re} \int_0^{\infty} \tilde{v}_i(\omega) e^{-\gamma l} e^{j\omega t} d\omega$$

$$\gamma = j\beta = j\omega\sqrt{LC}$$
$$= j\omega / v_p$$
$(v_p = \text{constant})$

$$v_o(t) = \frac{1}{\pi} \mathrm{Re} \int_0^{\infty} \tilde{v}_i(\omega)\, e^{-j\left(\omega/v_p\right)l} e^{j\omega t} d\omega$$
$$= \frac{1}{\pi} \mathrm{Re} \int_0^{\infty} \tilde{v}_i(\omega)\, e^{j\omega\left(t - l/v_p\right)} d\omega$$

Denote $t' = t - \dfrac{l}{v_p}$

Then $v_o(t) = \dfrac{1}{\pi} \mathrm{Re} \displaystyle\int_0^{\infty} \tilde{v}_i(\omega)\, e^{j\omega t'} d\omega$

Lossless Line (cont.)

$$v_o(t) = \frac{1}{\pi} \mathrm{Re} \int_0^\infty \tilde{v}_i(\omega)\, e^{j\omega t'} d\omega$$

Compare with

$$v_i(t) = \frac{1}{\pi} \int_0^\infty \mathrm{Re}\left(\tilde{v}_i(\omega)\, e^{j\omega t} \right) d\omega$$

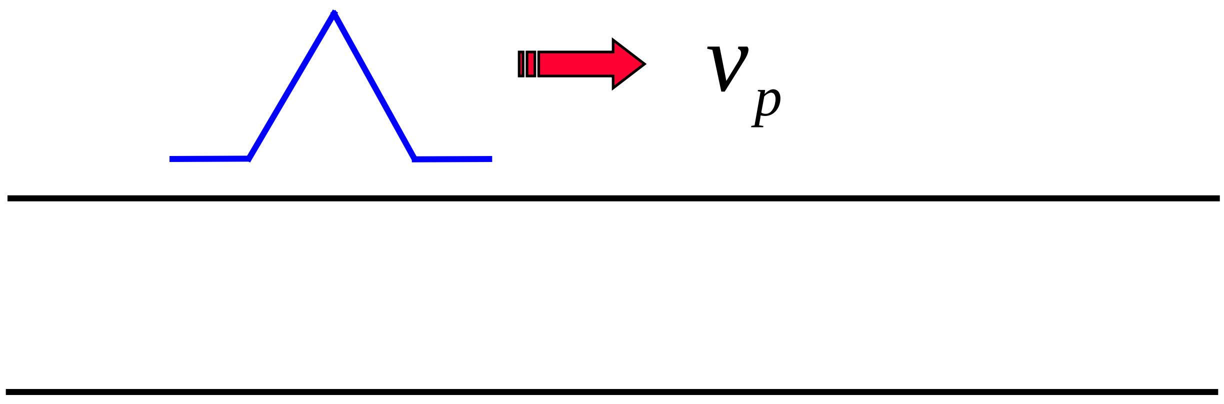
Hence $$v_o(t) = v_i(t')$$

or $$v_o(t) = v_i(t - l / v_p)$$

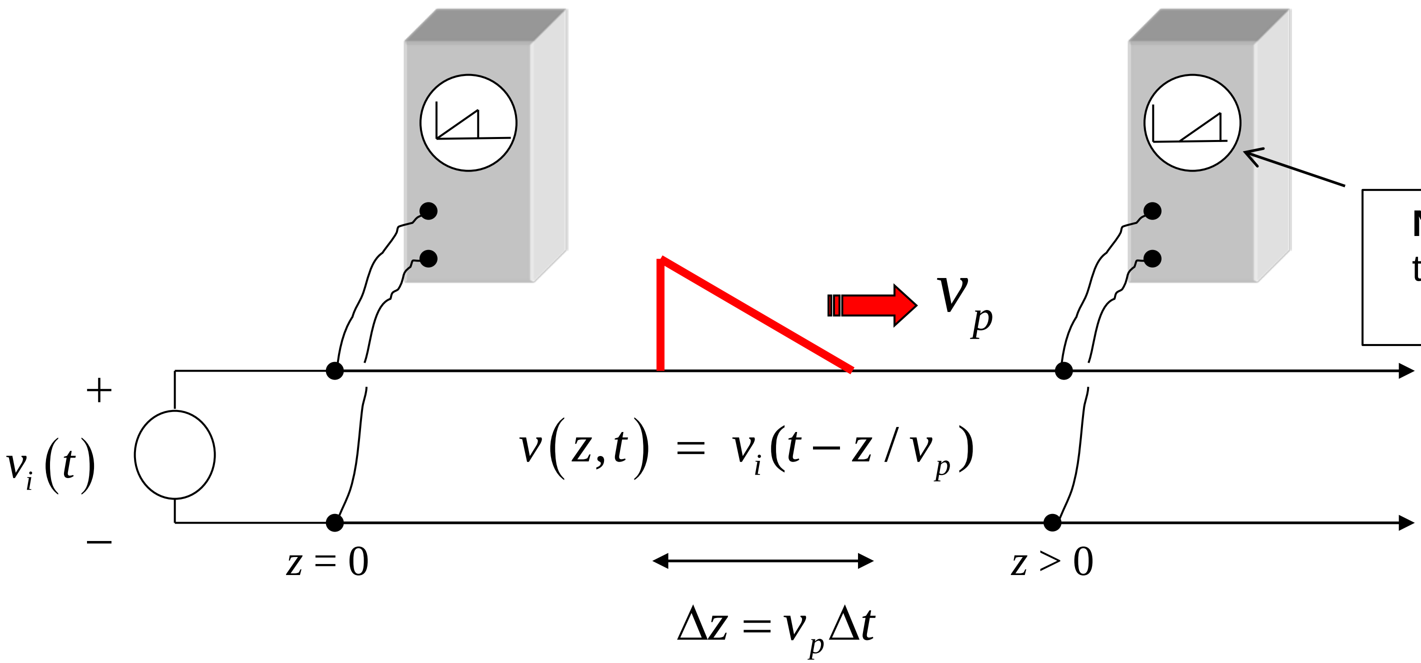
Lossless Line (cont.)

$$v_o\left(t\right) = v_i(t - l / v_p)$$

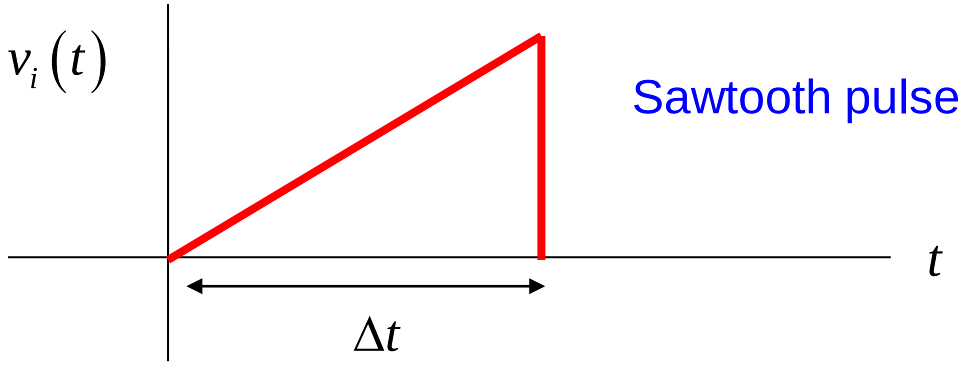
The pulse moves at the phase velocity without distortion.



Lossless Line (cont.)



Note: There is a time delay in the trace!

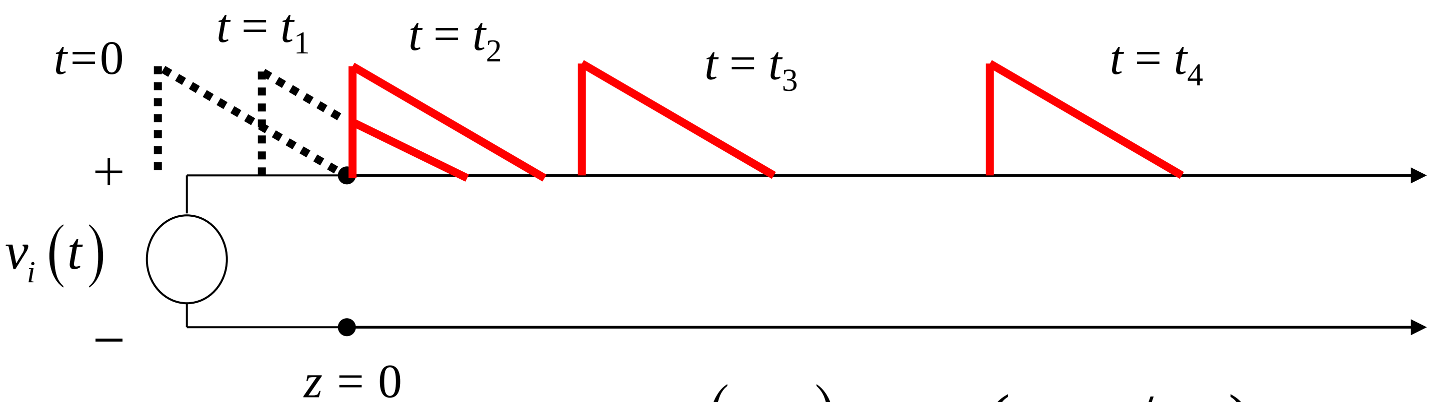


Sawtooth pulse

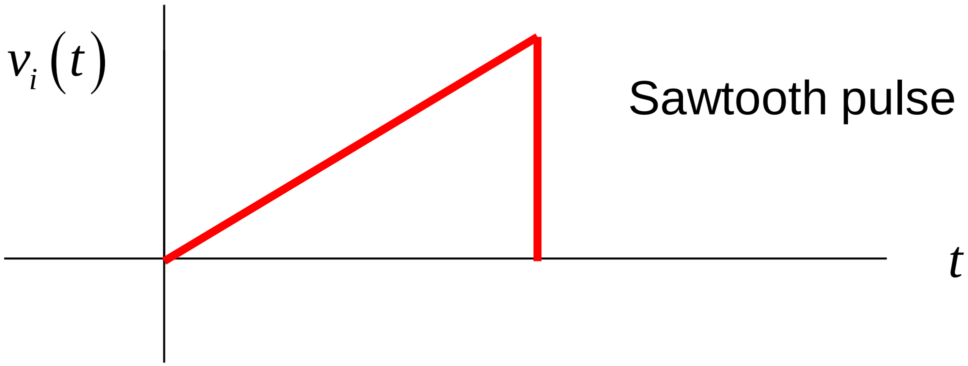
Note:
The shape of the pulse as a function of z is the mirror image of the pulse shape as a function of t.

Lossless Line (cont.)

The sawtooth pulse is shown emerging from the source end of the line.

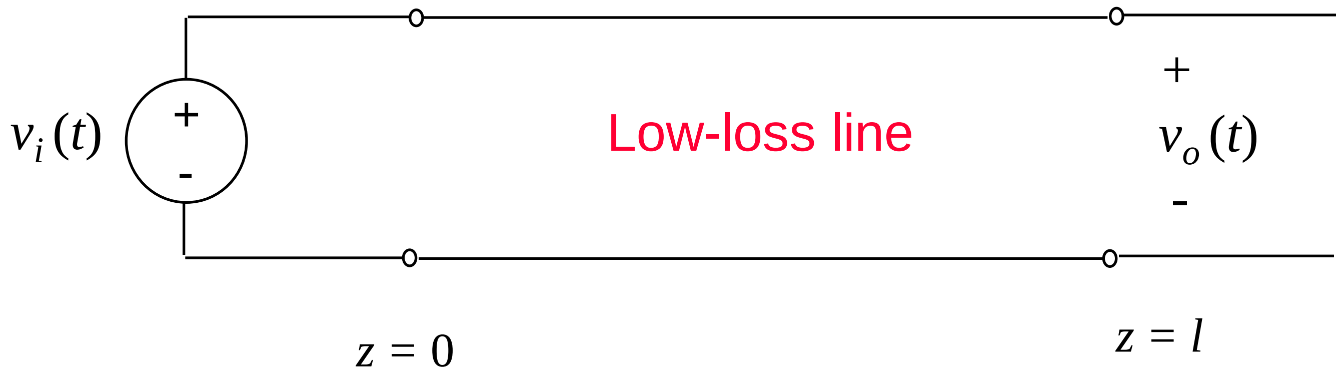


$$v(z,t) = v_i(t - z/v_p)$$



Sawtooth pulse

Signal Propagation with Dispersion



$$v_p = v_p(\omega)$$ (dispersion)

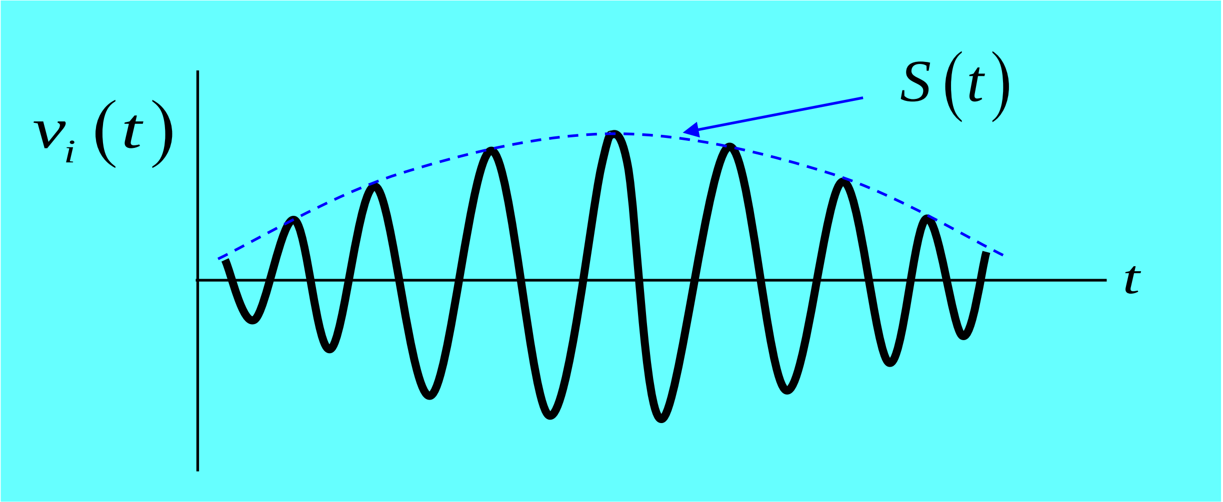
Assume $$v_i(t) = S(t)\cos(\omega_0 t)$$

$S(t)$ = slowly-varying envelope function

Signal Propagation with Dispersion (cont.)

This could be called a "wave packet":

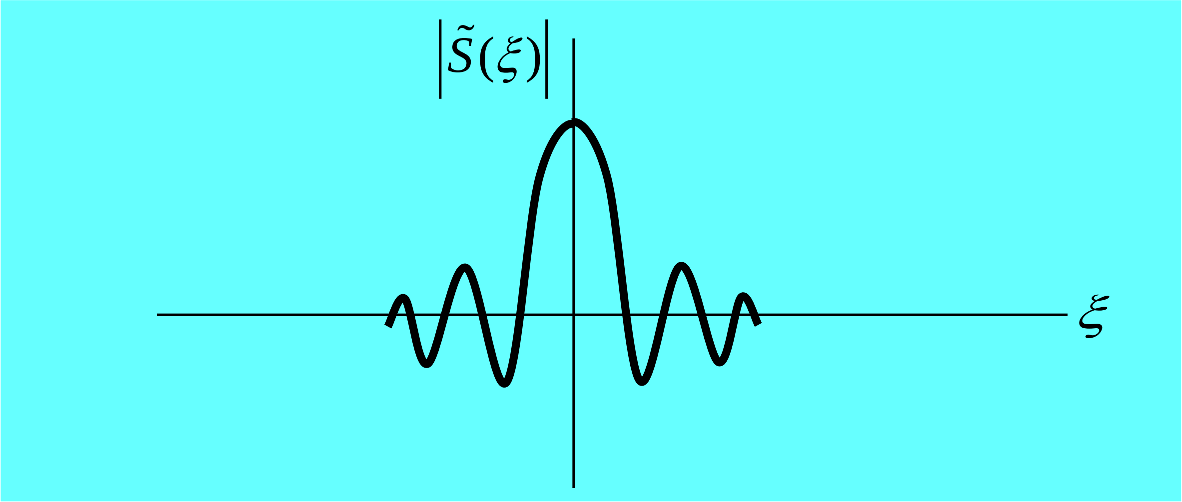
The signal consists of not a single frequency, but a group of closely-spaced frequencies, centered near the carrier frequency ω_0.



Signal Propagation with Dispersion (cont.)

The spectrum of $S(t)$ is very localized near zero frequency:

$$\tilde{S}(\xi) = \int_{-\infty}^{\infty} S(t)\, e^{-j\xi t} dt$$



The envelope function is narrow in the ξ (transform) domain.

Signal Propagation with Dispersion (cont.)

Use

$$v_o(t) = \frac{1}{\pi} \operatorname{Re} \int_0^{\infty} \tilde{v}_i(\omega)\; e^{-\gamma l} e^{j\omega t} d\omega$$

$$v_i(t) = S(t) \cos\left(\omega_0 t\right)$$

Hence

$$\tilde{v}_i(\omega) = \mathscr{F}\left\{ S\left(t\right)\left(\frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}\right)\right\}$$

$$= \frac{1}{2}\tilde{S}(\omega - \omega_0) + \frac{1}{2}\tilde{S}(\omega + \omega_0)$$

Signal Propagation with Dispersion (cont.)

$$\tilde{v}_i(\omega) = \frac{1}{2}\tilde{S}(\omega - \omega_0) + \frac{1}{2}\tilde{S}(\omega + \omega_0)$$

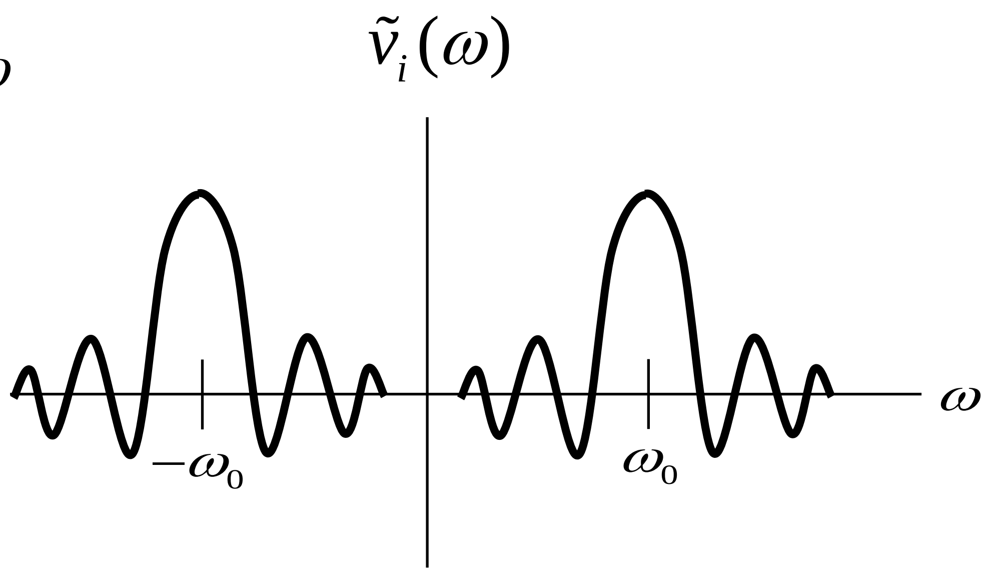
Peak near ω_0 Peak near $-\omega_0$

Recall:

$$v_o(t) = \frac{1}{\pi}\mathrm{Re}\int_0^{\infty} \tilde{v}_i(\omega)\, e^{-\gamma l} e^{j\omega t} d\omega$$

Hence, we have

$$\tilde{v}_i(\omega) \approx \frac{1}{2}\tilde{S}(\omega - \omega_0)$$
$$(\omega > 0)$$



Signal Propagation with Dispersion (cont.)

Hence

$$v_o(t) \approx \frac{1}{2\pi} \mathrm{Re} \int_0^\infty \tilde{S}(\omega - \omega_0)\, e^{-\gamma l} e^{j\omega t} d\omega$$

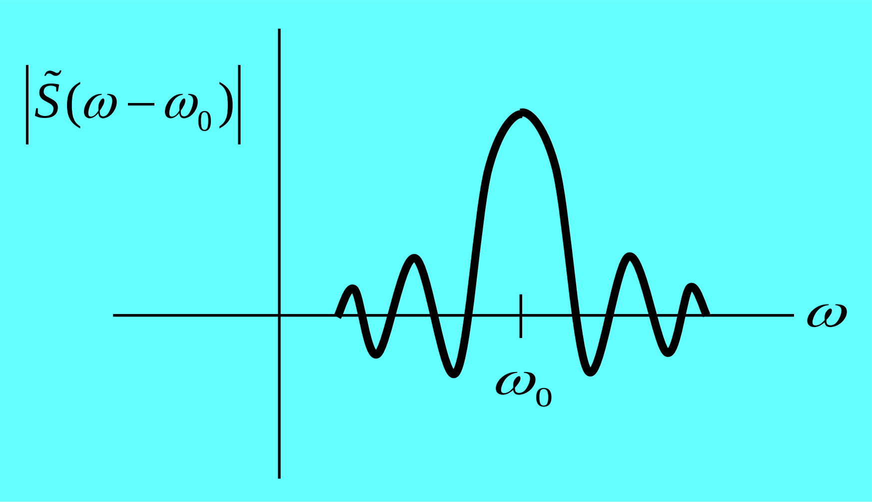
with $\gamma = \alpha + j\beta$

Next, use

$$\beta \approx \beta(\omega_0) + \left(\frac{d\beta}{d\omega}\right)_{\omega_0} (\omega - \omega_0)$$

$$\alpha \approx \alpha(\omega_0) + \left(\frac{d\alpha}{d\omega}\right)_{\omega_0} (\omega - \omega_0)$$

Neglect (second order)



Signal Propagation with Dispersion (cont.)

$$v_o(t) \approx \frac{1}{2\pi} \mathrm{Re} \int_0^\infty \tilde{S}(\omega - \omega_0) e^{-\gamma l} e^{j\omega t} d\omega$$

$$\beta \approx \beta_0 + \left(\frac{d\beta}{d\omega}\right)_{\omega_0} (\omega - \omega_0)$$

$$\alpha \approx \alpha_0$$

Denote

$$\beta_0 = \beta(\omega_0)$$

$$\alpha_0 = \alpha(\omega_0)$$

Hence

$$v_o(t) \approx \frac{1}{2\pi} \mathrm{Re} \int_0^\infty \tilde{S}(\omega - \omega_0) e^{-\alpha_0 l} e^{-j\beta_0 l}$$

$$\cdot e^{-j\left(\frac{d\beta}{d\omega}\right)_{\omega_0} (\omega - \omega_0) l} e^{j\omega t} d\omega$$

Signal Propagation with Dispersion (cont.)

$$v_o(t) \approx \frac{1}{2\pi} \mathrm{Re} \int_0^\infty \tilde{S}(\omega - \omega_0) e^{-\alpha_0 l} e^{-j\beta_0 l} \cdot e^{-j\left(\frac{d\beta}{d\omega}\right)_{\omega_0}(\omega - \omega_0) l} e^{j\omega t} d\omega$$

Multiply and divide by $\exp(j\omega_0 t)$:

$$v_o(t) \approx \frac{1}{2\pi} \mathrm{Re} \int_0^\infty \tilde{S}(\omega - \omega_0) e^{-\alpha_0 l} e^{-j\beta_0 l} \cdot e^{-j\left(\frac{d\beta}{d\omega}\right)_{\omega_0}(\omega - \omega_0) l} e^{j\omega_0 t} e^{j(\omega - \omega_0) t} d\omega$$

Signal Propagation with Dispersion (cont.)

$$v_o(t) \approx \frac{1}{2\pi} \mathrm{Re} \int_0^{\infty} \tilde{S}(\omega - \omega_0) e^{-\alpha_0 l} e^{-j\beta_0 l}$$

$$\cdot e^{-j\left(\frac{d\beta}{d\omega}\right)_{\omega_0} (\omega - \omega_0) l} e^{j\omega_0 t} e^{j(\omega - \omega_0)t} d\omega$$

Let $\xi = \omega - \omega_0$

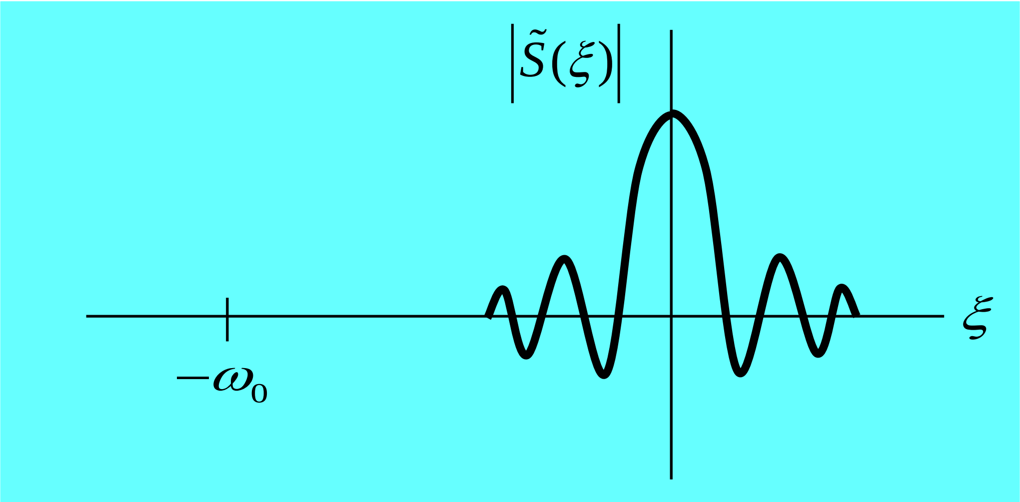
$$v_o(t) \approx \frac{1}{2\pi} \mathrm{Re} \left\{ e^{-\alpha_0 l} e^{-j\beta_0 l} e^{j\omega_0 t} \int_{-\omega_0}^{\infty} \tilde{S}(\xi) e^{-j\left(\frac{d\beta}{d\omega}\right)_{\omega_0} \xi l} e^{j\xi t} d\xi \right\}$$

Signal Propagation with Dispersion (cont.)

$$v_o(t) \approx \frac{1}{2\pi} \mathrm{Re}\left\{ e^{-\alpha_0 l} e^{-j\beta_0 l} e^{j\omega_0 t} \int_{-\omega_0}^{\infty} \tilde{S}(\xi) e^{-j\left(\frac{d\beta}{d\omega}\right)_{\omega_0} \xi l} e^{j\xi t} d\xi \right\}$$

Extend the lower limit to minus infinity:

(since the spectrum of the envelope function is concentrated near zero frequency).



Hence

$$v_o(t) \approx \frac{1}{2\pi} \mathrm{Re}\left(e^{-\alpha_0 l} e^{-j\beta_0 l} e^{j\omega_0 t} \int_{-\infty}^{\infty} \tilde{S}(\xi)\, e^{j\xi\left(t - \left(\frac{d\beta}{d\omega}\right)_{\omega_0} l\right)} d\xi \right)$$

Signal Propagation with Dispersion (cont.)

$$v_o(t) \approx \mathrm{Re}\left(e^{-\alpha_0 l} e^{-j\beta_0 l} e^{j\omega_0 t} \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{S}(\xi)\, e^{j\xi\left(t - \left(\frac{d\beta}{d\omega}\right)_{\omega_0} l\right)} d\xi \right)$$

Hence

$$v_o(t) \approx \mathrm{Re}\left\{ e^{-\alpha_0 l} e^{-j\beta_0 l} e^{j\omega_0 t} S\left(t - \left(\frac{d\beta}{d\omega}\right)_{\omega_0} l \right) \right\}$$

Signal Propagation with Dispersion (cont.)

Hence we have

$$v_o(t) \approx S\left(t - \left(\frac{d\beta}{d\omega} \right)_{\omega_0} l \right) e^{-\alpha_0 l} \cos\left(\omega_0 t - \beta_0 l\right)$$

Define group velocity:

$$v_g \equiv \left. \frac{d\omega}{d\beta} \right|_{\omega_0}$$

Then we have

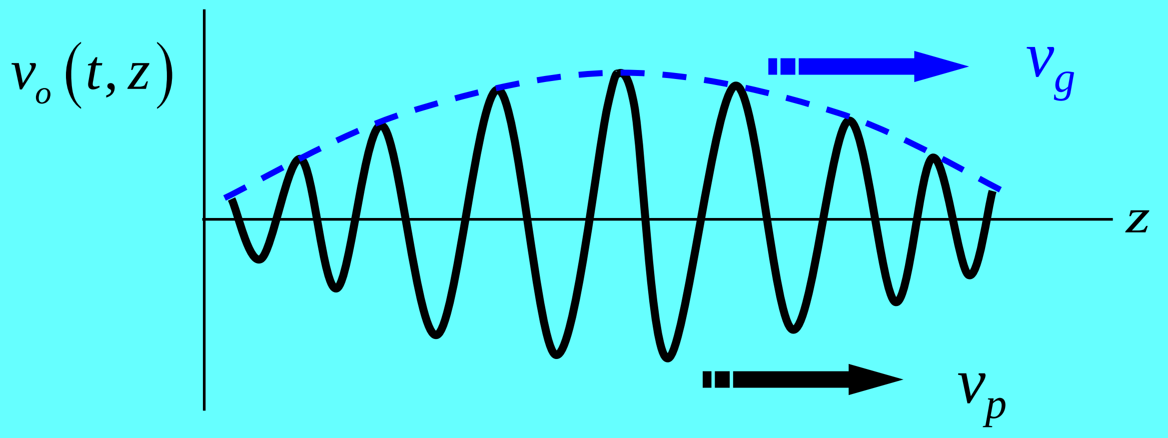
$$v_o(t) \approx S\left(t - l / v_g\right) \cos\left(\omega_0 \left(t - l / v_p\right)\right) e^{-\alpha_0 l}$$

Signal Propagation with Dispersion (cont.)

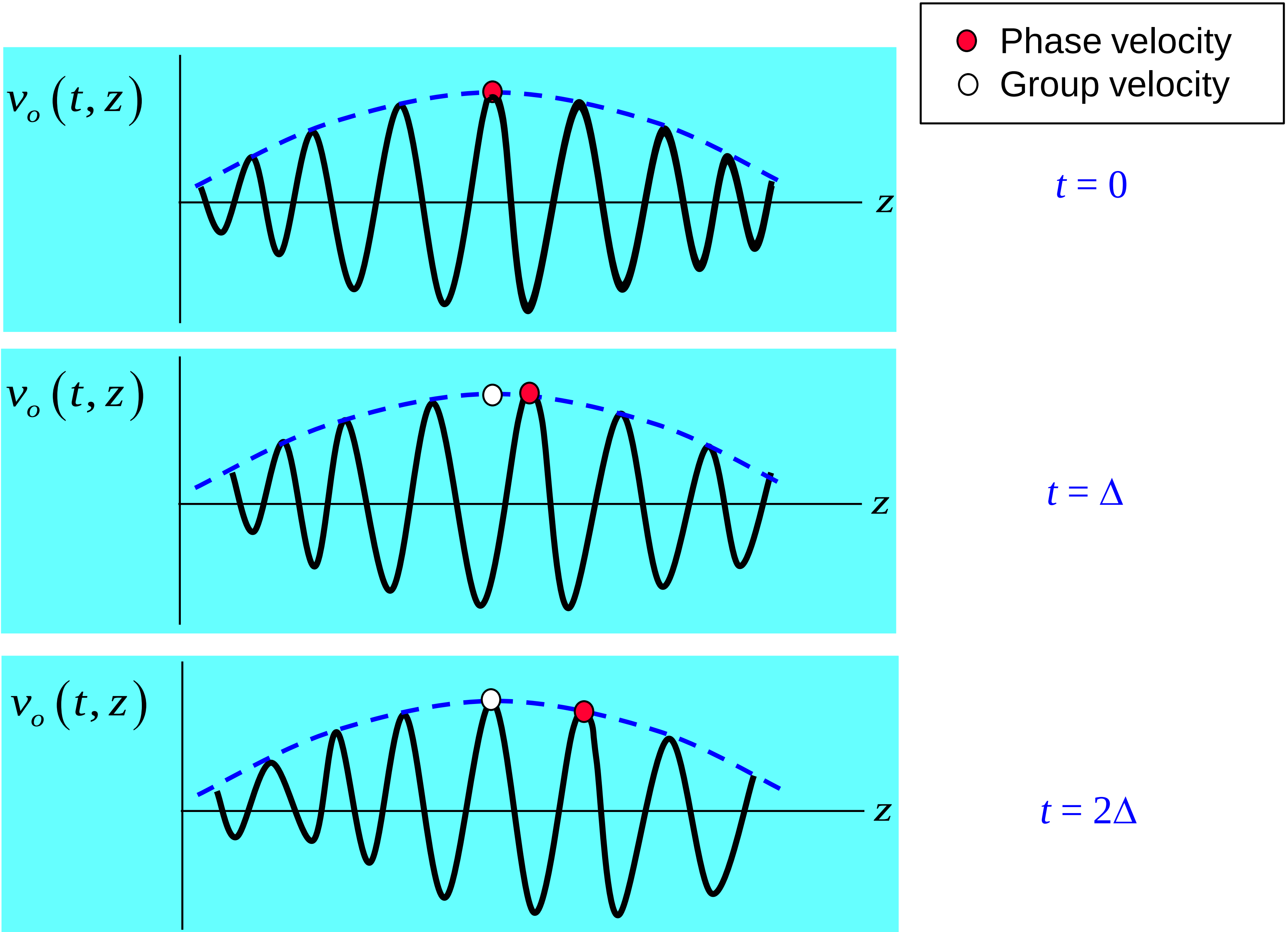
Summary

$$v_i(t) = S(t)\cos\left(\omega_0 t\right)$$

$$v_o(t,z) \approx S\left(t - z / v_g\right)\cos\left(\omega_0\left(t - z / v_p\right)\right) e^{-\alpha_0 l}$$

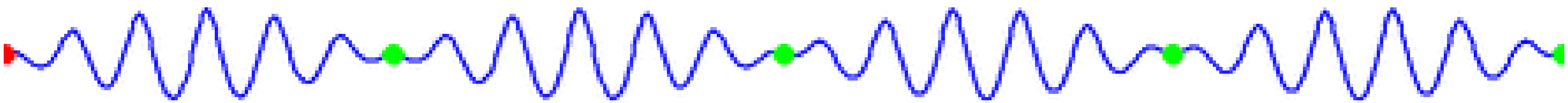


Example: $v_g = 0,\ v_p > 0$



Example: $v_p > v_g$

Example from Wikipedia (view in full-screen mode with pptx)



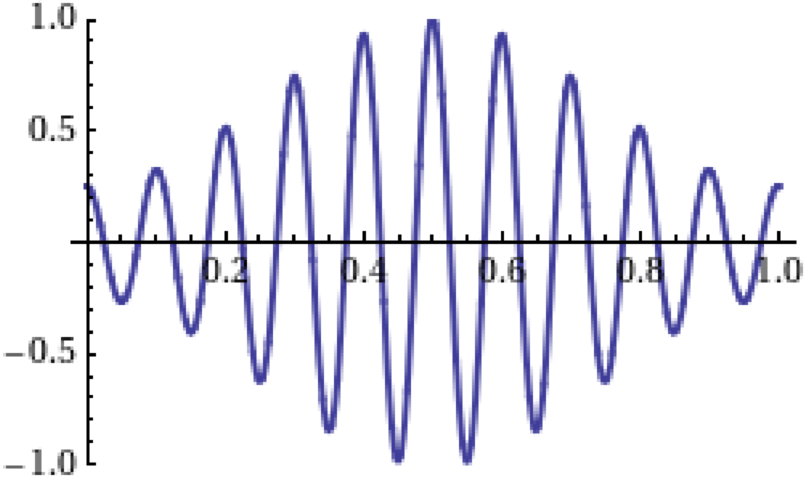
Red dot: phase velocity

Green dot: group velocity

http://en.wikipedia.org/wiki/Group_velocity

Example: $v_g > 0$, $v_p < 0$

Example from Wikipedia (view in full-screen mode with pptx)



"Backward wave"

(The phase and group velocities are in opposite directions.)

This shows a wave with the group velocity and phase velocity going in different directions. (The group velocity is positive and the phase velocity is negative.)

http://en.wikipedia.org/wiki/Group_velocity

Notes on Group Velocity

- In many cases, the group velocity represents the velocity of information flow (the velocity of the baseband signal).
- This is true when the dispersion is sufficiently small over the frequency spectrum of the signal, and the group velocity is less than the speed of light.

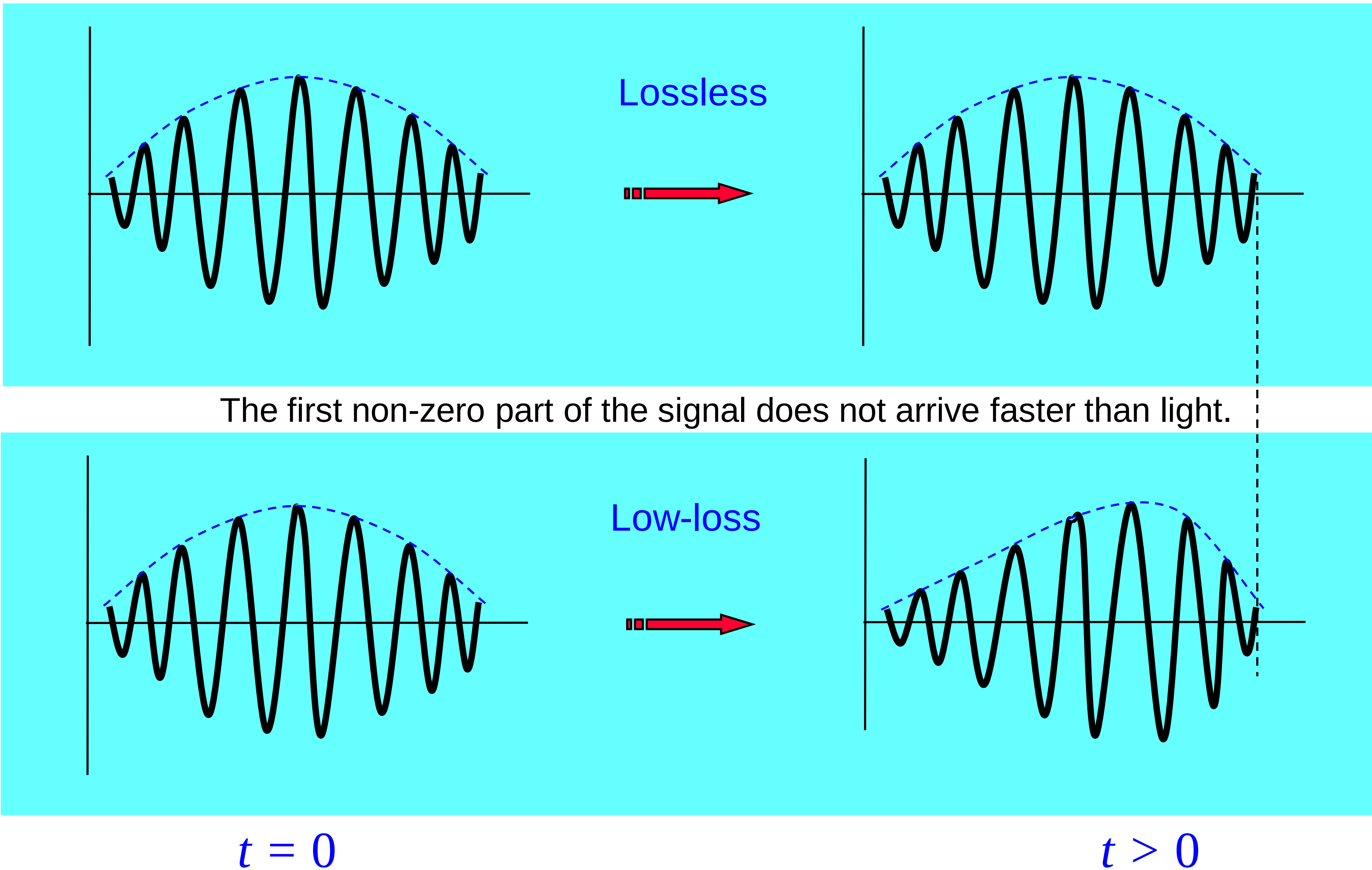
Example: a narrow-band signal propagating in a rectangular waveguide.

Note:

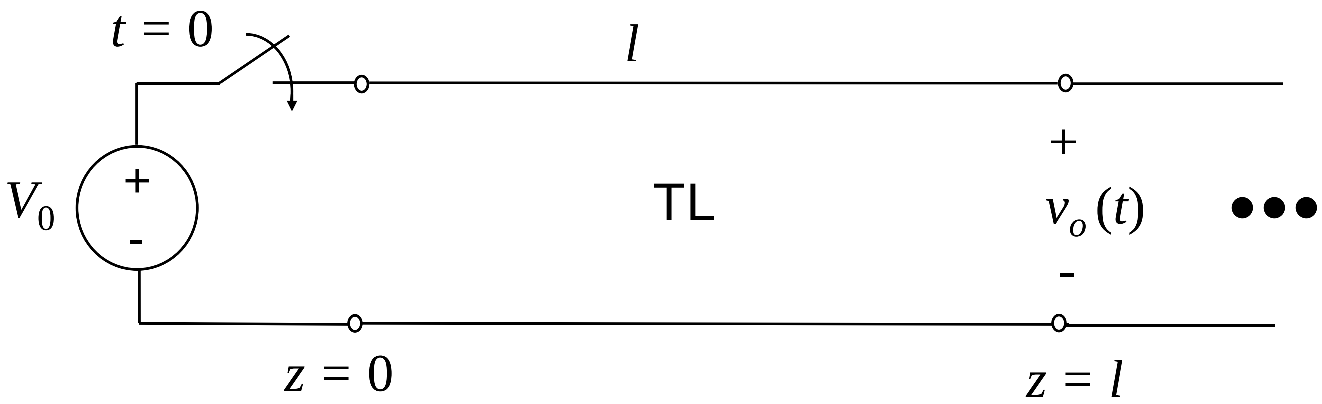
In some cases the group velocity exceeds the speed of light. In such cases, the waveform always distorts sufficiently as it propagates so that the signal never arrives fast than light.

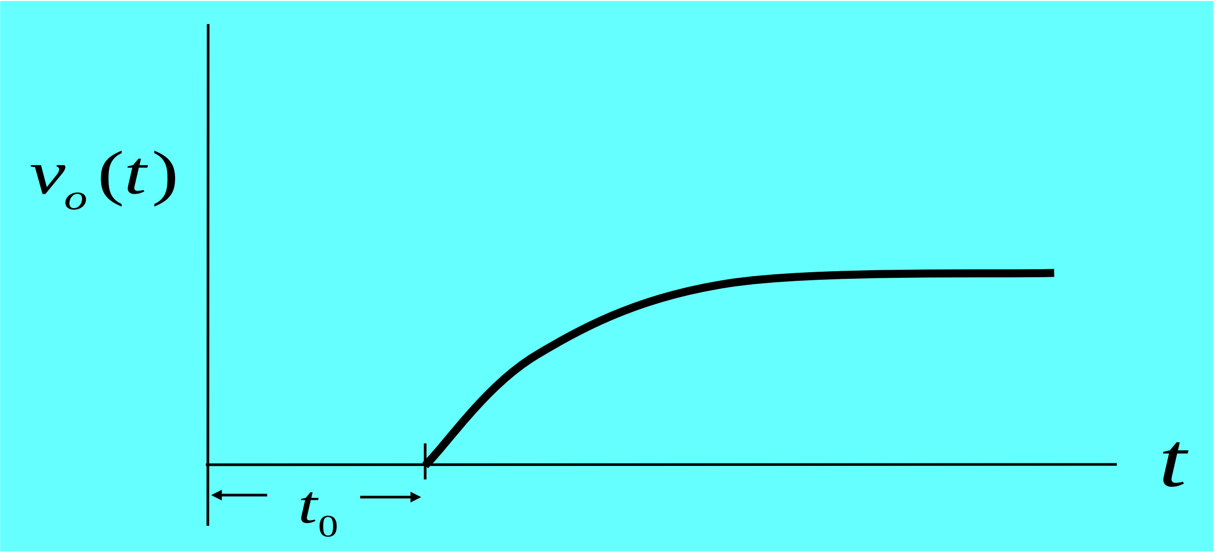
Notes on Group Velocity (cont.)

Sometimes $v_g > c$ (e.g., hypothetical low-loss TL filled with air, with constant R and/or G).



Signal Velocity





$$v_s \equiv \frac{l}{t_0}$$

Relativity:

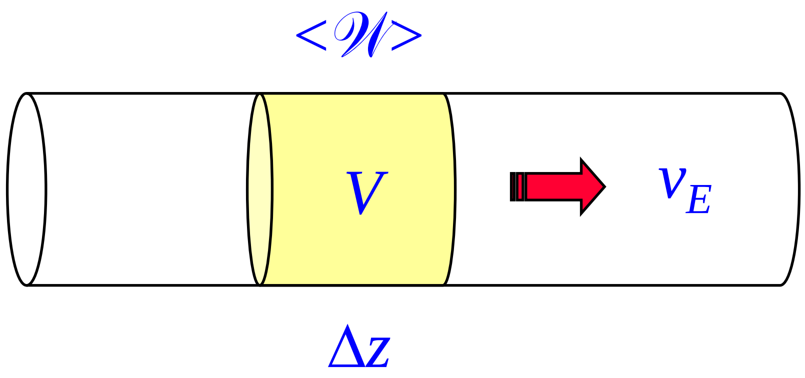
$$v_s < c$$

Energy Velocity

Definition of energy velocity v_E :

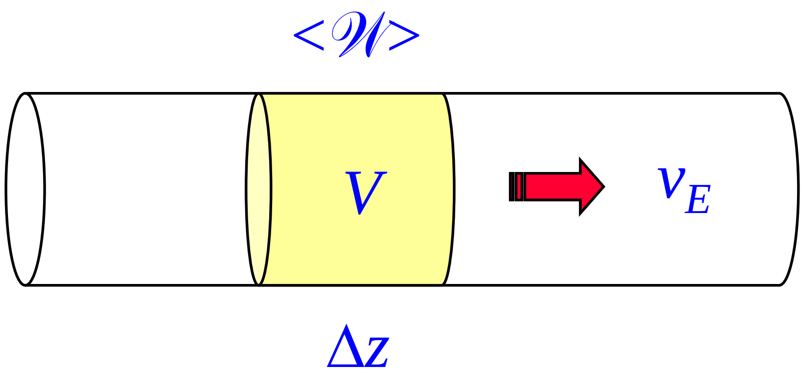
$$\langle \mathscr{P}_z \rangle = \frac{\langle \mathscr{W} \rangle}{\Delta t} = \frac{\langle \mathscr{W}_l \rangle \Delta z}{\Delta t} = \langle \mathscr{W}_l \rangle v_E \quad \Rightarrow \quad v_E \equiv \frac{\langle \mathscr{P}_z \rangle}{\langle \mathscr{W}_l \rangle}$$

$< \mathscr{W}_l >$ = time-average energy stored per unit length in the z direction.



Here we think of stored energy moving down the system.

Energy Velocity



$$v_E \equiv \frac{\langle \mathscr{P}_z \rangle}{\langle \mathscr{W}_l \rangle}$$

where

$$\langle P_z \rangle = \mathrm{Re} \int_S \frac{1}{2} \left(\underline{E} \times \underline{H}^* \right) \cdot \underline{\hat{z}} \, dS$$

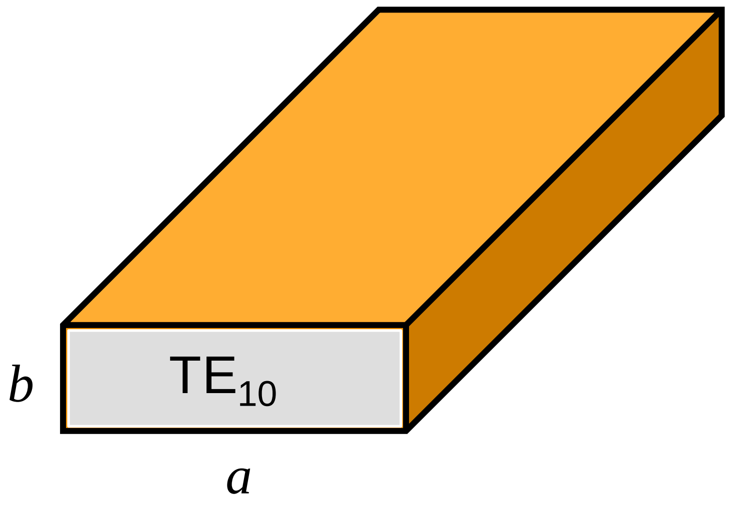
$$\langle \mathscr{W}_l \rangle = \frac{1}{\Delta z} \int_V \left(\frac{1}{4} \varepsilon_c' \left| \underline{E} \right|^2 + \frac{1}{4} \mu_c' \left| \underline{H} \right|^2 \right) dV = \int_S \left(\frac{1}{4} \varepsilon_c' \left| \underline{E} \right|^2 + \frac{1}{4} \mu_c' \left| \underline{H} \right|^2 \right) dS$$

Note: In many systems the energy velocity is equal to the group velocity.

Example

Rectangular Waveguide

(air-filled)



$$\beta = \sqrt{k_0^{\,2} - \left(\frac{\pi}{a}\right)^2}$$

$$= \sqrt{\omega^2 \mu_0 \varepsilon_0 - \left(\frac{\pi}{a}\right)^2}$$

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\omega^2 \mu_0 \varepsilon_0 - \left(\frac{\pi}{a}\right)^2}}$$

Multiply top and bottom by $c/\omega = 1/k_0$:

$$v_p = \frac{c}{\sqrt{1 - \left(\frac{\pi}{k_0 a}\right)^2}}$$

Example (cont.)

To calculate the group velocity, use

$$\beta^2 = \omega^2 \mu_0 \varepsilon_0 - \left(\frac{\pi}{a}\right)^2$$

$$2\beta d\beta = 2\omega d\omega (\mu_0 \varepsilon_0)$$

$$\Rightarrow v_g = \frac{d\omega}{d\beta} = \frac{\beta}{\omega}\left(\frac{1}{\mu_0 \varepsilon_0}\right) = \frac{1}{v_p}(c^2)$$

Hence

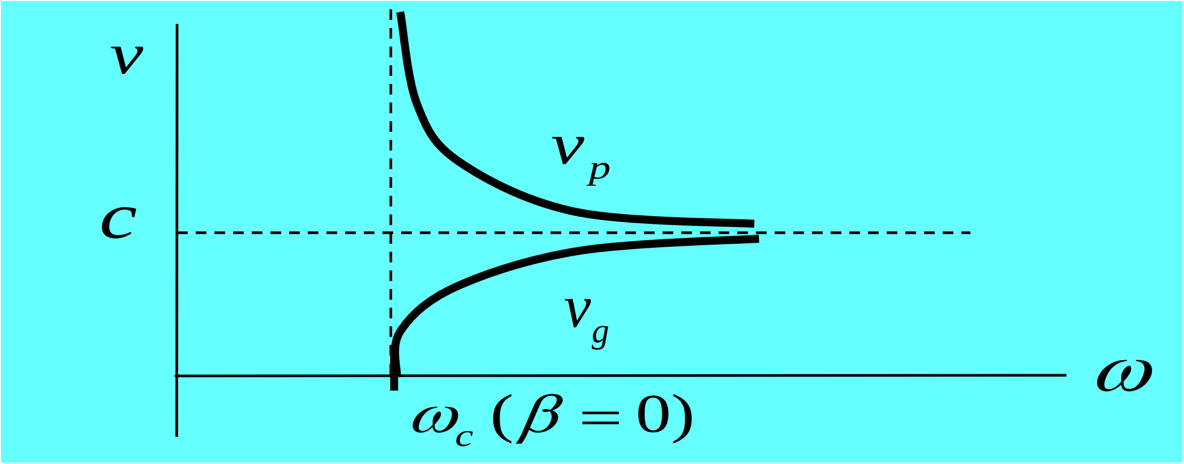
$$v_g v_p = c^2$$

Note: This property holds for all lossless waveguides.

Example (cont.)

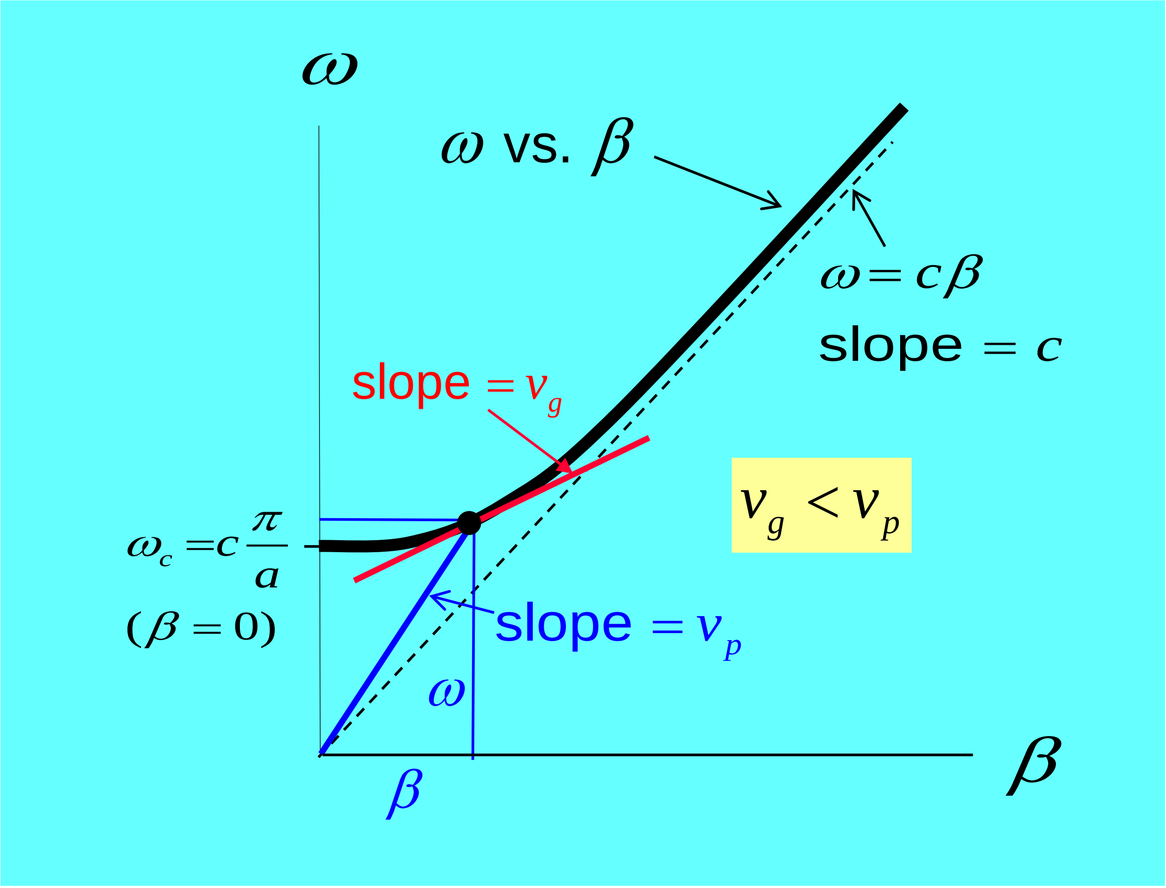
$$v_p = \frac{c}{\sqrt{1-\left(\dfrac{\pi}{k_0 a}\right)^2}}$$

$$v_g = c\sqrt{1-\left(\dfrac{\pi}{k_0 a}\right)^2}$$



Example (cont.)

Graphical representation (ω - β diagram):



$$\beta = \sqrt{\omega^2 \mu_0 \varepsilon_0 - \left(\frac{\pi}{a}\right)^2}$$

$$v_p = \frac{\omega}{\beta}$$

$$v_g = \left. \frac{d\omega}{d\beta} \right|_{\omega_0}$$

Dispersion

Theorem: If there is no dispersion, then $v_p = v_g$ (a constant).

An example is a lossless transmission line (no dispersion).

Note: For a lossless TL we have $v_p = \frac{1}{\sqrt{LC}}$

Proof:

$$v_p(\omega) = \text{constant (from the definition of dispersion)}$$

Hence:

$$\frac{\omega}{\beta} = a_1$$

$$\Rightarrow \omega = a_1 \beta$$

$$\Rightarrow d\omega = a_1 d\beta$$

$$\Rightarrow \frac{d\omega}{d\beta} = a_1 \quad \Rightarrow \quad v_g = v_p$$

Dispersion (cont.)

Theorem: If $v_p = v_g$ for all frequencies, then there is no dispersion.

Proof:

$$\frac{\omega}{\beta} = \frac{d\omega}{d\beta}$$

$$\Rightarrow \frac{d\beta}{\beta} = \frac{d\omega}{\omega}$$

$$\Rightarrow \ln\beta = \ln\omega + C$$

$$\Rightarrow \beta = e^{C}\omega$$

$$\Rightarrow \quad v_p = \frac{\omega}{\beta} = \text{constant}$$

Dispersion (cont.)

$$v_p = \text{constant (no dispersion)}$$

$$v_p = v_g$$

for all frequencies

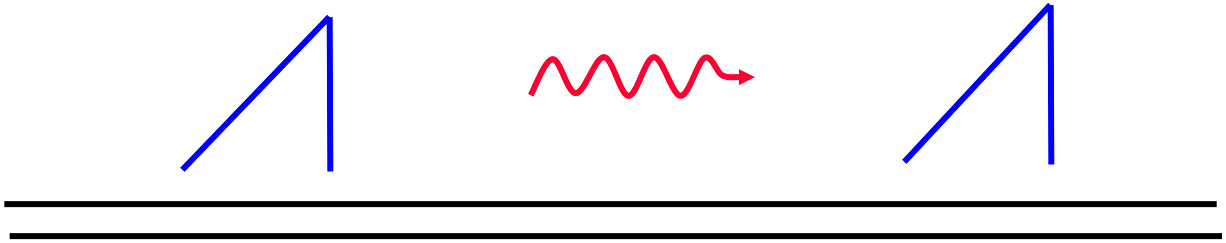
EXAMPLES:

- Plane wave in free space
- Lossless TL
- Distortionless (lossy)TL

Dispersion (cont.)

No dispersion and Attenuation is frequency independent ⇒ No distortion

(The phase velocity is frequency independent.)



Note: Loss on a TL causes dispersion and it <u>also</u> causes the attenuation to be frequency dependent.

Dispersion (cont.)

"Normal" Dispersion: $v_g < v_p$ Example: waveguide

This is equivalent to $\dfrac{d\left(\beta / k_0\right)}{d\omega} > 0$ (proof omitted)

"Anomalous" Dispersion: $v_g > v_p$ Example: low-loss transmission line

This is equivalent to $\dfrac{d\left(\beta / k_0\right)}{d\omega} < 0$ (proof omitted)

Backward Wave

Definition of backward wave: $$v_g v_p < 0$$

The group velocity has the opposite sign as the phase velocity.

This type of wave will never exist on a TEM transmission line filled with usual dielectric materials, but may exist on a periodic artificial transmission line.

Note:
Do not confuse “backward wave" with “a wave traveling in the backward direction.”