ECE 6340
Intermediate EM Waves

Fall 2016

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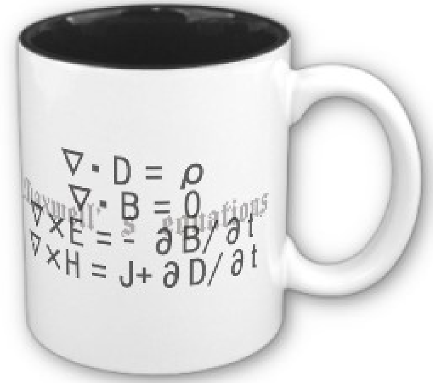

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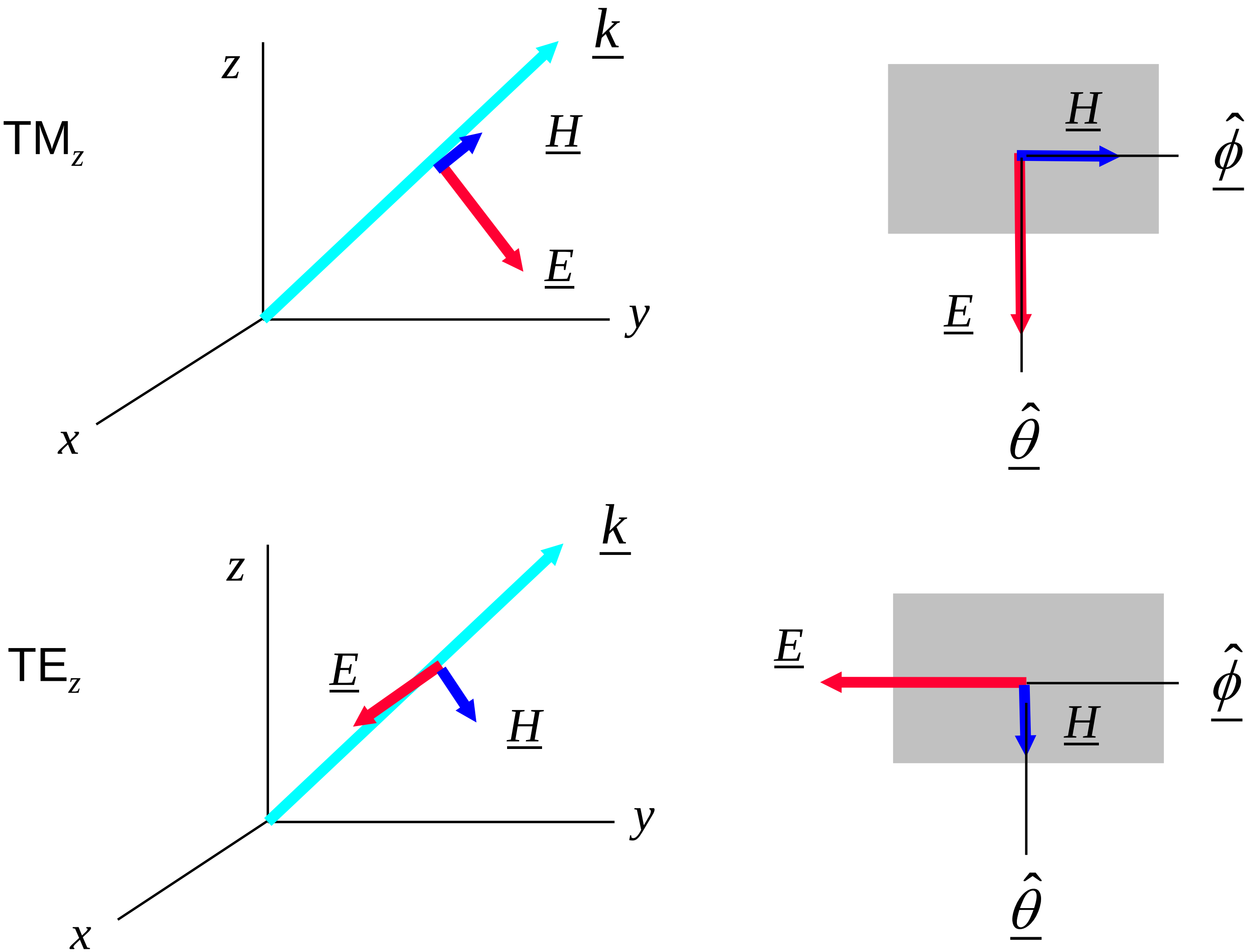
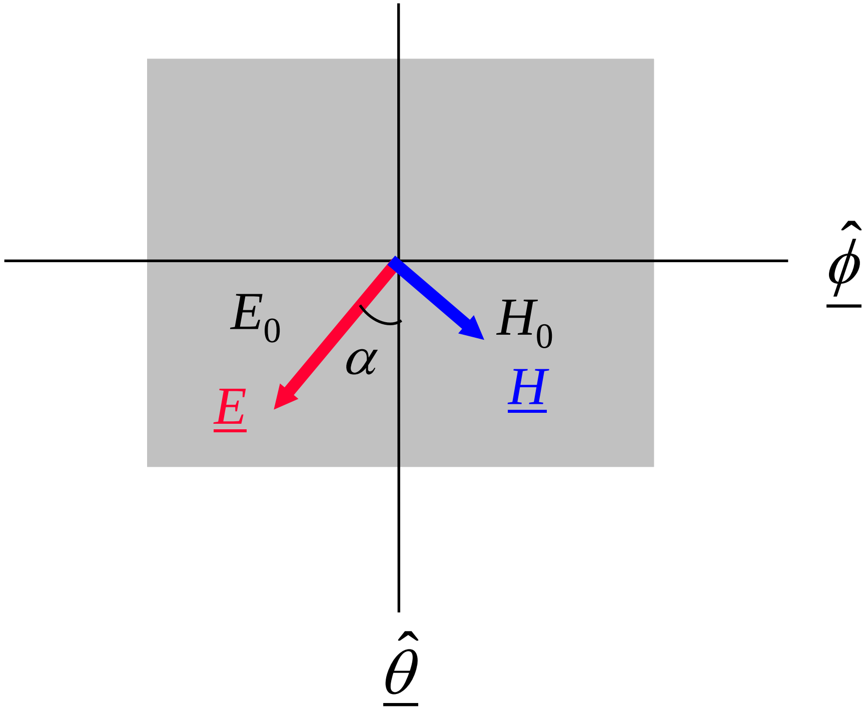
TE_z-TM_z Plane Waves



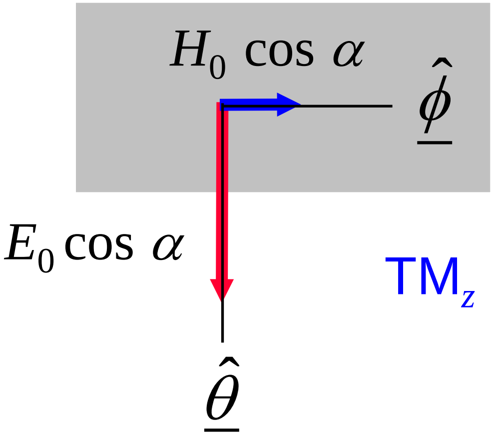
A homogeneous plane wave is shown for simplicity (but the principle is general).

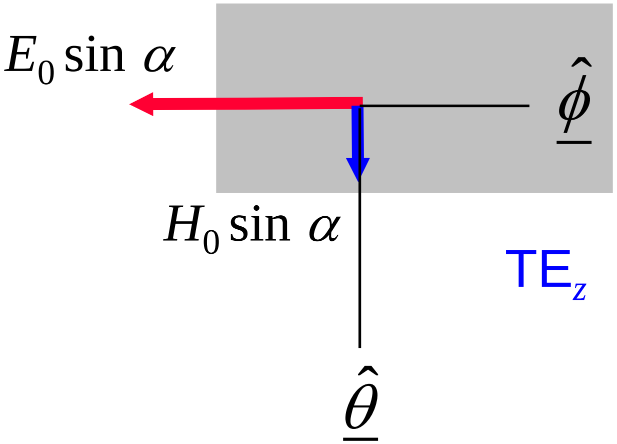
Arbitrary Polarization: Decomposition

Assume that $\underline{E}$ and $\underline{H}$ are real vectors in the figure for simplicity.



TM$_z$ and TE$_z$ parts:





TM$_z$ Plane Wave

$$\nabla \times \underline{H} = j\omega\varepsilon_c \underline{E}$$

or

$$-j\underline{k} \times \underline{H} = j\omega\varepsilon_c \underline{E}$$

or

$$-\underline{k} \times \underline{H} = \omega\varepsilon_c \underline{E}$$

Take x component:

$$\omega\varepsilon_c E_x = \underline{\hat{x}} \cdot \left[-(\underline{\hat{x}} k_x + \underline{\hat{y}} k_y + \underline{\hat{z}} k_z) \times (\underline{\hat{x}} H_x + \underline{\hat{y}} H_y) \right]$$

$$= k_z H_y$$

so

$$\frac{E_x}{H_y} = \frac{k_z}{\omega\varepsilon_c}$$

Assume that the wave is propagating upward (+z direction) so that k_z is positive (or has a positive real part for a lossy medium).

TM$_z$ Plane Wave (cont.)

Take y component:

$$\omega\,\varepsilon_c\,E_y = \underline{\hat{y}}\cdot\left[-(\underline{\hat{x}}\,k_x + \underline{\hat{y}}\,k_y + \underline{\hat{z}}\,k_z)\times(\underline{\hat{x}}\,H_x + \underline{\hat{y}}\,H_y)\right]$$

$$= -\,k_z\,H_x$$

$$\frac{E_y}{-H_x} = \frac{k_z}{\omega\varepsilon_c}$$

Define:

$$\boxed{Z^{TM} = \frac{k_z}{\omega\,\varepsilon_c}}$$

TM_z Plane Wave (cont.)

Both results are summarized in a vector equation:

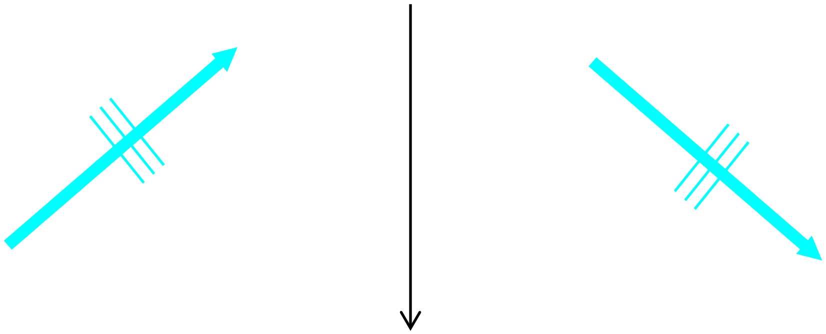
$$\underline{E}_t = -Z^{TM}\left(\underline{\hat{z}} \times \underline{H}_t\right)$$

or

$$\underline{H}_t = \frac{1}{Z^{TM}}\left(\underline{\hat{z}} \times \underline{E}_t\right)$$

Note:
“ *t* ” stands for “transverse,” meaning the x and y components.

Consider replacing $k_z \rightarrow -k_z$



$$Z^{TM} \rightarrow -Z^{TM}$$

Recall: $Z^{TM} = \dfrac{k_z}{\omega \varepsilon_c}$

TM$_z$ Plane Wave (cont.)

so

$$\underline{H}_t = -\frac{1}{Z^{TM}}\left(\underline{\hat{z}} \times \underline{E}_t\right)$$

(Here Z^{TM} is the same as before, defined for a $+z$ wave.)

Summary for both cases:

$$\underline{H}_t = \pm\frac{1}{Z^{TM}}\left(\underline{\hat{z}} \times \underline{E}_t\right)$$

$$\underline{E}_t = \mp Z^{TM}\left(\underline{\hat{z}} \times \underline{H}_t\right)$$

TE$_z$ Plane Wave

$$\nabla \times \underline{E} = -j\omega\mu\underline{H}$$

or

$$-j\,\underline{k} \times \underline{E} = -j\omega\mu\underline{H}$$

or

$$\underline{k} \times \underline{E} = \omega\mu\underline{H}$$

Take *x* component:

$$\omega\,\mu\,H_x = \underline{\hat{x}} \cdot \left[(\underline{\hat{x}}\,k_x + \underline{\hat{y}}\,k_y + \underline{\hat{z}}\,k_z) \times (\underline{\hat{x}}\,E_x + \underline{\hat{y}}\,E_y) \right]$$

$$= -k_z\,E_y$$

so

$$\frac{E_y}{-H_x} = \frac{\omega\mu}{k_z}$$

TE$_z$ Plane Wave (cont.)

Take *y* component:

$$\omega\mu H_y = \underline{\hat{y}} \cdot \left[\left(\underline{\hat{x}} k_x + \underline{\hat{y}} k_y + \underline{\hat{z}} k_z \right) \times \left(\underline{\hat{x}} E_x + \underline{\hat{y}} E_y \right) \right]$$

$$= k_z E_x$$

so

$$\frac{E_x}{H_y} = \frac{\omega\mu}{k_z} = \eta \frac{k}{k_z}$$

Define:

$$Z^{TE} = \frac{\omega\mu}{k_z}$$

TE$_z$ Plane Wave (cont.)

Then

$$\underline{E}_t = -Z^{TE}\left(\underline{\hat{z}} \times \underline{H}_t\right)$$

$$\underline{H}_t = \frac{1}{Z^{TE}}\left(\underline{\hat{z}} \times \underline{E}_t\right)$$

Allowing for both directions, $+z$ and $-z$, we have:

$$\underline{H}_t = \pm \frac{1}{Z^{TE}}\left(\underline{\hat{z}} \times \underline{E}_t\right)$$

$$\underline{E}_t = \mp Z^{TE}\left(\underline{\hat{z}} \times \underline{H}_t\right)$$

Transverse Equivalent Network

Assume a TM_z plane wave going upward
(the z propagation is in the $+z$ direction).

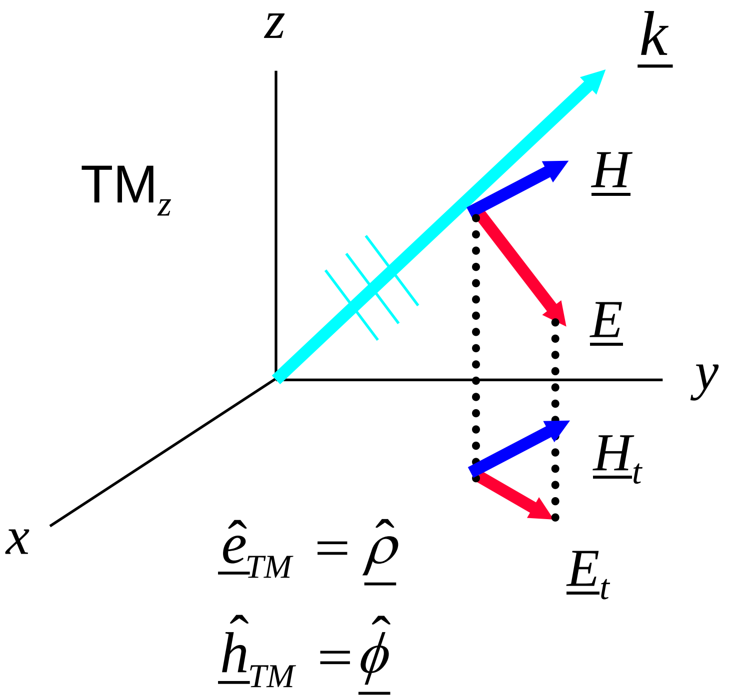
Denote

$$\underline{E}_t(x,y,z) = \underline{\hat{e}}_{TM}\, V_{TM}(z)\, \psi_t(x,y)$$

$$\underline{H}_t(x,y,z) = \underline{\hat{h}}_{TM}\, I_{TM}(z)\, \psi_t(x,y)$$

where

$$\psi_t(x,y) = e^{-j(k_x x + k_y y)}$$



$$\underline{\hat{e}}_{TM} = \underline{\hat{\rho}}$$

$$\underline{\hat{h}}_{TM} = \underline{\hat{\phi}}$$

As we will see,

$V_{TM}(z)$ and $I_{TM}(z)$ behave as voltage and current on a TL

TEN (cont.)

Assume a +z wave:

$$V_{TM}(z) = Ae^{-jk_z z}$$

$$I_{TM}(z) = Be^{-jk_z z}$$

The form is the same as the waves on a TL.

Also $$\underline{H}_t = \frac{1}{Z^{TM}}(\underline{\hat{z}} \times \underline{E}_t)$$

Therefore

$$\psi_t(x,y)\, I_{TM}(z)\, \underline{\hat{h}}_{TM} = \frac{1}{Z^{TM}} \psi_t(x,y)\, V_{TM}(z)\, (\underline{\hat{z}} \times \underline{\hat{e}}_{TM})$$

We choose: $$\underline{\hat{h}}_{TM} = \underline{\hat{z}} \times \underline{\hat{e}}_{TM}$$

TEN (cont.)

We then have

$$I_{TM}(z) = \frac{1}{Z_{TM}} V_{TM}(z)$$

If we assume a -z (downward) wave:

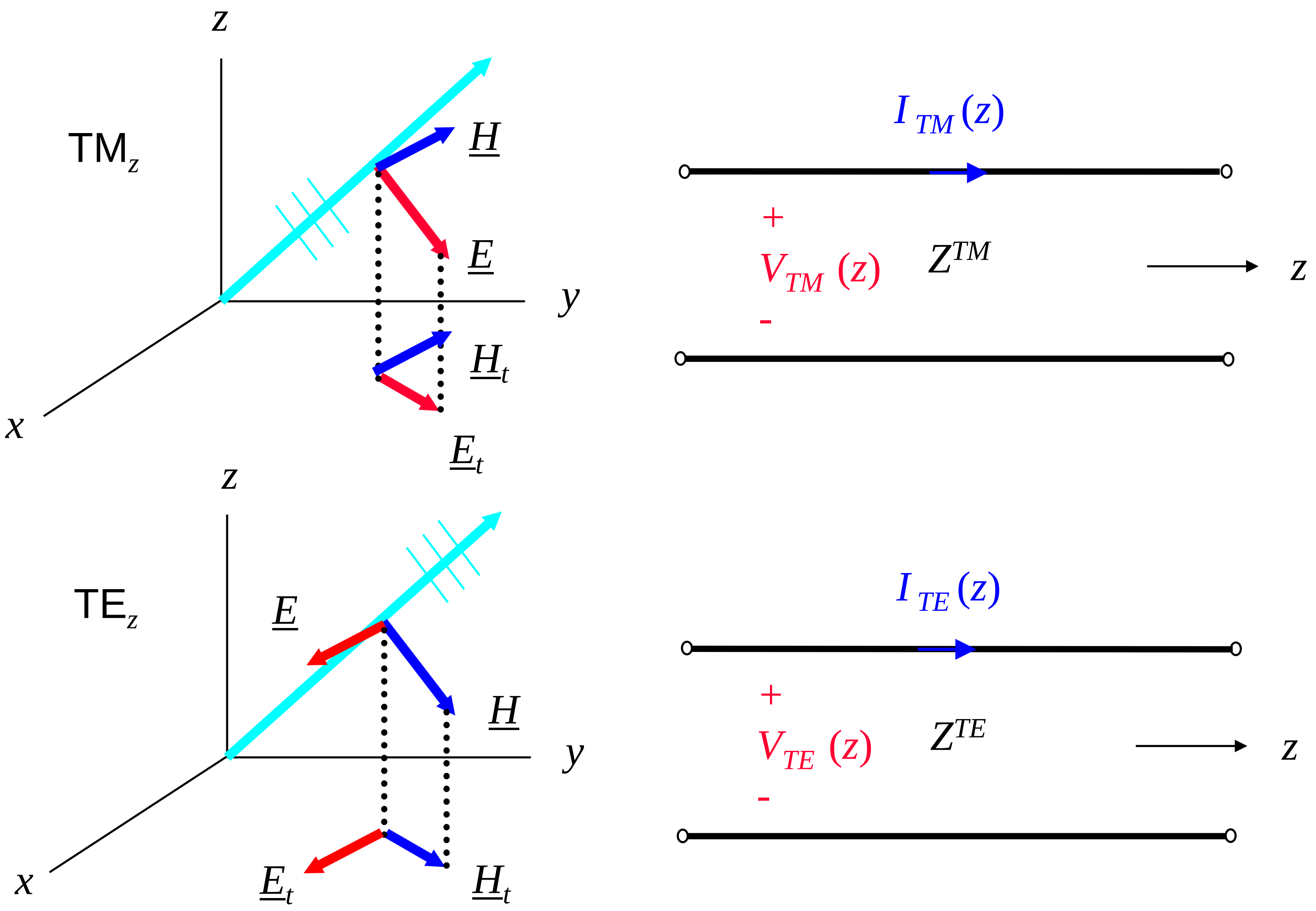
$$I_{TM}(z) = -\frac{1}{Z_{TM}} V_{TM}(z)$$

Hence, in summary:

$$I_{TM}(z) = \pm \frac{1}{Z_{TM}} V_{TM}(z)$$

This proves that the transverse fields behave as voltage and current on a TL.

TEN (cont.)


TM$_z$
z
y
x
$\underline{H}$
$\underline{E}$
$\underline{H}_t$
$\underline{E}_t$
$I_{TM}(z)$
+
$V_{TM}(z)$
-
Z^{TM}
z
TE$_z$
z
y
x
$\underline{E}$
$\underline{H}$
$\underline{E}_t$
$\underline{H}_t$
$I_{TE}(z)$
+
$V_{TE}(z)$
-
Z^{TE}
z

TEN (cont.)

Note: $V(z)$ and $I(z)$ model only the transverse fields, but we can obtain the z component of the fields from these.

Example: Find E_z for a TM$_z$ plane wave:

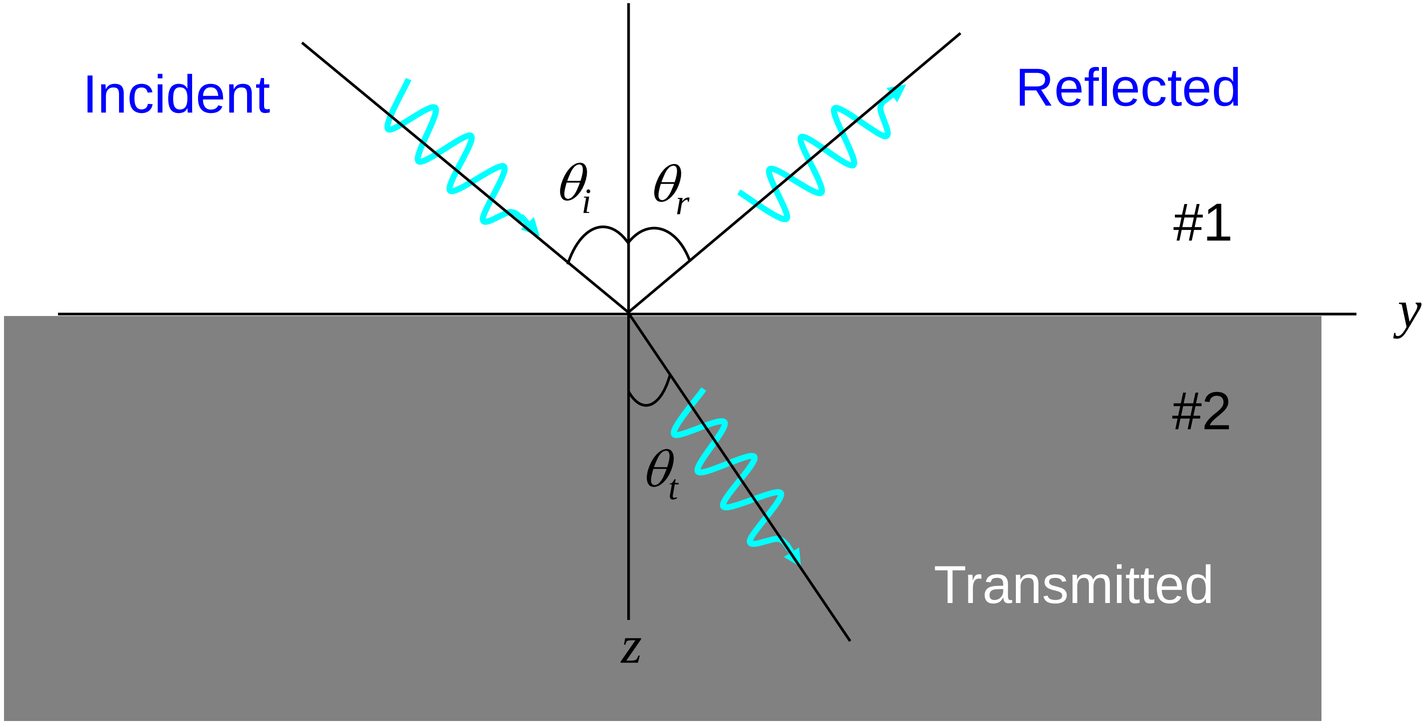
$$\nabla \times \underline{H} = j\omega\varepsilon_c \, \underline{E}$$

$$E_z = \frac{1}{j\omega\varepsilon_c} \underline{\hat{z}} \cdot (\nabla \times \underline{H})$$

$$E_z = \frac{1}{j\omega\varepsilon_c}\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]$$

$$= \frac{1}{j\omega\varepsilon_c}\left[(-jk_x)H_y + (jk_y)H_x\right]$$

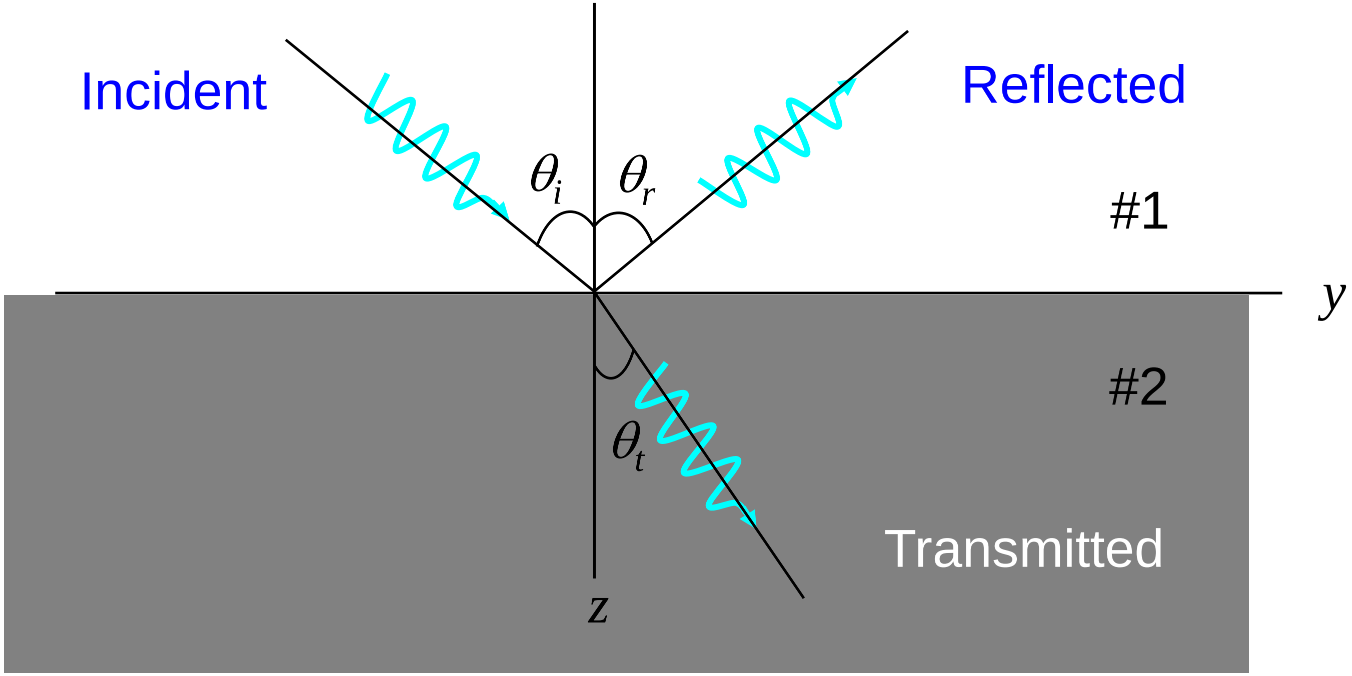
Reflection From Interface



We want to find the directions of the reflected and transmitted waves, and the reflection and transmission coefficients.

Note: The yz plane is the "plane of incidence"
(the plane containing the incident wave vector and the unit normal to the boundary).

Reflection From Interface



$$k_x = k \sin\theta \cos\phi$$

$$k_y = k \sin\theta \sin\phi$$

$$k_z = k \cos\theta$$

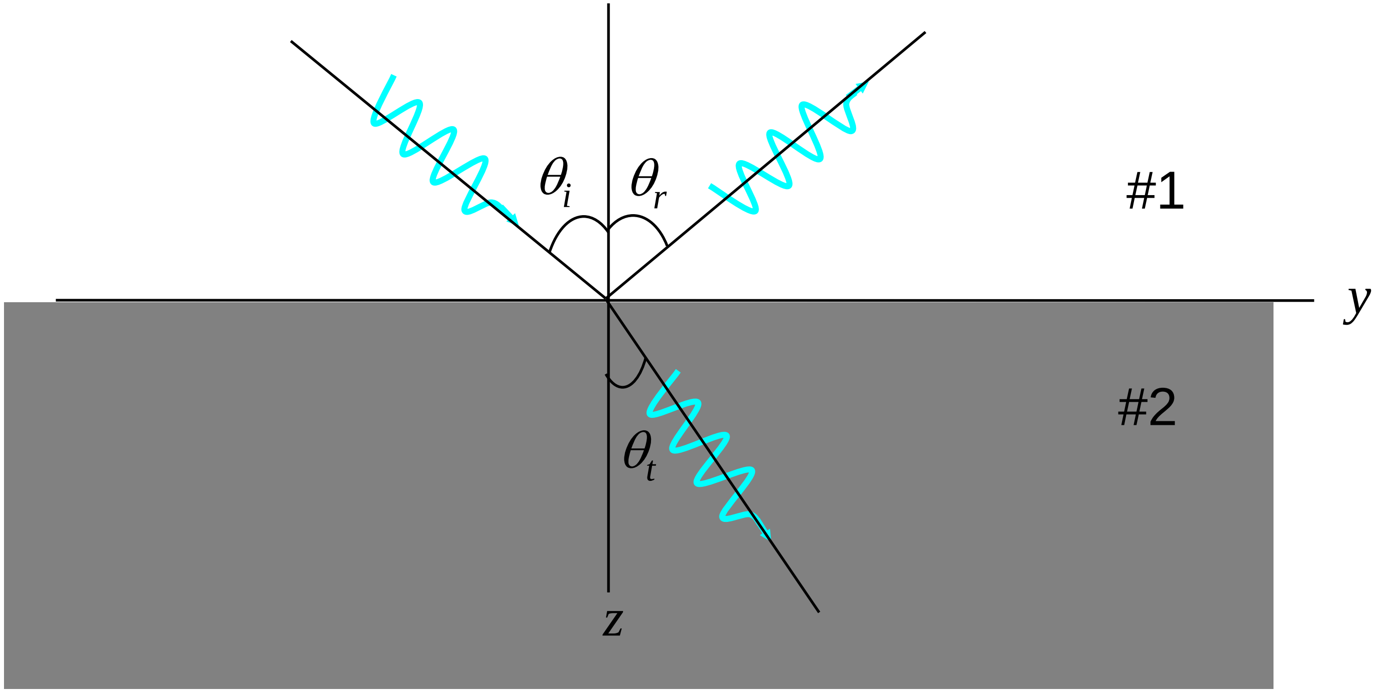
$$\xrightarrow{\phi = \pi / 2}$$

$$k_x = 0$$

$$k_y = k \sin\theta$$

$$k_z = k \cos\theta$$

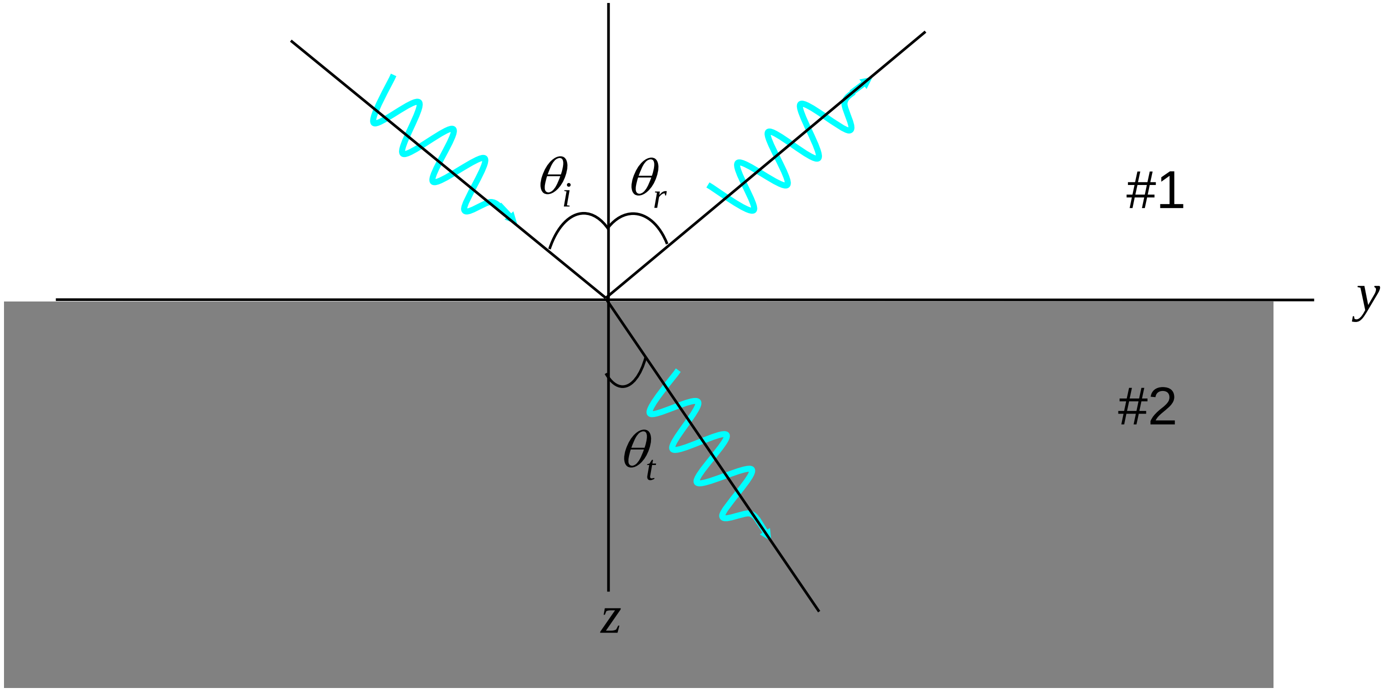
Reflection From Interface (cont.)



Phase matching condition: $\underbrace{k_{yi} = k_{yr}} = k_{yt} \Rightarrow k_1 \sin\theta_i = k_1 \sin\theta_r$

Hence $\boxed{\theta_i = \theta_r}$ (law of reflection)

Reflection From Interface (cont.)



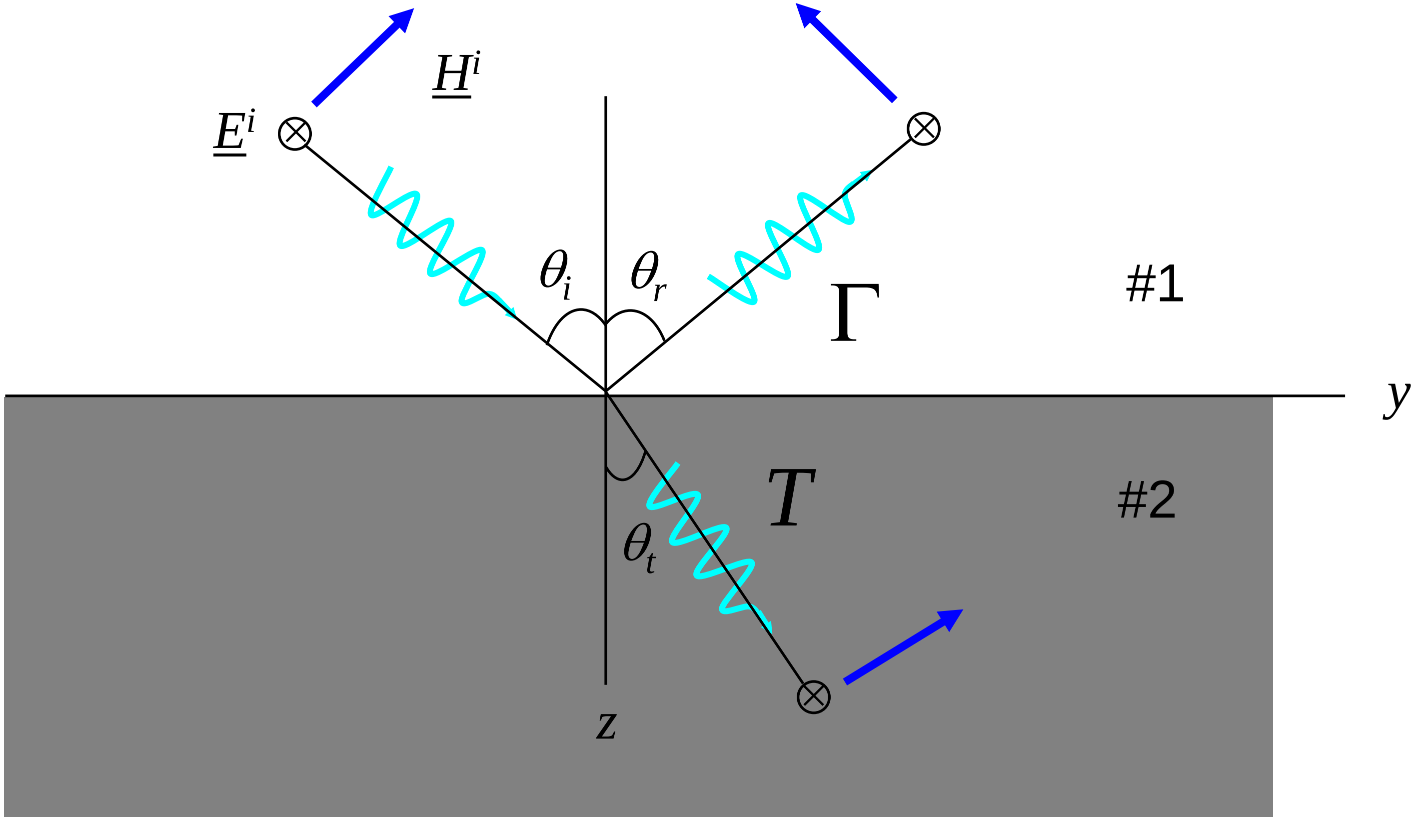
Phase matching condition: $k_{yi} = k_{yr} = k_{yt}$

Hence $k_1 \sin\theta_i = k_2 \sin\theta_t$ (Snell's law)

Reflection From Interface (cont.)

Determine reflection and transmission coefficients.

(Assume a TE_z wave)



Reflection From Interface (cont.)

Modeling equations:

$$E_x \leftrightarrow V \qquad H_y \leftrightarrow I$$

$$E_x^i = V^i(z)\,e^{-jk_y y}$$

$$E_x^r = V^r(z)\,e^{-jk_y y}$$

$$E_x^t = V^t(z)\,e^{-jk_y y}$$

$$V^i(z) = e^{-jk_{z1}z}$$

$$V^r(z) = \Gamma e^{+jk_{z1}z}$$

$$V^t(z) = Te^{-jk_{z2}z}$$

$$k_y = k_1 \sin\theta_i$$

$$k_{z1} = \sqrt{k_1{}^2 - k_y{}^2} = k_1 \cos\theta_i$$

$$k_{z2} = \sqrt{k_2{}^2 - k_y{}^2} = k_2 \cos\theta_t$$

Reflection From Interface (cont.)

Practical note:

When dealing with lossy media, the wave in region 2 will be inhomogeneous. Therefore the transmitted angle will be complex. In this case it is usually easier to work with the separation equation (the square-root formula for k_{z2}) rather than the transmitted-angle formula.

$$k_{y2} = k_2 \sin\theta_t = k_{y1} = k_1 \sin\theta_i = \text{real}$$

complex

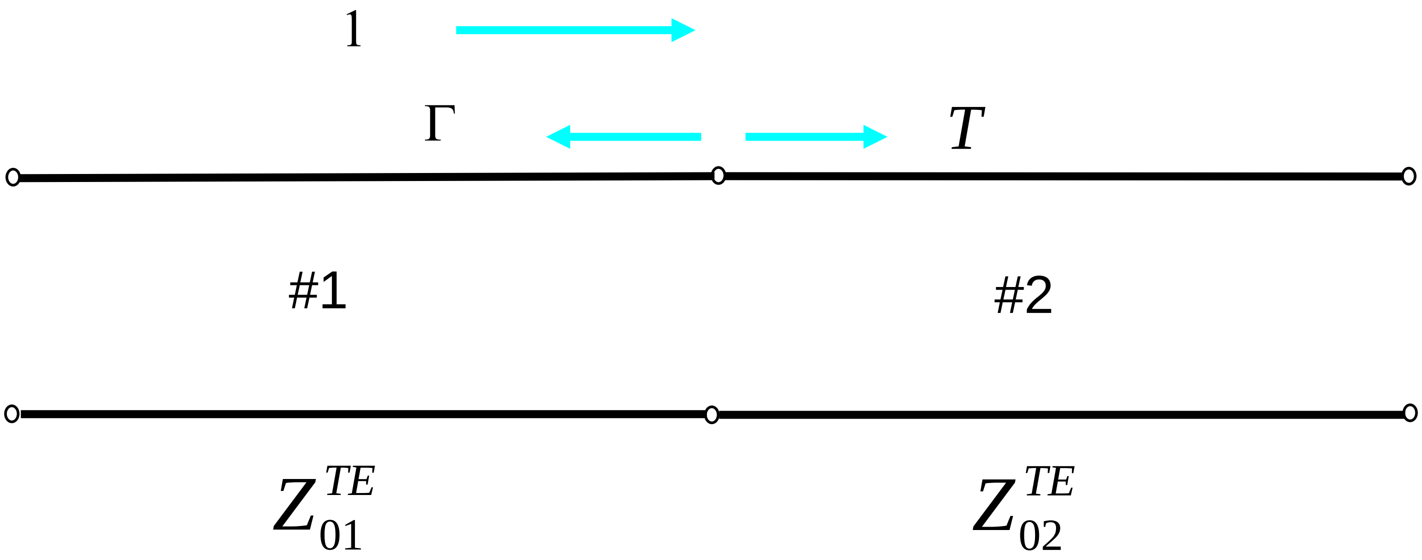
It is difficult to solve for θ_t

$$k_{z2} = \sqrt{k_2^2 - k_{y2}^2} = k_2 \cos\theta_t$$

Easy

Hard (needs complex angle)

Reflection From Interface (cont.)



$$\Gamma = \frac{Z_{02}^{TE} - Z_{01}^{TE}}{Z_{02}^{TE} + Z_{01}^{TE}}$$

$$T = 1 + \Gamma = \frac{2\,Z_{02}^{TE}}{Z_{02}^{TE} + Z_{01}^{TE}}$$

Reflection From Interface (cont.)

Hence

$$\Gamma = \frac{\dfrac{\omega\mu_2}{k_{z2}} - \dfrac{\omega\mu_1}{k_{z1}}}{\dfrac{\omega\mu_2}{k_{z2}} + \dfrac{\omega\mu_1}{k_{z1}}}$$

or

$$\Gamma = \frac{\mu_2 k_{z1} - \mu_1 k_{z2}}{\mu_2 k_{z1} + \mu_1 k_{z2}} \qquad T = \frac{2\mu_2 k_{z1}}{\mu_2 k_{z1} + \mu_1 k_{z2}}$$

Reflection From Interface (cont.)

Percent power reflected:

$$P_r^{\%} = 100 \left|\Gamma\right|^2$$

Percent power transmitted:

$$P_t^{\%} = 100 - P_r^{\%}$$