ECE 6340
Intermediate EM Waves

Fall 2016

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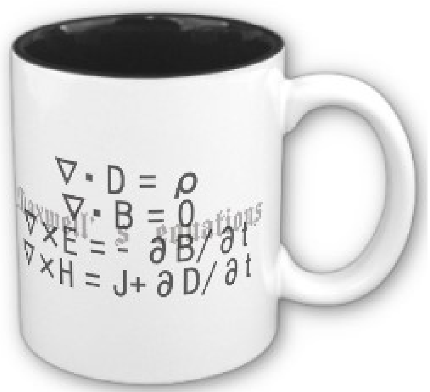
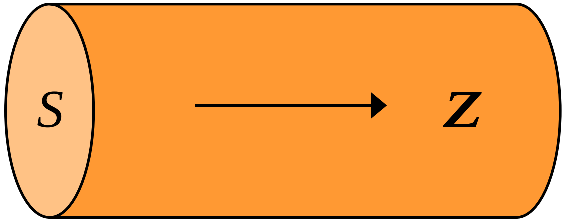
Dept. of ECE



Notes 15

Attenuation Formula

Waveguiding system (WG or TL):



Waveguiding system

$$\underline{E}(x,y,z) = \underline{E}_0(x,y)\,e^{-\gamma z} = \underline{E}_0(x,y)\,e^{-j\beta z}e^{-\alpha z}$$

$$\underline{H}(x,y,z) = \underline{H}_0(x,y)\,e^{-\gamma z} = \underline{H}_0(x,y)\,e^{-j\beta z}e^{-\alpha z}$$

At $z = 0$: $$P_f(0) = \int_S \frac{1}{2}\left(\underline{E}_0 \times \underline{H}_0^*\right)\cdot \underline{\hat{z}}\, dS$$

At $z = \Delta z$: $$P_f(\Delta z) = \int_S \frac{1}{2}\left(\underline{E}_0 \times \underline{H}_0^*\right)\, e^{-2\alpha\,\Delta z} \cdot \underline{\hat{z}}\, dS$$

Attenuation Formula (cont.)

Hence

$$P_f(\Delta z) = P_f(0)\, e^{-2\alpha\,\Delta z}$$

$$\operatorname{Re} P_f(\Delta z) = \operatorname{Re} P_f(0)\, e^{-2\alpha\,\Delta z}$$

so

$$\left\langle \mathscr{P}_f(\Delta z) \right\rangle = \left\langle \mathscr{P}_f(0) \right\rangle e^{-2\alpha\,\Delta z}$$

If $\alpha\,\Delta z \ll 1$

$$\left\langle \mathscr{P}_f(\Delta z) \right\rangle \approx \left\langle \mathscr{P}_f(0) \right\rangle (1 - 2\alpha\,\Delta z)$$

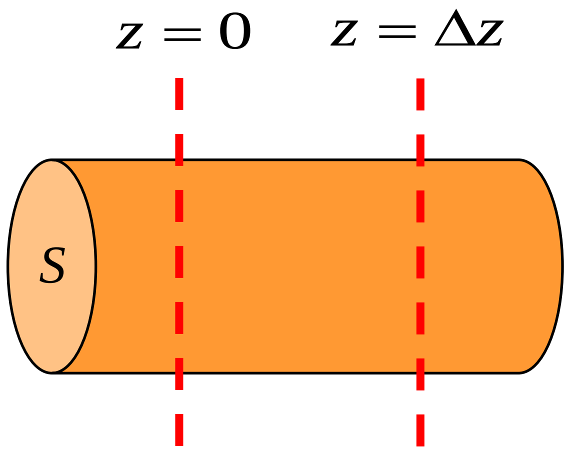
$$= \left\langle \mathscr{P}_f(0) \right\rangle - 2\alpha\,\Delta z \left\langle \mathscr{P}_f(0) \right\rangle$$

Attenuation Formula (cont.)

$$\left\langle \mathscr{P}_f(\Delta z) \right\rangle \approx \left\langle \mathscr{P}_f(0) \right\rangle - 2\alpha\,\Delta z \left\langle \mathscr{P}_f(0) \right\rangle$$

so

$$\alpha \approx \frac{\left\langle \mathscr{P}_f(0) \right\rangle - \left\langle \mathscr{P}_f(\Delta z) \right\rangle}{2\,\Delta z \left\langle \mathscr{P}_f(0) \right\rangle}$$



From conservation of energy:

$$\left\langle \mathscr{P}_f(0) \right\rangle - \left\langle \mathscr{P}_f(\Delta z) \right\rangle \approx \Delta z \left\langle \mathscr{P}_d^{\,l}(\Delta z/2) \right\rangle$$

where

$$\left\langle \mathscr{P}_d^{\,l}(z) \right\rangle = \text{power dissipated per length at point } z$$

Attenuation Formula (cont.)

Hence

$$\alpha \approx \frac{\Delta z \left\langle \mathscr{P}_d^{\,l}(\Delta z/2) \right\rangle}{2\,\Delta z \left\langle \mathscr{P}_f(0) \right\rangle}$$

As $\Delta z \rightarrow 0$:

$$\alpha = \frac{\left\langle \mathscr{P}_d^{\,l}(0) \right\rangle}{2\left\langle \mathscr{P}_f(0) \right\rangle}$$

Note: Where the point $z = 0$ is located is arbitrary.

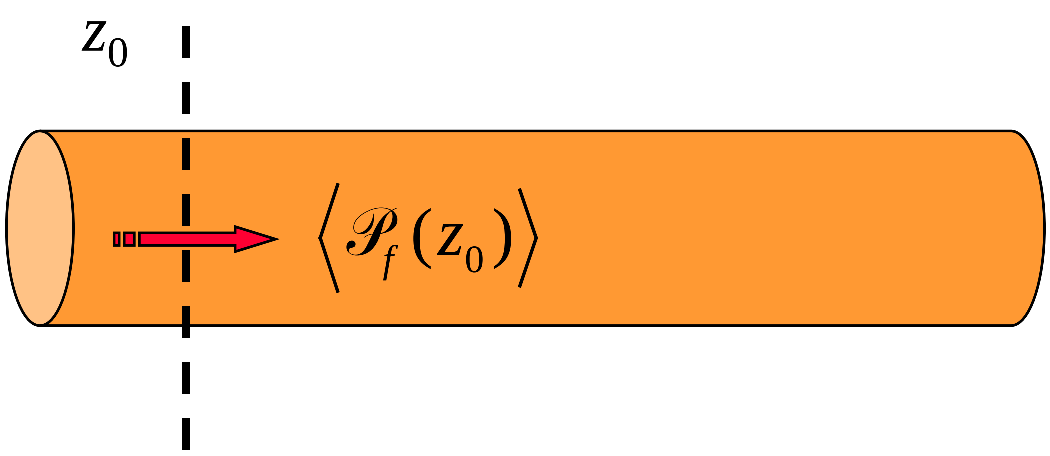
Attenuation Formula (cont.)

General formula:

$$\alpha = \frac{\left\langle \mathscr{P}_d^{\,l}(z_0) \right\rangle}{2\left\langle \mathscr{P}_f(z_0) \right\rangle}$$

This is a <u>perturbational formula</u> for the conductor attenuation.

The power flow and power dissipation are usually calculated assuming the fields are those of the mode with PEC conductors.



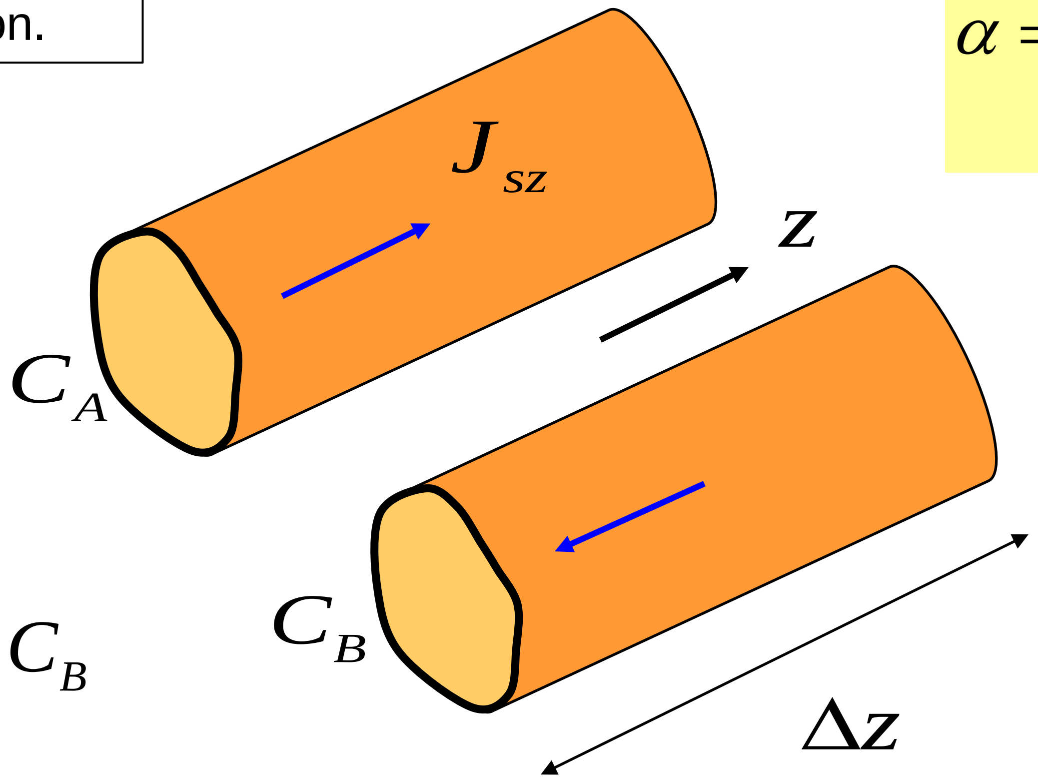
Attenuation on Transmission Line

Attenuation due to Conductor Loss

$$\alpha = \alpha_c$$

The current of the TEM mode flows in the z direction.

$$\alpha = \frac{\left\langle \mathscr{P}_d^l \right\rangle}{2\left\langle \mathscr{P}_f \right\rangle}$$



$$C = C_A + C_B$$

Attenuation on Line (cont.)

Power dissipation due to conductor loss:

$$\left\langle \mathscr{P}_d^l \right\rangle = \frac{1}{\Delta z} \int_S \frac{1}{2} R_s \left| J_{sz} \right|^2 dS$$

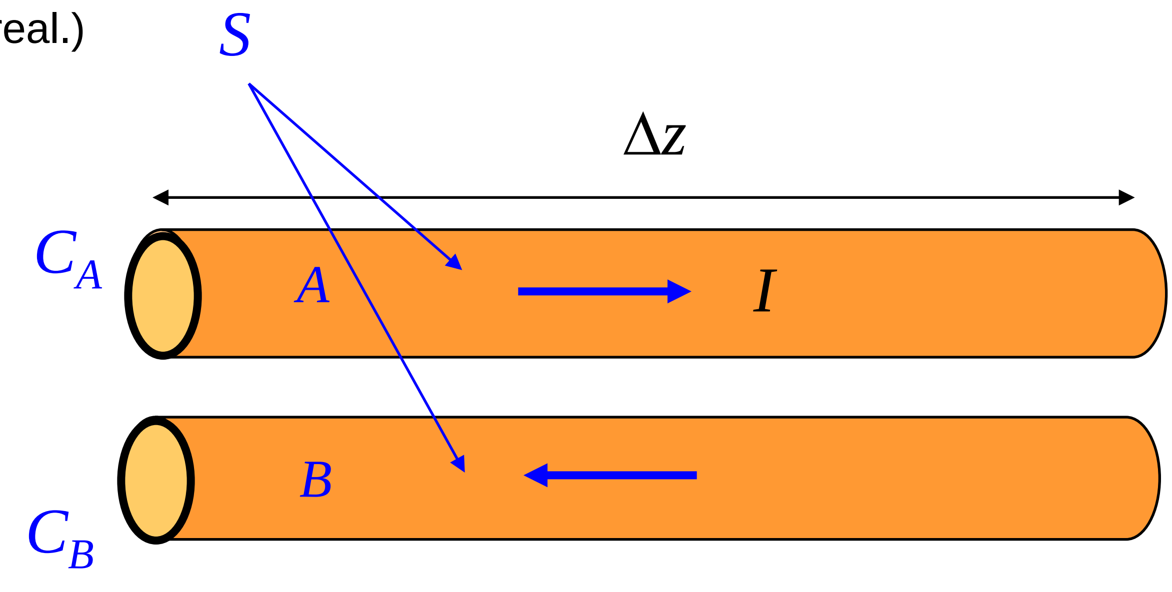
$$= \frac{1}{\Delta z} (\Delta z) \int_C \frac{1}{2} R_s \left| J_{sz} (\ell) \right|^2 dl$$

$$= \int_C \frac{1}{2} R_s \left| J_{sz} (\ell) \right|^2 dl$$

Power flowing on line:

$$\left\langle \mathscr{P}_f \right\rangle = \frac{1}{2} Z_0 \left| I \right|^2$$

(Z_0 is assumed to be approximately real.)



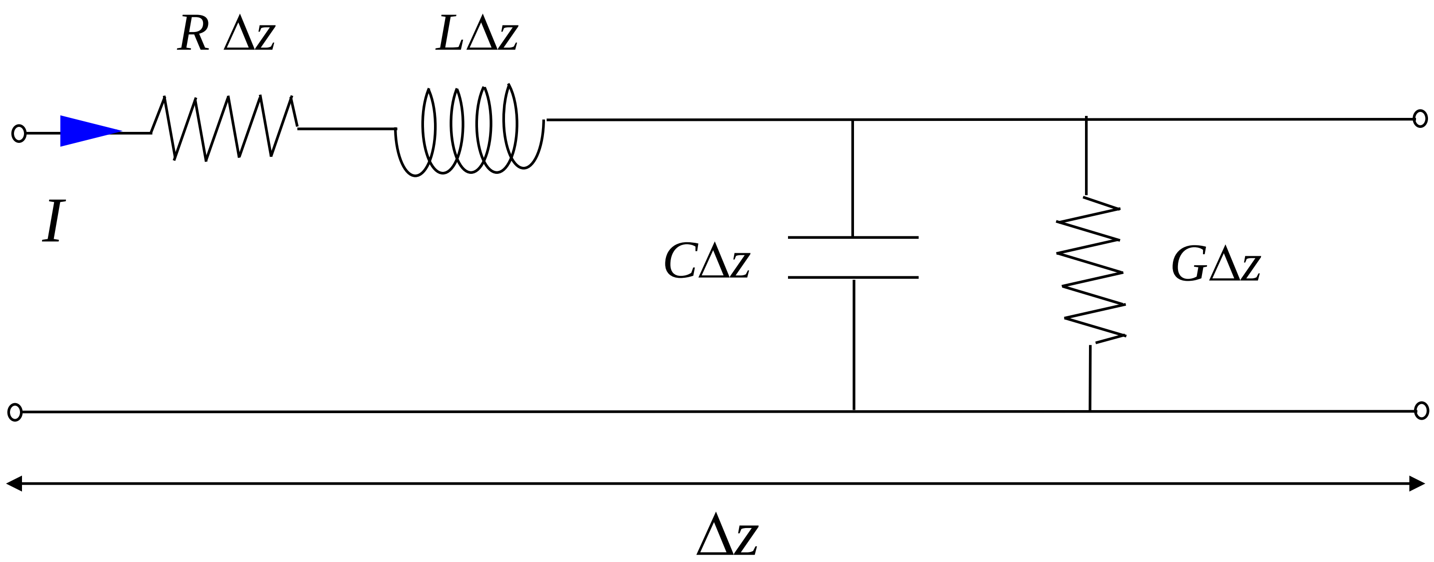
$C = C_A + C_B$

Attenuation on Line (cont.)

Hence

$$\alpha_c = \left(\frac{R_s}{2Z_0}\right)\left[\frac{1}{|I|^2}\int_{C_A+C_B} \left|J_{sz}(\ell)\right|^2 dl\right]$$

R on Transmission Line



Ignore G for the R calculation ($\alpha = \alpha_c$):

$$\alpha_c = \frac{\left\langle \mathscr{P}_d^l \right\rangle}{2\left\langle \mathscr{P}_f \right\rangle}$$

$$\left\langle \mathscr{P}_d^l \right\rangle = \frac{1}{2} R \left| I \right|^2$$

$$\left\langle \mathscr{P}_f \right\rangle = \frac{1}{2} Z_0 \left| I \right|^2$$

R on Transmission Line (cont.)

We then have

$$\alpha_c = \frac{R}{2Z_0}$$

Hence

$$R = \alpha_c \left(2Z_0\right)$$

Substituting for α_c,

$$R = R_s \left[\frac{1}{\left|I\right|^2} \oint_C \left|J_{sz}(l)\right|^2 dl \right]$$

Total Attenuation on Line

Method #1

$$\alpha = \alpha_c + \alpha_d$$

$$\alpha_d = \alpha_{TEM}$$

When we ignore conductor loss to calculate α_d, we have a TEM mode.

$$k_z^{TEM} = \beta - j\alpha_d = k = k' - jk''$$

so

$$\alpha_d = k''$$

Hence,

$$\alpha = \alpha_c + k''$$

Total Attenuation on Line (cont.)

Method #2

$$\begin{aligned}\alpha &= \operatorname{Re}\gamma \\ &= \operatorname{Re}\left(\sqrt{(R+j\omega L)(G+j\omega C)}\right)\end{aligned}$$

where

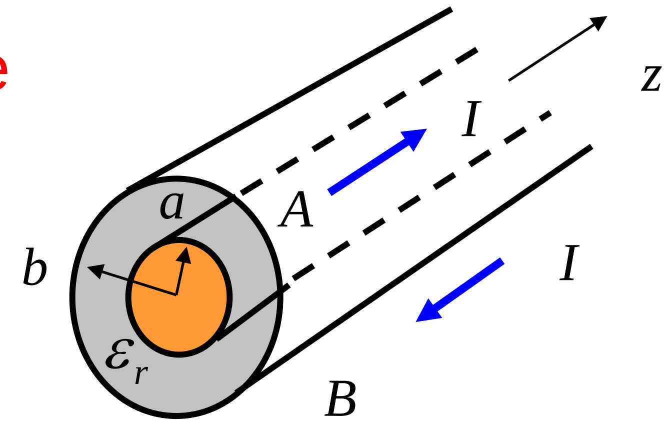
$$R = \alpha_c (2Z_0)$$

$$G = (\omega C)\left(\frac{\varepsilon_c''}{\varepsilon_c'}\right)$$

The two methods give approximately the same results.

Example: Coax

Coaxial Cable



$$\alpha_c = \left(\frac{R_s}{2Z_0}\right)\frac{1}{|I|^2}\left\{\int_{C_A} |J_{sz}(\ell)|^2\, dl + \int_{C_B} |J_{sz}(\ell)|^2\, dl\right\}$$

A) $\quad J_{sz} = \dfrac{I}{2\pi a}$

B) $\quad J_{sz} = \dfrac{-I}{2\pi b}$

Example (cont.)

Hence

$$\alpha_c = \left(\frac{R_s}{2Z_0}\right)\frac{1}{|I|^2}\left\{\int_0^{2\pi}\left|\frac{I}{2\pi a}\right|^2 a\,d\phi + \int_0^{2\pi}\left|\frac{-I}{2\pi b}\right|^2 b\,d\phi\right\}$$

$$= \left(\frac{R_s}{2Z_0}\right)\left\{\frac{1}{2\pi a} + \frac{1}{2\pi b}\right\}$$

Also,

$$Z_0 = \frac{\eta_0}{2\pi\sqrt{\varepsilon_r}}\ln\left(\frac{b}{a}\right)$$

Hence

$$\alpha_c = \frac{R_s}{b}\left[\frac{\sqrt{\varepsilon_r}\left(1+\frac{b}{a}\right)}{2\eta_0\ln\left(\frac{b}{a}\right)}\right]$$

(nepers/m)

Example (cont.)

Calculate R:

$$R = \alpha_c (2Z_0)$$

$$= \left[\left(\frac{R_s}{2Z_0} \right) \left\{ \frac{1}{2\pi a} + \frac{1}{2\pi b} \right\} \right] (2Z_0)$$

$$= \left(\frac{R_s}{2\pi} \right) \left(\frac{1}{a} + \frac{1}{b} \right)$$

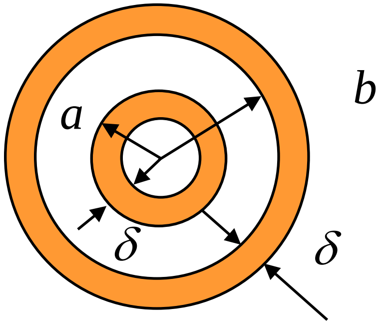
$$= \frac{1}{2\pi\sigma\delta} \left(\frac{1}{a} + \frac{1}{b} \right)$$

Example (cont.)

$$R = \frac{1}{2\pi a\sigma\delta} + \frac{1}{2\pi b\sigma\delta}$$

This agrees with the formula obtained from the “DC equivalent model.”

(The DC equivalent model assumes that the current is uniform around the boundary, so it is a less general method.)



DC equivalent model of coax

Internal Inductance

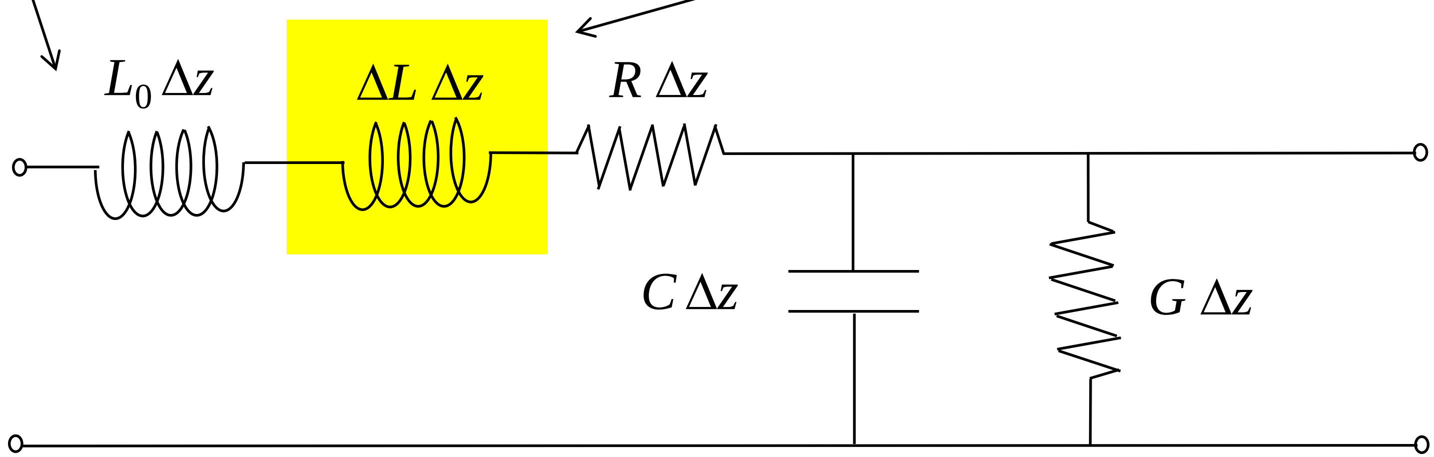
An extra inductance per unit length ΔL is added to the TL model in order to account for the internal inductance of the conductors.

This extra (internal) inductance consumes imaginary (reactive) power.

The "external inductance" L_0 accounts for magnetic energy only in the external region (between the conductors). This is what we get by assuming PEC conductors.

$$L = L_0 + \Delta L$$

Internal inductance



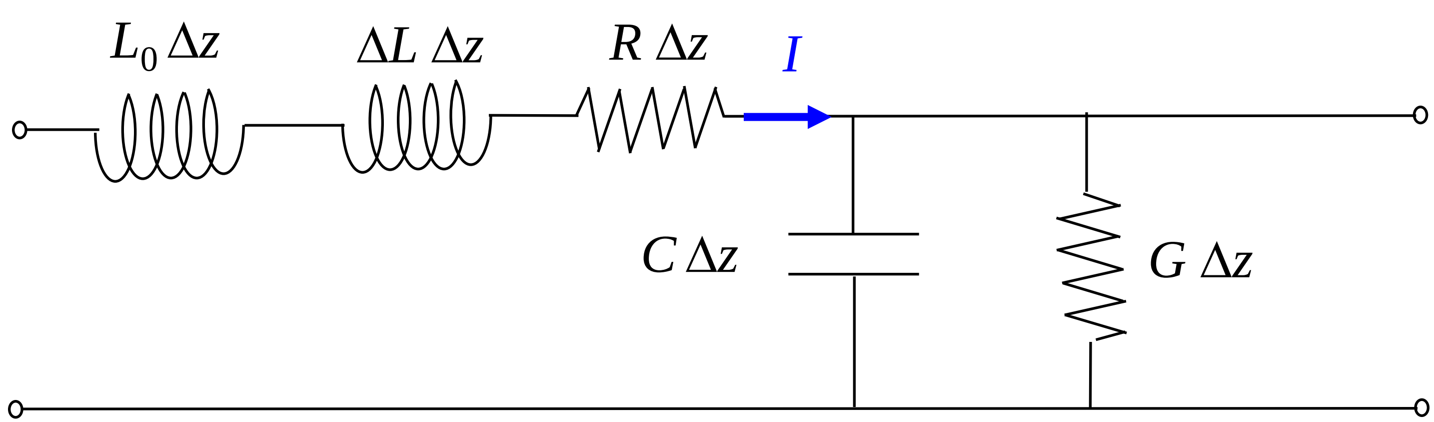
Skin Inductance (cont.)

Imaginary (reactive) power per meter consumed by the extra inductance:

Circuit model: $$P_I = \frac{1}{2}(\omega \Delta L)|I|^2$$

Equate

Skin-effect formula: $$P_I = \frac{1}{2} X_s \int_{C_A + C_B} |J_{sz}(\ell)|^2 dl$$



Skin Inductance (cont.)

Hence:

$$\frac{1}{2}\omega\Delta L = \frac{1}{2}X_s \frac{1}{|I|^2}\int_{C_A+C_B}|J_{sz}(\ell)|^2 dl$$

$$= \frac{1}{2}R_s \frac{1}{|I|^2}\int_{C_A+C_B}|J_{sz}(\ell)|^2 dl$$

$$= \frac{1}{2}R$$

Skin Inductance (cont.)

$$\frac{1}{2}\omega\Delta L = \frac{1}{2}R$$

Hence

$$\Delta X = R$$

or

$$\Delta L = \frac{R}{\omega}$$

Summary of High-Frequency Formulas for Coax

Assumption: $\delta << a$

$$R_a^{HF} = \frac{1}{2\pi a\sigma\delta}$$

$$R_b^{HF} = \frac{1}{2\pi b\sigma\delta}$$

$$R^{HF} = R_a^{HF} + R_b^{HF}$$

$$\Delta X_a^{HF} = \omega\left(\Delta L_a^{HF}\right) = \frac{1}{2\pi a\sigma\delta}$$

$$\Delta X_b^{HF} = \omega\left(\Delta L_b^{HF}\right) = \frac{1}{2\pi b\sigma\delta}$$

$$\Delta L^{HF} = \Delta L_a^{HF} + \Delta L_b^{HF}$$

Low Frequency (DC) Coax Model

At low frequency (DC) we have:

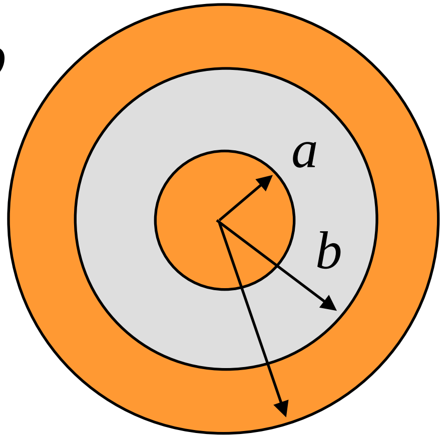
$$R^{DC} = R_a^{DC} + R_b^{DC}$$

$$\Delta L^{DC} = \Delta L_a^{DC} + \Delta L_b^{DC}$$

$$R_a^{DC} = \frac{1}{\sigma\left(\pi a^2\right)}$$

$$R_b^{DC} = \frac{1}{\sigma\left(2\pi b t\right)}$$

$t = c - b$



Derivation omitted

$$\Delta L_a^{DC} = \frac{\mu_0}{8\pi}$$

$$\Delta L_b^{DC} = \frac{\mu_0}{2\pi}\left[\frac{c^4 \ln\left(\frac{c}{b}\right)}{\left(c^2 - b^2\right)^2} + \frac{b^2 - 3c^2}{4\left(c^2 - b^2\right)}\right]$$

Tesche Model

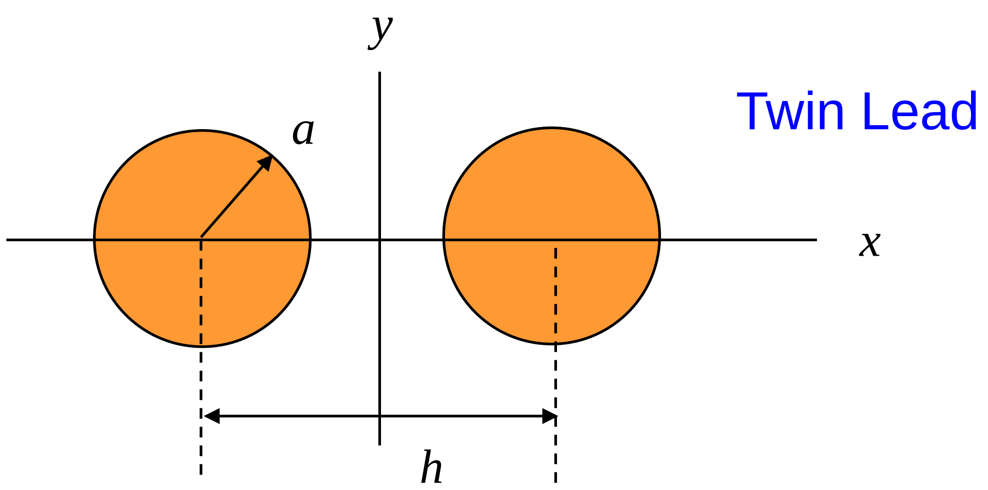
This empirical model combines the low-frequency (DC) and the high-frequency (HF) skin-effect results together into one result by using an approximate circuit model to get $R(\omega)$ and $\Delta L(\omega)$.

F. M. Tesche, "A Simple model for the line parameters of a lossy coaxial cable filled with a nondispersive dielectric," IEEE Trans. EMC, vol. 49, no. 1, pp. 12-17, Feb. 2007.

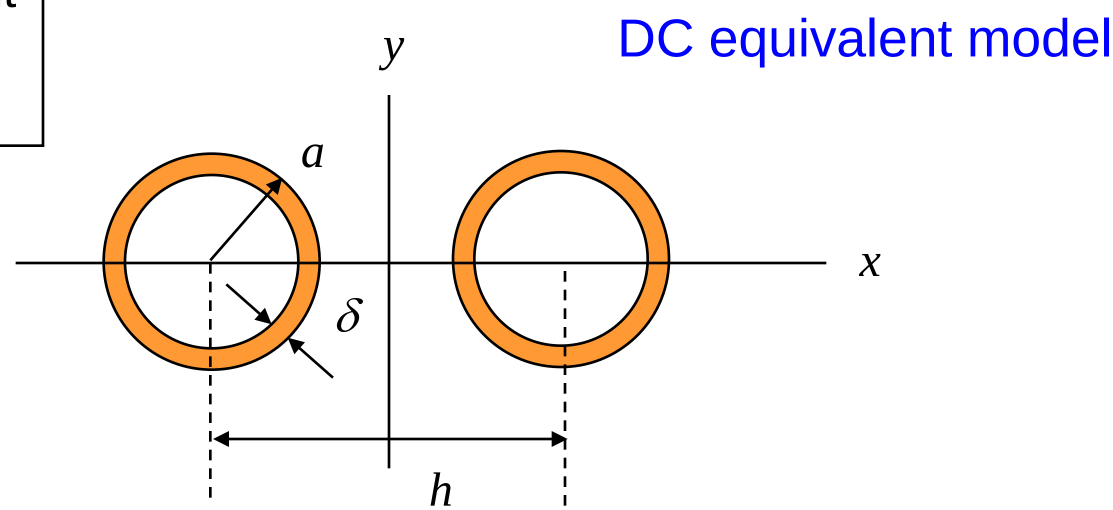
Note: The method was applied in the above reference for a coaxial cable, but it should work for any type of transmission line.

(Please see the Appendix for a discussion of the Tesche model.)

Twin Lead

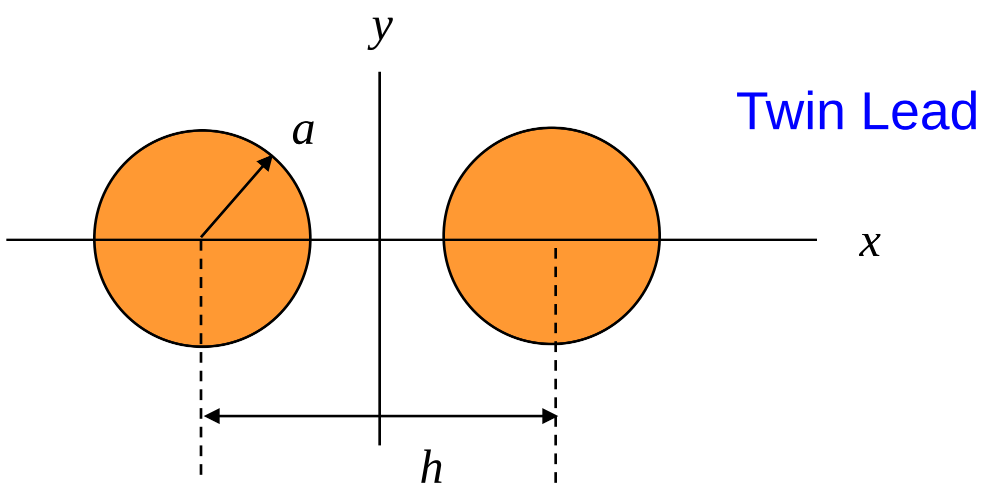


Assume uniform current density on each conductor ($h >> a$).



$$R \approx \frac{1}{2\pi a \sigma \delta} + \frac{1}{2\pi a \sigma \delta}$$

Twin Lead



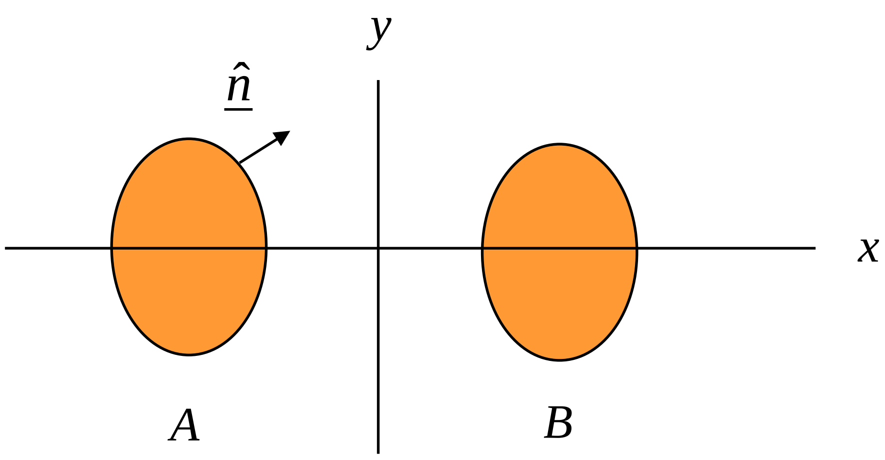
$$R \approx \frac{1}{2\pi a \sigma \delta} + \frac{1}{2\pi a \sigma \delta} = \frac{1}{\pi a \sigma \delta}$$

or

$$R \approx \frac{R_s}{\pi a}$$

(A more accurate formula will come later.)

Wheeler Incremental Inductance Rule



$$R = R_s \left[\frac{1}{|I|^2} \oint_C |J_{sz}(l)|^2 \, dl \right]$$

Wheeler showed that R could be expressed in a way that is easy to calculate (provided we have a formula for L_0):

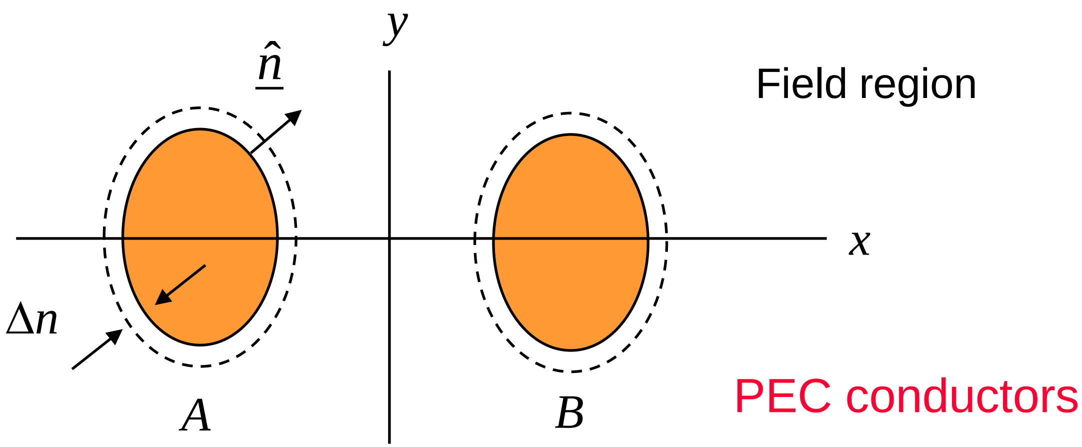
$$R = R_s \left[-\frac{1}{\mu_0} \frac{\partial L_0}{\partial n} \right]$$

L_0 is the external inductance (calculated assuming PEC conductors) and ∂n is an increase in the dimension of the conductors (expanded into the active field region).

H. Wheeler, "Formulas for the skin-effect," Proc. IRE, vol. 30, pp. 412-424, 1942.

Wheeler Incremental Inductance Rule (cont.)

The boundaries are expanded a small amount Δn into the field region.



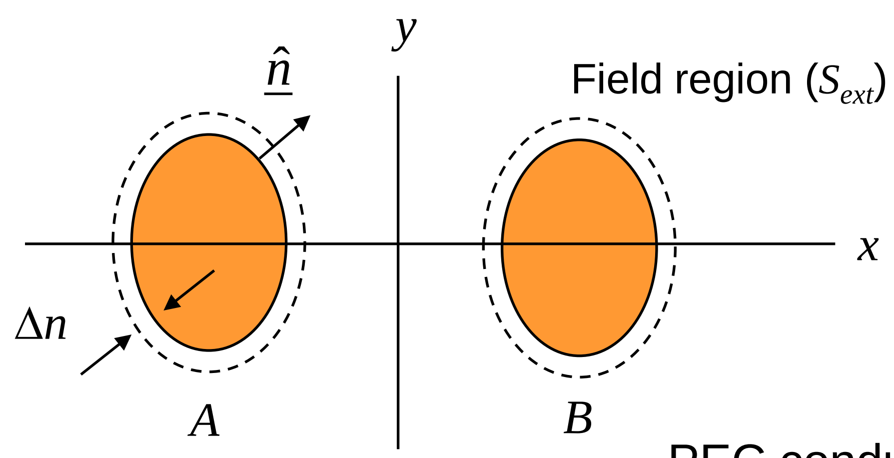
L_0 = external inductance (assuming perfect conductors).

$$R = R_s \left[-\frac{1}{\mu_0} \frac{\partial L_0}{\partial n} \right]$$

Wheeler Incremental Inductance Rule (cont.)

Derivation of Wheeler Incremental Inductance rule

$$R = R_s \left[\frac{1}{|I|^2} \oint_C |J_{sz}(l)|^2 \, dl \right]$$



PEC conductors

$$W_H = \frac{1}{4} L_0 |I|^2$$

$$W_H = \frac{1}{4} \mu_0 \int_{S_{ext}} |\underline{H}|^2 \, dS$$

$$L_0 = \left[\frac{\mu_0}{|I|^2} \int_{S_{ext}} |\underline{H}|^2 \, dS \right]$$

Hence

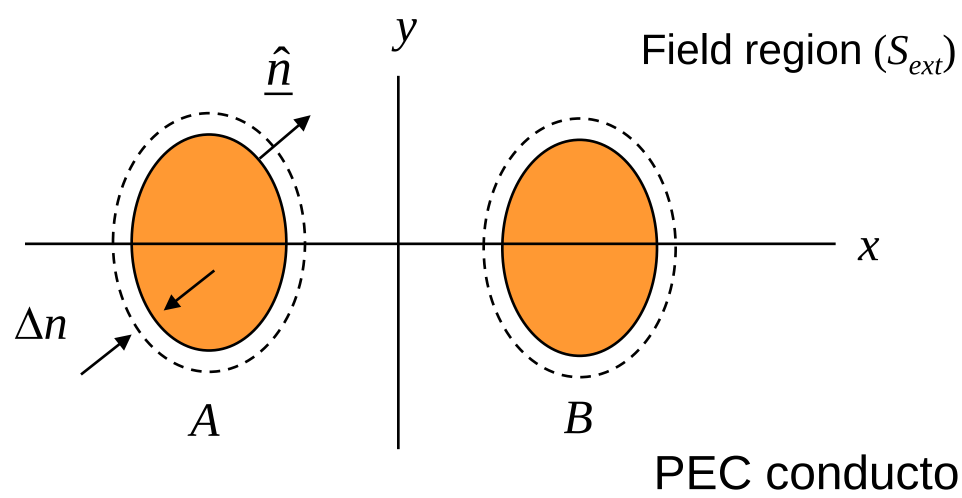
$$\Delta L_0 = -(\Delta n) \frac{\mu_0}{|I|^2} \oint_C |\underline{H}|^2 \, dl$$

We then have

$$\frac{\Delta L_0}{\Delta n} = -\frac{\mu_0}{|I|^2} \oint_C |\underline{H}|^2 \, dl = -\frac{\mu_0}{|I|^2} \oint_C |J_{sz}(l)|^2 \, dl$$

Wheeler Incremental Inductance Rule (cont.)

$$R = R_s\left[\frac{1}{|I|^2}\oint_C |J_{sz}(l)|^2\, dl\right]$$



From the last slide,

$$\frac{\partial L_0}{\partial n} = -\frac{\mu_0}{|I|^2}\oint_C |J_{sz}(l)|^2\, dl \qquad \Rightarrow \frac{1}{|I|^2}\oint_C |J_{sz}(l)|^2\, dl = -\frac{1}{\mu_0}\frac{\partial L_0}{\partial n}$$

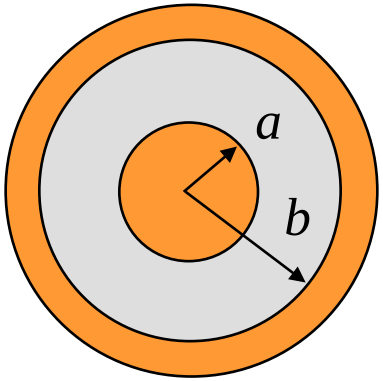
Hence

$$R = R_s\left[-\frac{1}{\mu_0}\frac{\partial L_0}{\partial n}\right]$$

Wheeler Incremental Inductance Rule (cont.)

Example 1: Coax

$$L_0 = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$$



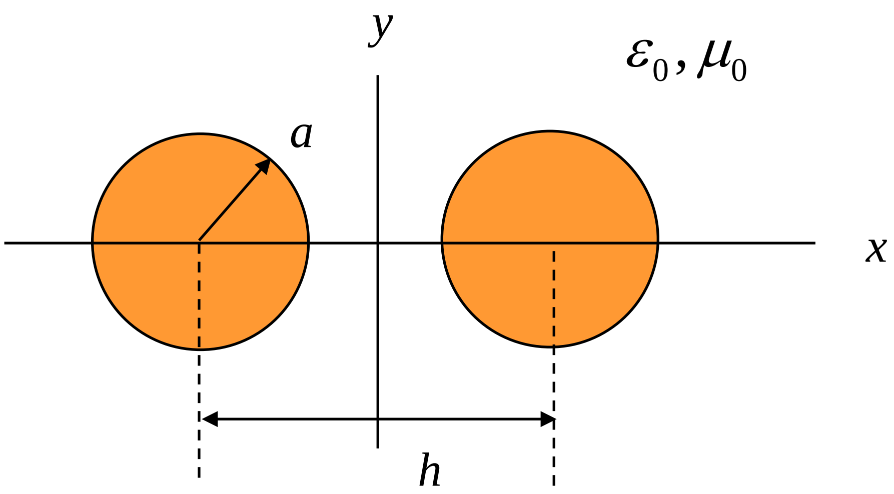
$$\frac{\partial L_0}{\partial n} = \frac{\partial L_0}{\partial a} + (-1)\frac{\partial L_0}{\partial b} = \frac{\mu_0}{2\pi}\left(\frac{b}{a}\right)^{-1}(b)\left(\frac{-1}{a^2}\right) - \frac{\mu_0}{2\pi}\left(\frac{b}{a}\right)^{-1}\left(\frac{1}{a}\right)$$

$$= -\frac{\mu_0}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$R = R_s\left[-\frac{1}{\mu_0}\frac{\partial L_0}{\partial n}\right] \quad \Rightarrow \quad R = R_s\left(\frac{1}{2\pi a} + \frac{1}{2\pi b}\right)$$

Wheeler Incremental Inductance Rule (cont.)

Example 2: Twin Lead



From image theory (or conformal mapping):

$$C = \pi\varepsilon_0 \frac{1}{\cosh^{-1}\left(\dfrac{h}{2a}\right)}$$

$$L_0 C = \mu\varepsilon$$

$$L_0 = \frac{\mu_0}{\pi}\cosh^{-1}\left(\frac{h}{2a}\right)$$

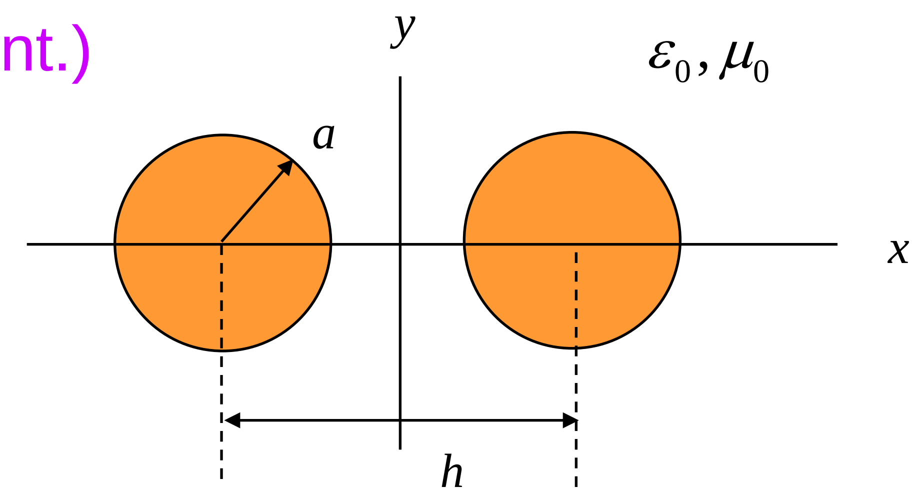
$$Z_0 = \frac{\eta_0}{\pi}\cosh^{-1}\left(\frac{h}{2a}\right)$$

$$Z_0 \approx \frac{\eta_0}{\pi}\ln\left(\frac{h}{a}\right), \quad a << h$$

Wheeler Incremental Inductance Rule (cont.)

Example 2: Twin Lead (cont.)

$$L_0 = \frac{\mu_0}{\pi} \cosh^{-1}\left(\frac{h}{2a}\right)$$



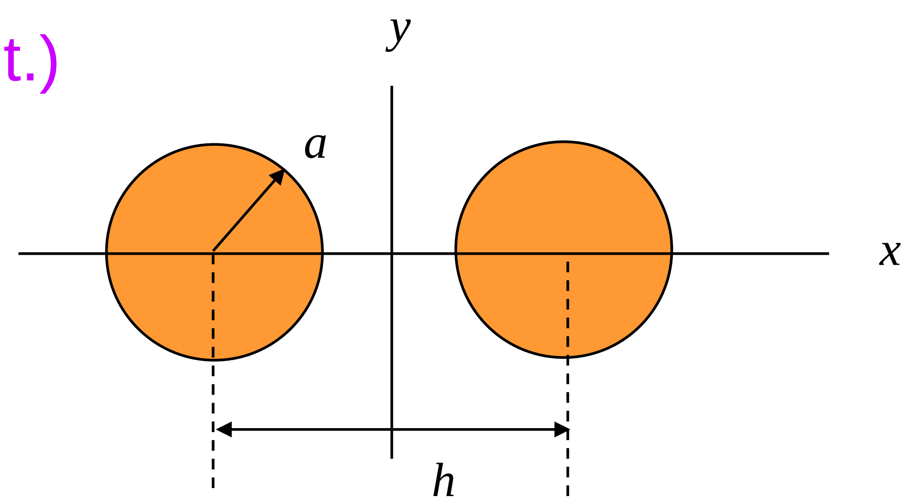
Note: By incrementing a, we increment <u>both</u> conductors simultaneously.

$$\frac{\partial L_0}{\partial n} = \frac{\partial L_0}{\partial a} = \frac{\mu_0}{\pi}\frac{\partial}{\partial a}\cosh^{-1}\left(\frac{h}{2a}\right) = \frac{\mu_0}{\pi}\left(\frac{1}{\sqrt{\left(\frac{h}{2a}\right)^2 - 1}}\right)\left(\frac{h}{2}\right)\left(\frac{-1}{a^2}\right) = -\frac{\mu_0}{\pi a}\frac{\left(\frac{h}{2a}\right)}{\sqrt{\left(\frac{h}{2a}\right)^2 - 1}}$$

$$R = R_s\left[-\frac{1}{\mu_0}\frac{\partial L_0}{\partial n}\right] \quad \Rightarrow \quad R = R_s\left[\frac{1}{\pi a}\frac{\left(\frac{h}{2a}\right)}{\sqrt{\left(\frac{h}{2a}\right)^2 - 1}}\right]$$

Wheeler Incremental Inductance Rule (cont.)

Example 2: Twin Lead (cont.)



Summary

$$Z_0 = \frac{\eta_0}{\pi}\cosh^{-1}\left(\frac{h}{2a}\right)$$

$$R = R_s\left[\frac{1}{\pi a}\frac{\left(\frac{h}{2a}\right)}{\sqrt{\left(\frac{h}{2a}\right)^2 - 1}}\right]$$

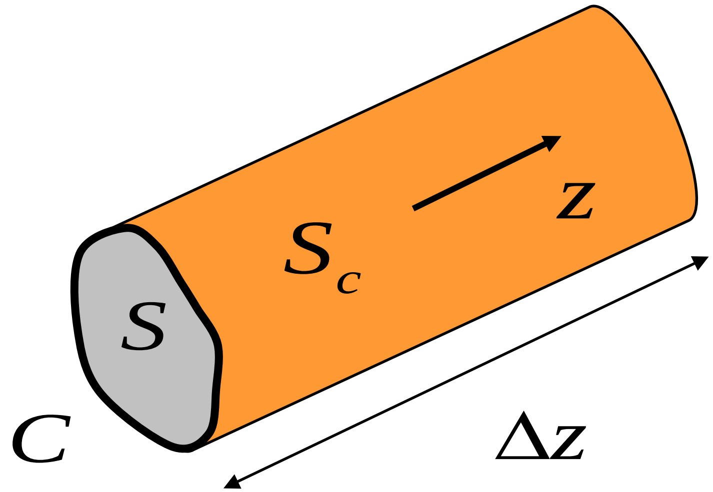
$$C = \pi\varepsilon_0 \frac{1}{\cosh^{-1}\left(\frac{h}{2a}\right)}$$

$$L_0 = \frac{\mu_0}{\pi}\cosh^{-1}\left(\frac{h}{2a}\right)$$

$$G = (\omega C)\tan\delta$$

Attenuation in Waveguide

We consider here conductor loss for a waveguide mode.



A waveguide mode is traveling in the positive z direction.

$$\alpha_c = \frac{\left\langle \mathscr{P}_d^l \right\rangle}{2\left\langle \mathscr{P}_f \right\rangle}$$

$$\left\langle \mathscr{P}_d^l \right\rangle = \frac{1}{\Delta z} \int_{S_c} \frac{1}{2} R_s \left| \underline{J}_s \right|^2 dS$$

$$= \oint_C \frac{1}{2} R_s \left| \underline{J}_s(\ell) \right|^2 dl$$

Attenuation in Waveguide (cont.)

or

$$\left\langle \mathscr{P}_d^l \right\rangle = \oint_C \frac{1}{2} R_s \left| \underline{\hat{n}} \times \underline{H} \right|^2 dl$$

Power flow:

$$\left\langle \mathscr{P}_f \right\rangle = \mathrm{Re} \int_S \frac{1}{2} (\underline{E}_t \times \underline{H}_t^*) \cdot \underline{\hat{z}} \, dS$$

Next, use

$$\underline{E}_t = -Z_0^{WG} (\underline{\hat{z}} \times \underline{H}_t) \qquad \left(Z_0^{WG} = Z^{TE} \;\; \text{or} \;\; Z^{TM} \right)$$

Hence

$$\left\langle \mathscr{P}_f \right\rangle = \int_S \mathrm{Re} \left\{ \frac{-1}{2} Z_0^{WG} (\underline{\hat{z}} \times \underline{H}_t) \times \underline{H}_t^* \right\} \cdot \underline{\hat{z}} \, dS$$

Attenuation in Waveguide (cont.)

Vector identity: $\underline{A}\times\left(\underline{B}\times\underline{C}\right)=\underline{B}\left(\underline{A}\cdot\underline{C}\right)-\underline{C}\left(\underline{A}\cdot\underline{B}\right)$

$$\left[(\hat{\underline{z}}\times\underline{H}_t)\times\underline{H}_t^*\right]\cdot\hat{\underline{z}}=-\left[\underline{H}_t^*\times(\hat{\underline{z}}\times\underline{H}_t)\right]\cdot\hat{\underline{z}}=\left[-\hat{\underline{z}}(\underline{H}_t\cdot\underline{H}_t^*)+\underline{H}_t(\hat{\underline{z}}\cdot\underline{H}_t^*)\right]\cdot\hat{\underline{z}}$$

$$=-\left|\underline{H}_t\right|^2$$

Hence

$$\left\langle \mathscr{P}_f \right\rangle=\int_S \mathrm{Re}\left\{\frac{1}{2}Z_0^{WG}\left|\underline{H}_t\right|^2\right\}dS$$

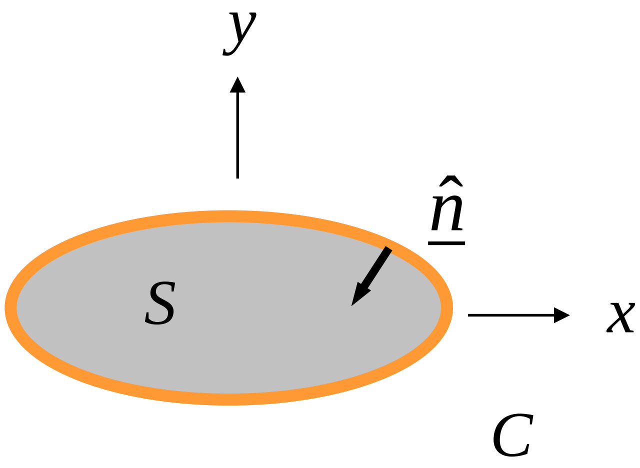
Assume Z_0^{WG} = real ($f > f_c$ and no dielectric loss)

$$\left\langle \mathscr{P}_f \right\rangle=\frac{1}{2}Z_0^{WG}\int_S\left|\underline{H}_t\right|^2 dS$$

Attenuation in Waveguide (cont.)

Then we have

$$\alpha_c = \left(\frac{R_s}{2Z_0^{WG}}\right)\left[\frac{\oint\limits_C \left|\underline{\hat{n}} \times \underline{H}\right|^2 dl}{\int\limits_S \left|\underline{H}_t\right|^2 dS}\right]$$



Attenuation in Waveguide (cont.)

Total Attenuation:

$$\alpha = \alpha_c + \alpha_d$$

Calculate α_d (assume PEC wall):

$$k_z = \beta - j\alpha = \sqrt{k^2 - k_c^2}$$

so

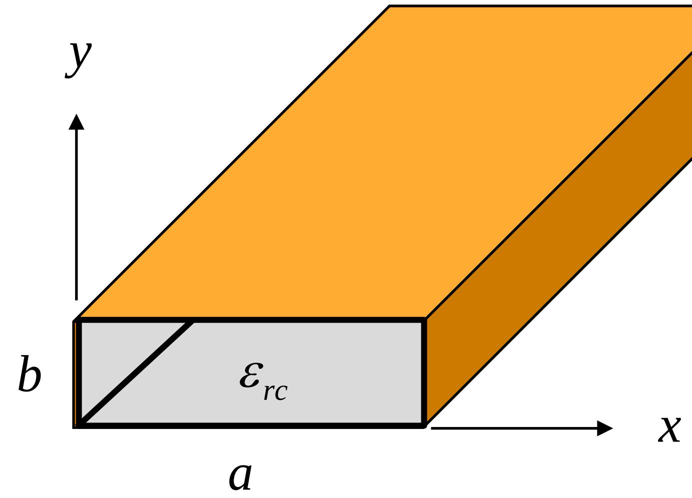
$$\alpha_d = -\operatorname{Im}\sqrt{k^2 - k_c^2}$$

where

$$k = \omega\sqrt{\mu\varepsilon_c} = k_0\sqrt{\mu_r\varepsilon_r'\left(1 - j\tan\delta\right)}$$

Attenuation in Waveguide (cont.)

TE_{10} Mode



$$\mu_r = 1$$

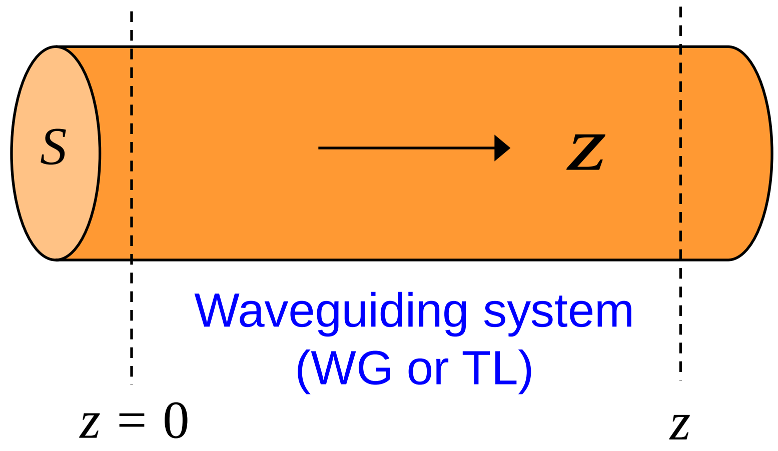
$$k = k_0 \sqrt{\varepsilon_{rc}}$$

$$\eta = \eta_0 \sqrt{\frac{1}{\varepsilon_{rc}}}$$

$$\alpha_c = \frac{R_s}{b\left(\mathrm{Re}\,\eta\right)\sqrt{1-\left(f_c / f\right)^2}}\left[1 + 2\frac{b}{a}\left(\frac{f_c}{f}\right)^2\right]$$

$$\alpha_d = -\mathrm{Im}\sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}$$

Attenuation in dB



$$V(z) = V(0)\, e^{-\alpha z} e^{-j\beta z}$$

$$\text{dB} = 20\log_{10}\left|\frac{V(z)}{V(0)}\right| = 20\log_{10}(e^{-\alpha z})$$

Use $\log_{10} x = \dfrac{\ln x}{\ln 10}$

Attenuation in dB (cont.)

so

$$\text{dB} = 20\frac{\ln(e^{-\alpha z})}{\ln 10}$$

$$= 20\frac{(-\alpha z)}{\ln 10}$$

Hence

$$\text{Attenuation} = \left(\frac{20}{\ln 10}\right)\alpha \qquad [\text{dB/m}]$$

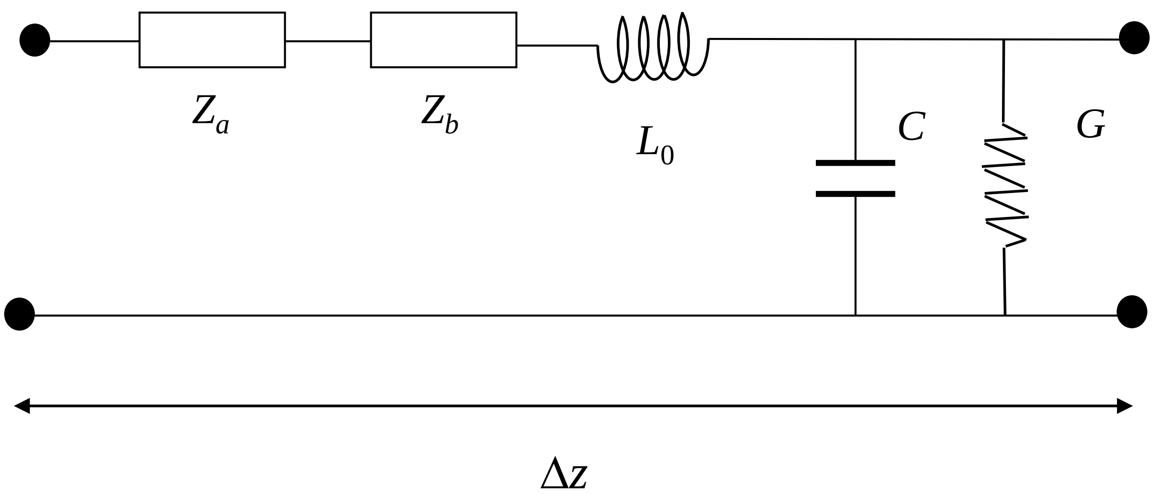
Attenuation in dB (cont.)

or

$$\text{Attenuation} = (8.6859)\alpha \qquad [\text{dB/m}]$$

Appendix: Tesche Model

The series elements Z_a and Z_b (defined on the next slide) account for the finite conductivity, and give us an accurate R and ΔL for each conductor at any frequency.



$$Z = Z_a + Z_b + j\omega L_0$$

$$Y = G + j\omega C$$

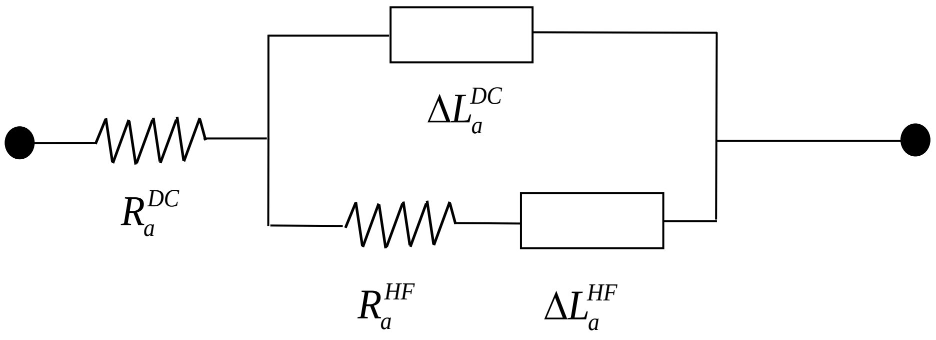
$$C = \frac{2\pi\varepsilon_0\varepsilon'_{rc}}{\ln\left(\frac{b}{a}\right)}$$

$$\frac{G}{\omega C} = \tan\delta = \frac{\varepsilon''_c}{\varepsilon'_c}$$

$$L_0 = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$$

Appendix: Tesche Model (cont.)

Z_a

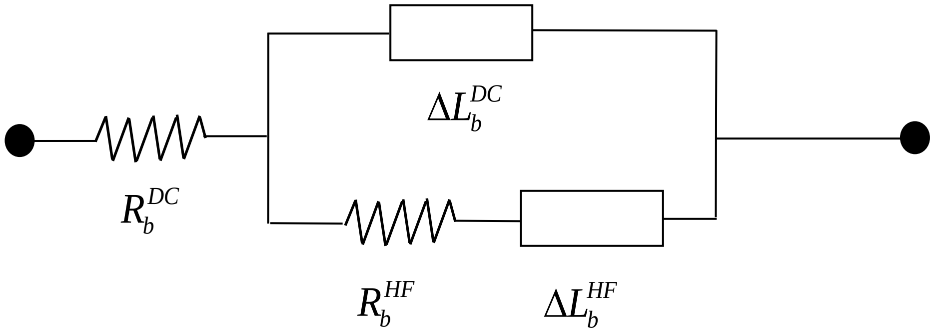


Inner conductor of coax

The impedance of this circuit is denoted as

$$Z_a = R_a + j\omega\left(\Delta L_a\right)$$

Z_b



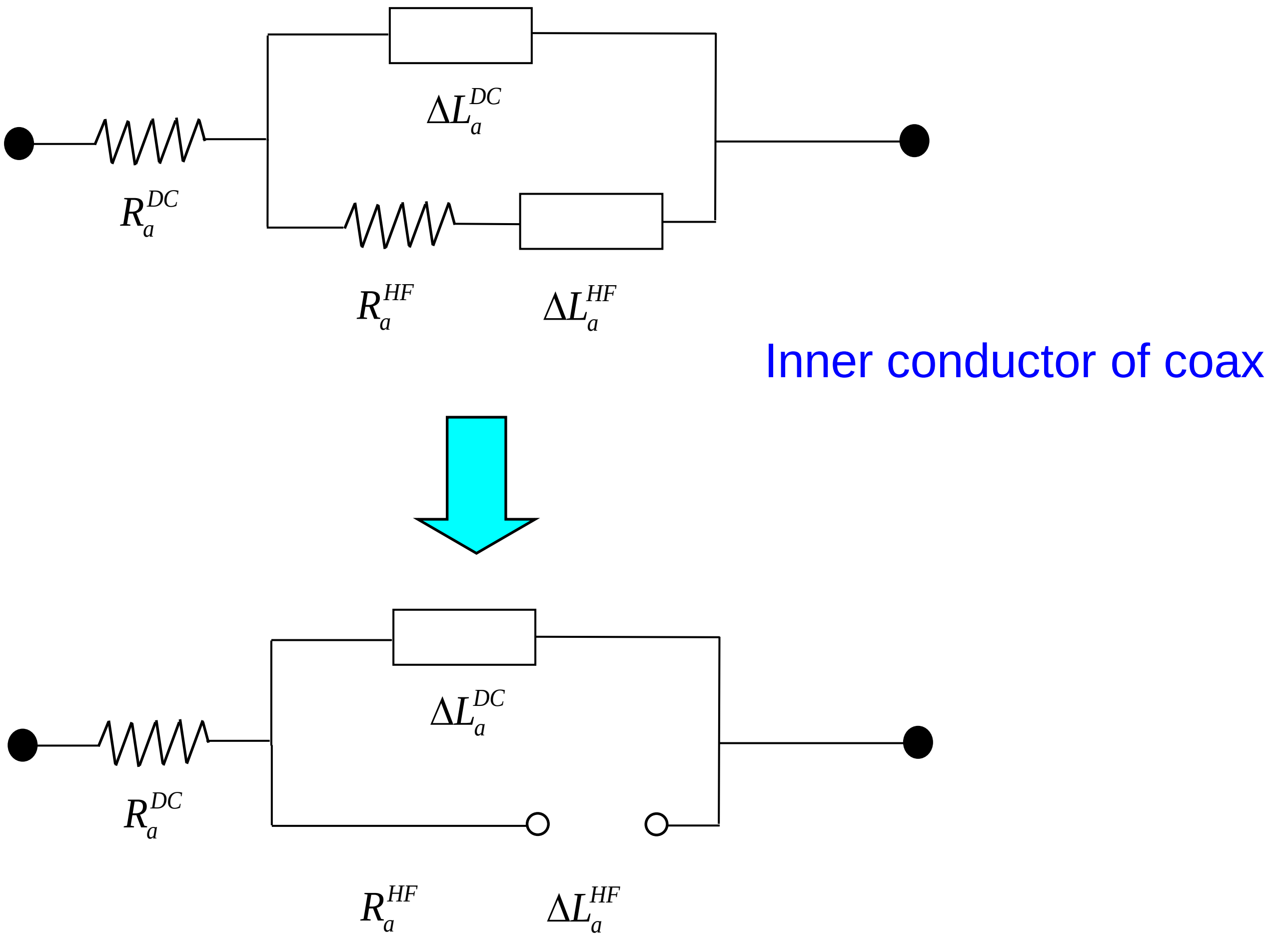
Outer conductor of coax

The impedance of this circuit is denoted as

$$Z_b = R_b + j\omega\left(\Delta L_b\right)$$

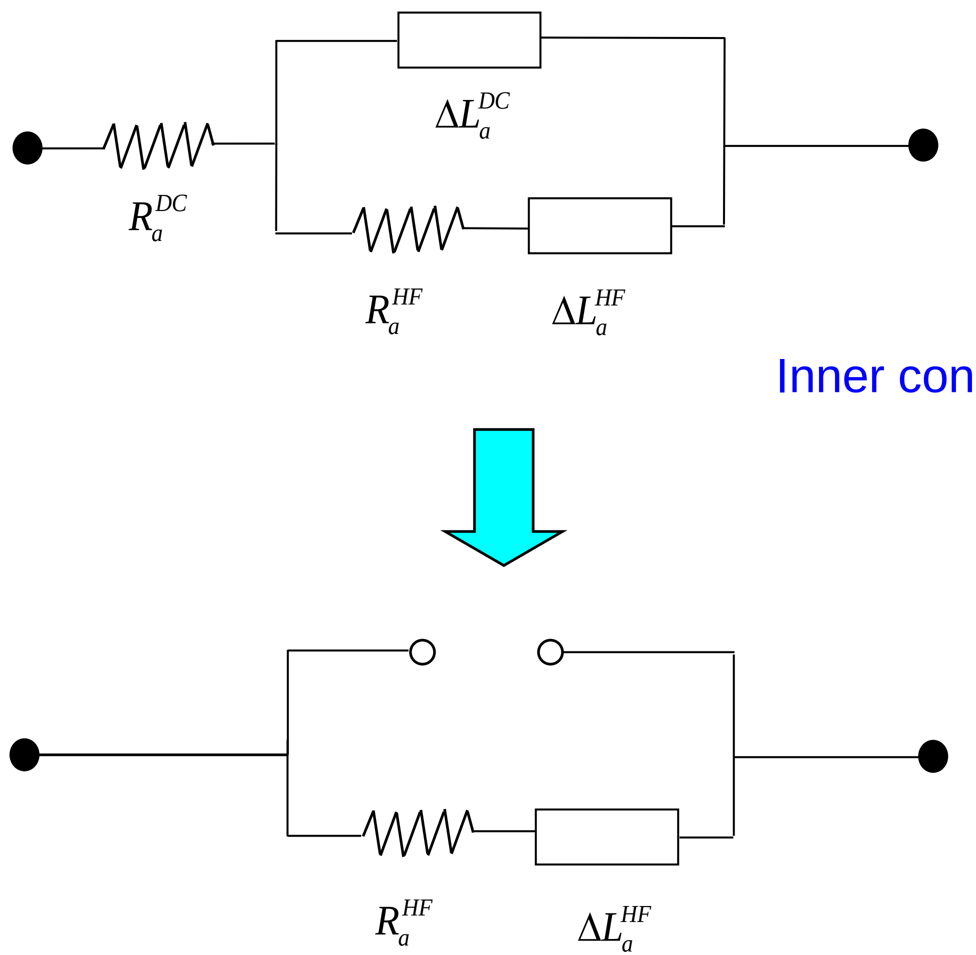
Appendix: Tesche Model (cont.)

- At <u>low frequency</u> the HF resistance gets small and the HF inductance gets large.



Appendix: Tesche Model (cont.)

- At <u>high frequency</u> the DC inductance gets very large compared to the HF inductance, and the DC resistance is small compared with the HF resistance.



Inner conductor of coax

Appendix: Tesche Model (cont.)

The formulas are summarized as follows:

$$R_a^{DC} = \frac{1}{\sigma\left(\pi a^2\right)}$$

$$R_b^{DC} = \frac{1}{\sigma\left(2\pi bt\right)}$$

$$\Delta L_a^{DC} = \frac{\mu_0}{8\pi}$$

$$\Delta L_b^{DC} = \frac{\mu_0}{2\pi}\left[\frac{c^4 \ln\left(\frac{c}{b}\right)}{\left(c^2 - b^2\right)^2} + \frac{b^2 - 3c^2}{4\left(c^2 - b^2\right)}\right]$$

$$R_a^{HF} = \omega\left(\Delta L_a^{HF}\right) = \frac{1}{2\pi a\sigma\delta}$$

$$R_b^{HF} = \omega\left(\Delta L_b^{HF}\right) = \frac{1}{2\pi b\sigma\delta}$$