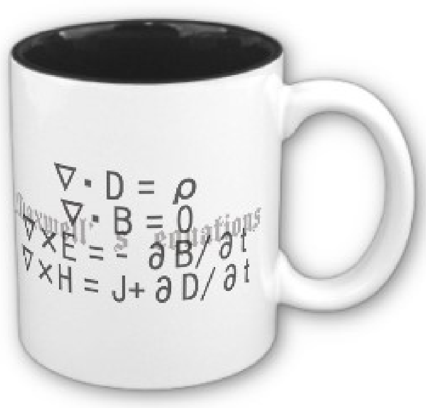
ECE 6340
Intermediate EM Waves

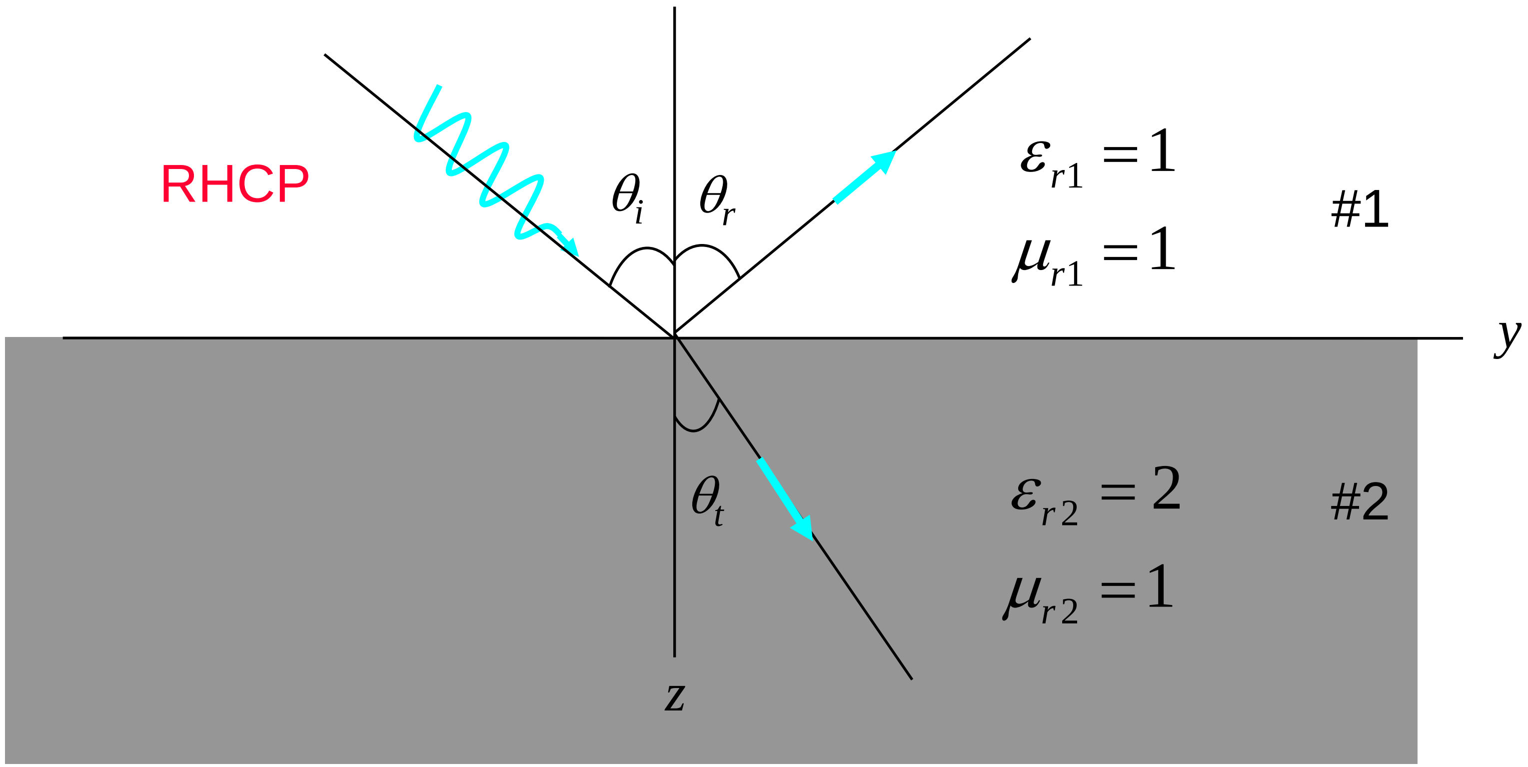
Fall 2016

Prof. David R. Jackson
Dept. of ECE



Notes 20

Example



$$\theta_i = 30^\circ$$

$$\theta_t = 20.70^\circ$$

(from Snells' law)

Find: % power reflected

AR of reflected wave

% power transmitted

AR of transmitted wave

Example (cont.)

First look at the TE$_z$ part:

$$\Gamma^{\text{TE}} = \frac{Z_2^{TE} - Z_1^{TE}}{Z_2^{TE} + Z_1^{TE}}$$

$$Z_2^{TE} = \frac{\omega \mu_0}{k_{z2}} = \frac{\omega \mu_0}{k_2 \cos \theta_t}$$

$$= \frac{\eta_0}{\sqrt{\varepsilon_{r2}}} \sec \theta_t$$

$$= 284.8 \ [\Omega]$$

$$Z_1^{TE} = \eta_0 \sec \theta_i$$

$$= 435.0 \ [\Omega]$$

$$\Gamma^{\text{TE}} = -0.2087$$

Example (cont.)

Now look at the TM_z part:

$$\Gamma^{TM} = \frac{Z_2^{TM} - Z_1^{TM}}{Z_2^{TM} + Z_1^{TM}}$$

$$Z_2^{TM} = \frac{k_{z2}}{\omega\varepsilon_2} = \frac{k_2\cos\theta_t}{\omega\varepsilon_0\varepsilon_{r2}}$$

$$= \frac{\eta_0}{\sqrt{\varepsilon_{r2}}}\cos\theta_t$$

$$= 249.2\ [\Omega]$$

$$Z_1^{TM} = \eta_0\cos\theta_i$$

$$= 326.3\ [\Omega]$$

$$\Gamma^{TM} = -0.1339$$

Example (cont.)

Note: If region 2 was lossy, then it is better <u>not</u> to solve for the (complex) angle θ_t. In this case, it is easier to use the separation equation to obtain $\cos(\theta_t)$:

$$k_{z2} = \sqrt{k_2^2 - k_{y2}^2} = \sqrt{k_2^2 - k_{y1}^2} = \sqrt{k_2^2 - k_0^2 \sin^2\theta_i}$$

Using the angle θ_t requires using complex angles:

$$k_{z2} = \sqrt{k_2^2 - k_0^2 \sin^2\theta_i}$$

Also

$$k_{z2} = k_2 \cos\theta_t = k_0 \sqrt{\varepsilon_{rc2}} \cos\theta_t$$

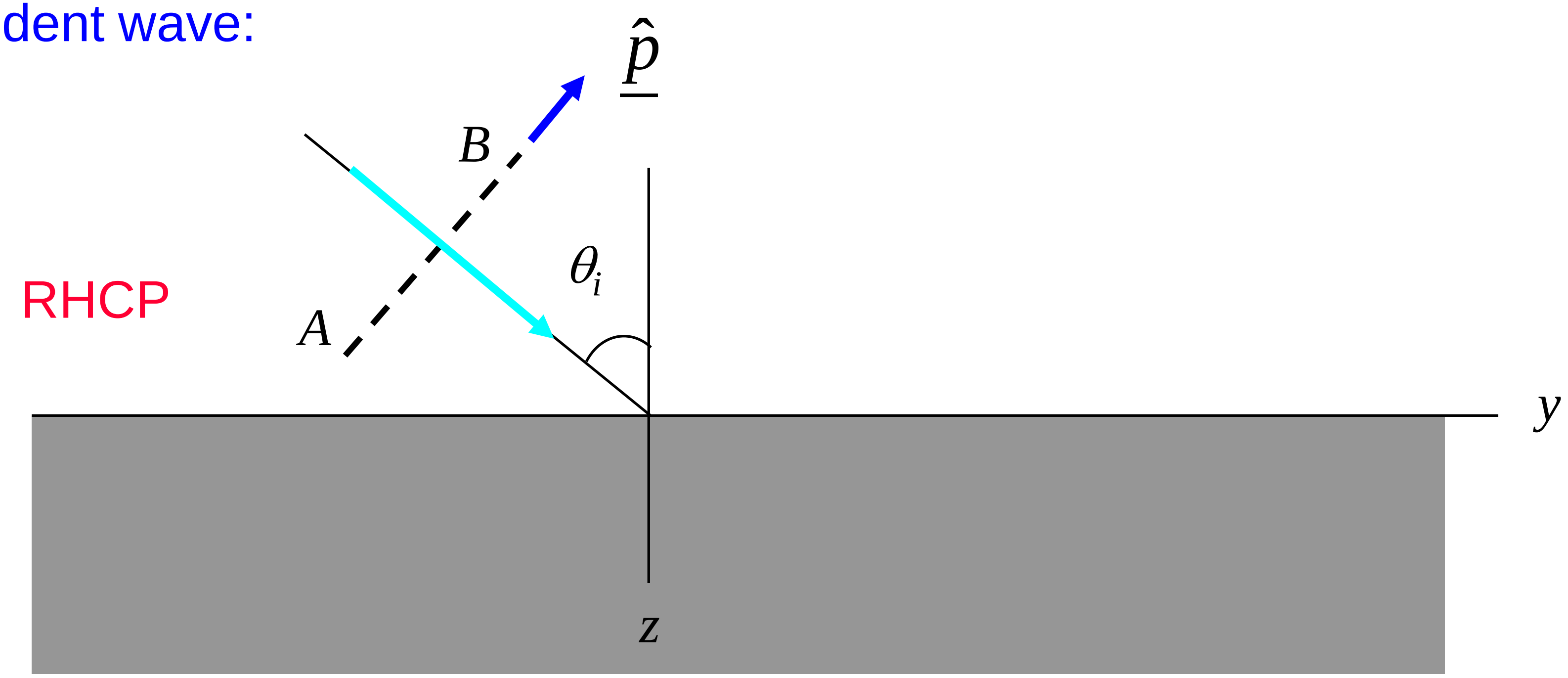
Hence

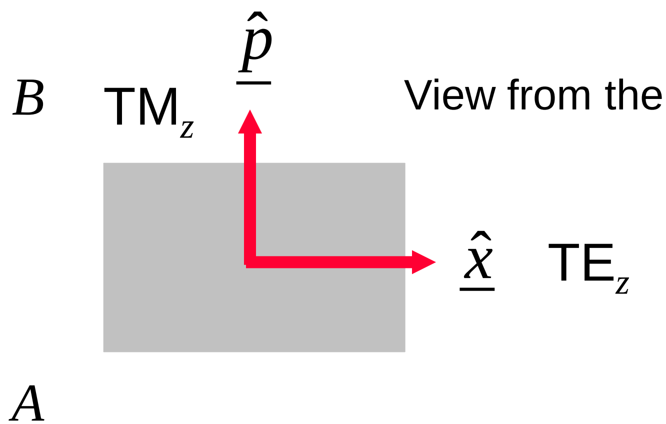
$$\cos\theta_t = \sqrt{1 - \left(\frac{1}{\varepsilon_{rc}}\right)^2 \sin^2\theta_i}$$

(complex)

Example (cont.)

Incident wave:

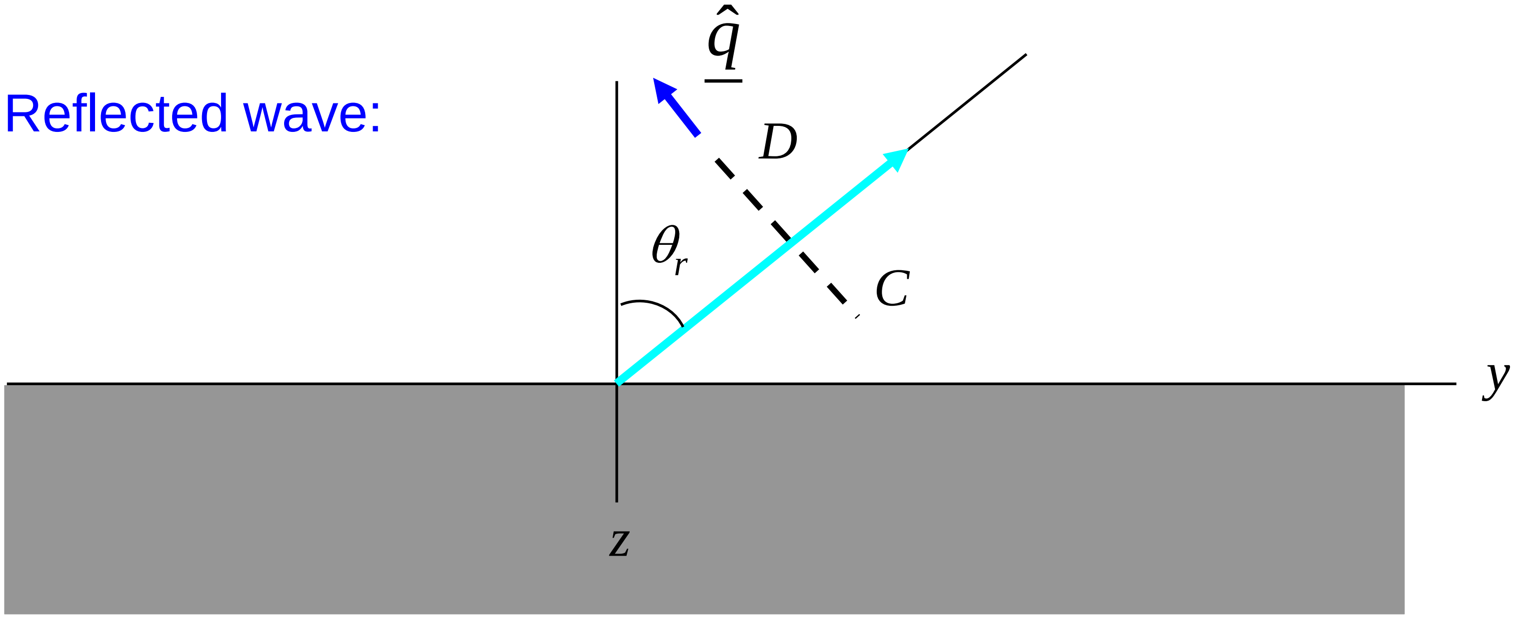


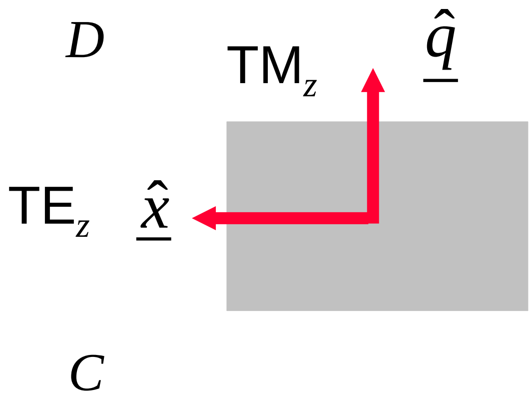


$$\underline{E}^i = [\,\hat{\underline{x}}\,E_0 + \hat{\underline{p}}\,E_0(-j)]\,e^{-j(k_{z1}z + k_y y)}$$

Example (cont.)

Reflected wave:





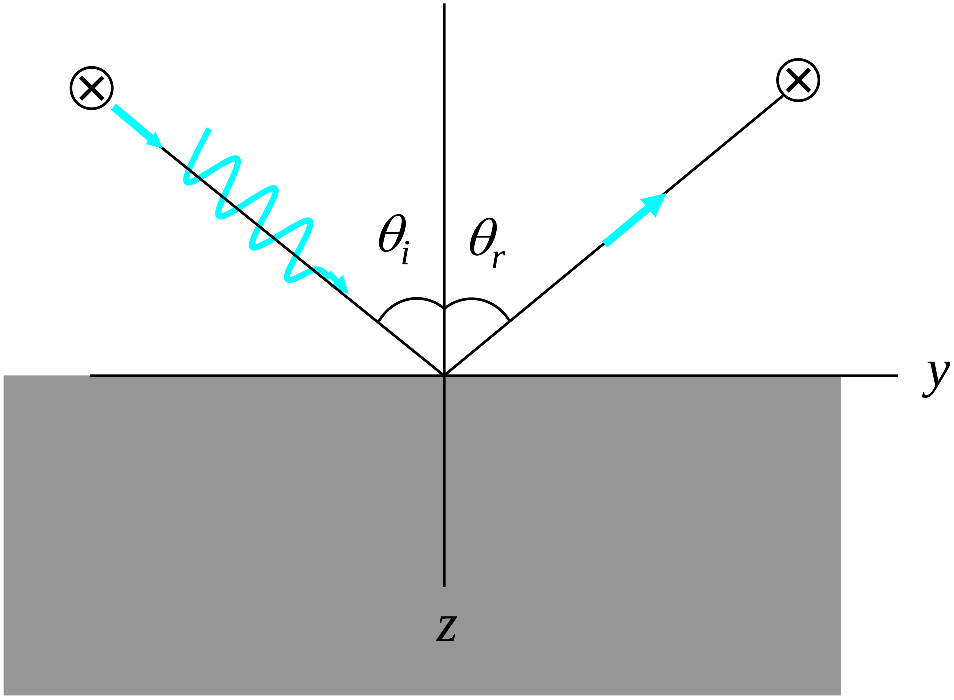
View from origin

$$\underline{E}^r = [\,\underline{\hat{x}}\,E_1 + \underline{\hat{q}}\,E_2\,]\,e^{+jk_{z1}z}e^{-jk_y y}$$

Example (cont.)

$$\underline{E}^i = [\,\hat{\underline{x}}\,E_0 + \hat{\underline{p}}\,E_0(-j)]e^{-j(k_{z1}z+k_y y)}$$

$$\underline{E}^r = [\,\hat{\underline{x}}\,E_1 + \hat{\underline{q}}\,E_2\,]e^{+jk_{z1}z}e^{-jk_y y}$$



TE$_z$ part:

$$E_x^{\,r} = \Gamma^{TE} E_x^{\,i}$$

$$\Rightarrow \quad E_1 = \Gamma^{TE} E_0$$

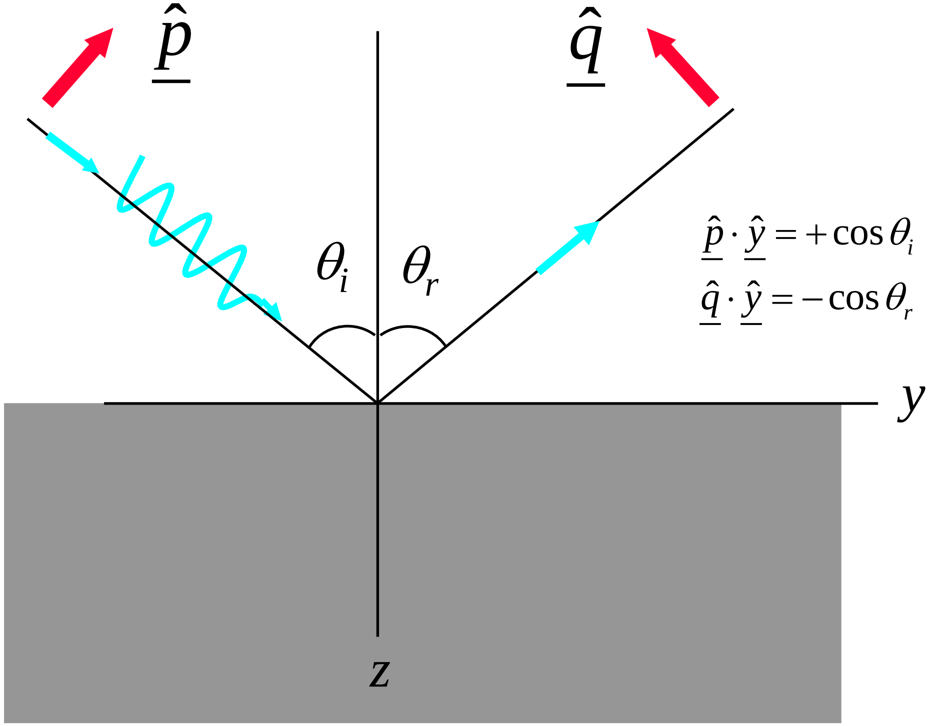
so

$$E_1 = (-0.2087)E_0$$

Example (cont.)

$$\underline{E}^i = [\,\hat{\underline{x}}\,E_0 + \hat{\underline{p}}\,E_0(-j)]e^{-j(k_{z1}z + k_y y)}$$

$$\underline{E}^r = [\,\hat{\underline{x}}\,E_1 + \hat{\underline{q}}\,E_2\,]e^{+jk_{z1}z}e^{-jk_y y}$$



TM$_z$ part:

$$E_y^r = \Gamma^{TM} E_y^i$$

$$\Rightarrow \quad -E_2\cos\theta_r = \Gamma^{TM}(-jE_0\cos\theta_i)$$

so

$$E_2 = jE_0\Gamma^{TM}$$

or

$$E_2 = j\,E_0(-0.1339)$$

Example (cont.)

$$\underline{E}^{r} = [\,\underline{\hat{x}}E_1 + \underline{\hat{q}}\,E_2\,]e^{+jk_{z1}z}e^{-jk_y y}$$

Hence

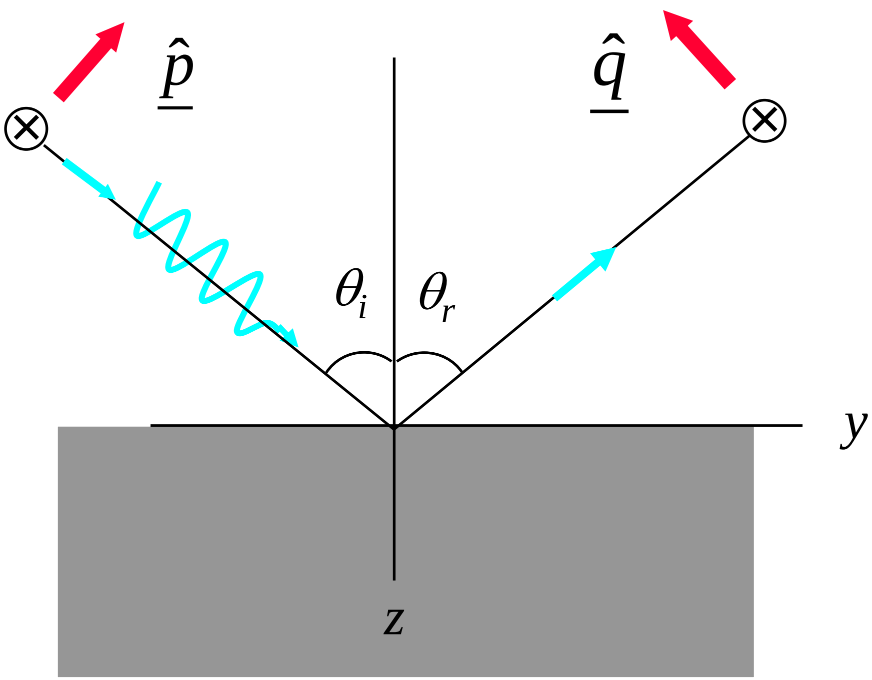
$$\underline{E}^{r} = E_0[\underline{\hat{x}}(-0.2087) + \underline{\hat{q}}(-j\,0.1339)]e^{+jk_{z1}z}\,e^{-jk_y y}$$

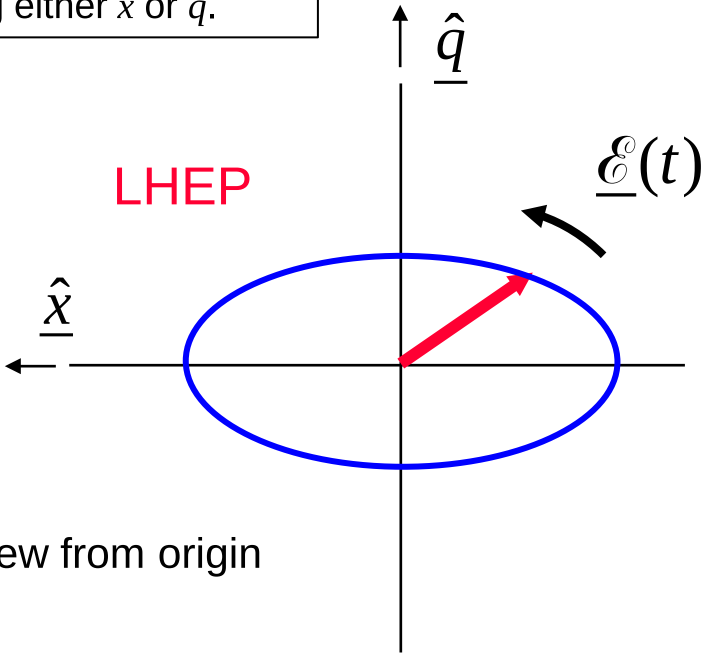
Example (cont.)

Axial Ratio of Reflected Wave

$$\underline{E}^r = -E_0[\hat{\underline{x}}(0.2087) + \hat{\underline{q}}(j\,0.1339)]e^{+jk_{z1}z}\,e^{-jk_y y}$$

Note: The phase difference between the components is 90°. The major axis will therefore lie along either x or q.





View from origin

$$AR^r = \frac{0.2087}{0.1339}$$

$$AR^r = 1.558$$

Example (cont.)

$P_r^{\%}$ = % power reflected

$$P_r^{\%} \equiv \frac{-\left\langle \mathscr{S}_z^{\,r} \right\rangle}{\left\langle \mathscr{S}_z^{\,i} \right\rangle}(100)$$

$$P_r^{\%} = 100 \left[\frac{\frac{1}{2\eta_0}\left[\left|E_x^r\right|^2 + \left|E_q^r\right|^2 \right]\cos\theta_r}{\frac{1}{2\eta_0}\left[\left|E_x^i\right|^2 + \left|E_p^i\right|^2 \right]\cos\theta_i} \right]$$

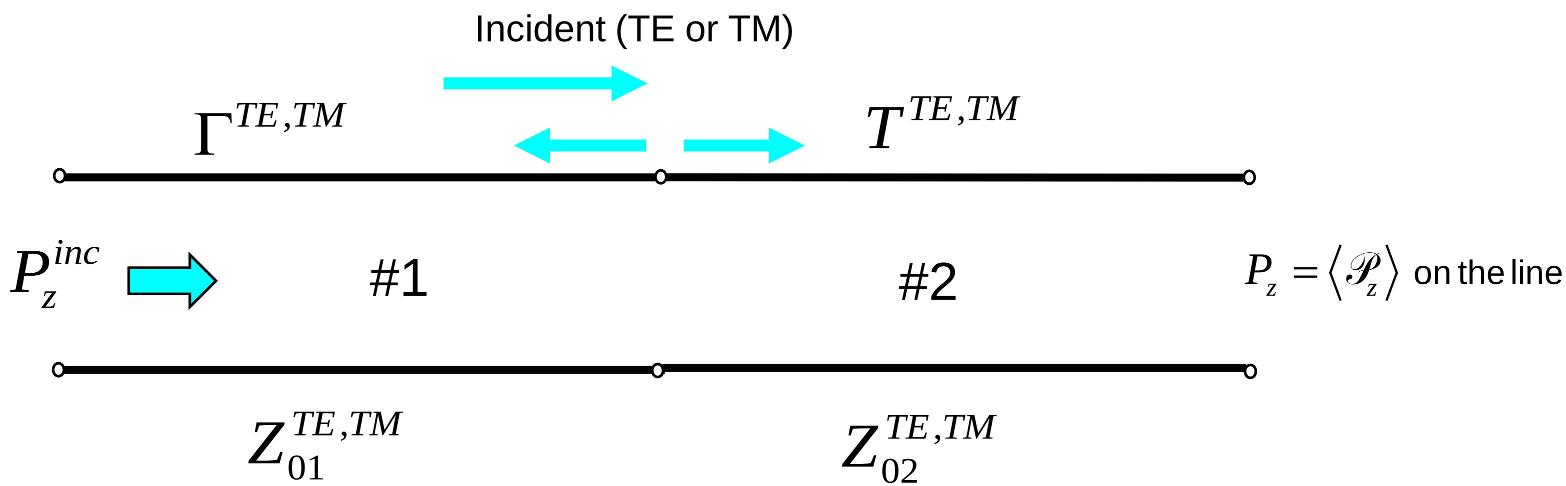
$$= 100 \left[\frac{\left[\left|-0.2087\right|^2 + \left|-0.1339\,j\right|^2 \right]}{\left[\left|1\right|^2 + \left|-j\right|^2 \right]} \right]$$

$$P_r^{\%} = 3.075$$

Example (cont.)

We can also use the TEN:

The incident wave has equal powers in the TM and TE parts.



$$P_r^{\%} = (100)\left[\frac{P_z^{inc,TE}\left|\Gamma^{TE}\right|^2 + P_z^{inc,TM}\left|\Gamma^{TM}\right|^2}{P_z^{inc,TE} + P_z^{inc,TM}}\right] = (100)\left[\frac{\left(P_z^{inc}/2\right)\left|\Gamma^{TE}\right|^2 + \left(P_z^{inc}/2\right)\left|\Gamma^{TM}\right|^2}{\left(P_z^{inc}/2\right) + \left(P_z^{inc}/2\right)}\right]$$

$$P_r^{\%} = (100)\left(\frac{1}{2}\right)\left(\left|\Gamma^{TE}\right|^2 + \left|\Gamma^{TM}\right|^2\right)$$

$$P_r^{\%} = 3.075$$

Example (cont.)

$$P_t^{\%} = \text{percent power transmitted}$$

$$P_t^{\%} \equiv \frac{\left\langle \mathscr{S}_z^{\,t} \right\rangle}{\left\langle \mathscr{S}_z^{\,i} \right\rangle}(100)$$

Conservation of energy: $\left\langle \mathscr{S}_z^{\,t} \right\rangle = \left\langle \mathscr{S}_z^{\,i} \right\rangle + \left\langle \mathscr{S}_z^{\,r} \right\rangle$

so

$$P_t^{\%} = 100 + 100\left(\frac{\left\langle \mathscr{S}_z^{\,r} \right\rangle}{\left\langle \mathscr{S}_z^{\,i} \right\rangle} \right)$$

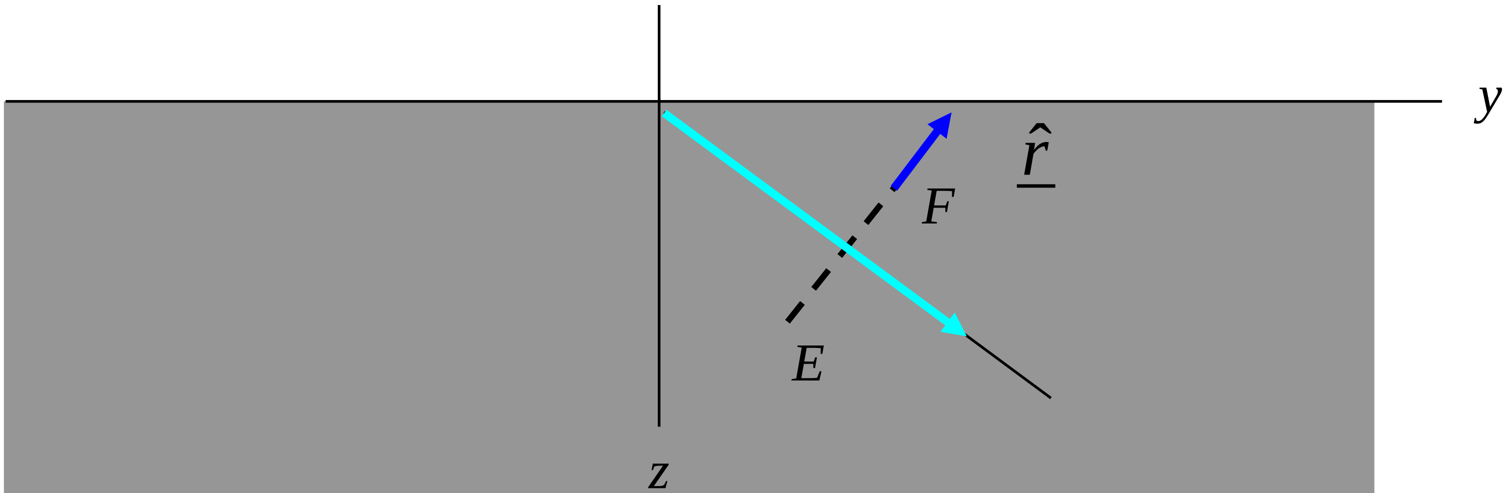
$$= 100 - P_r^{\%}$$

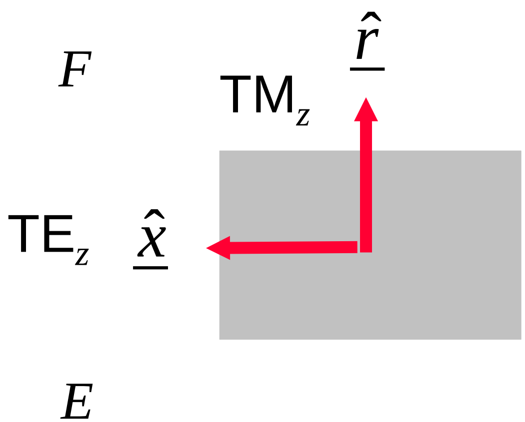
$$= 96.93$$

$$P_t^{\%} = 96.93$$

Example (cont.)

Transmitted wave:





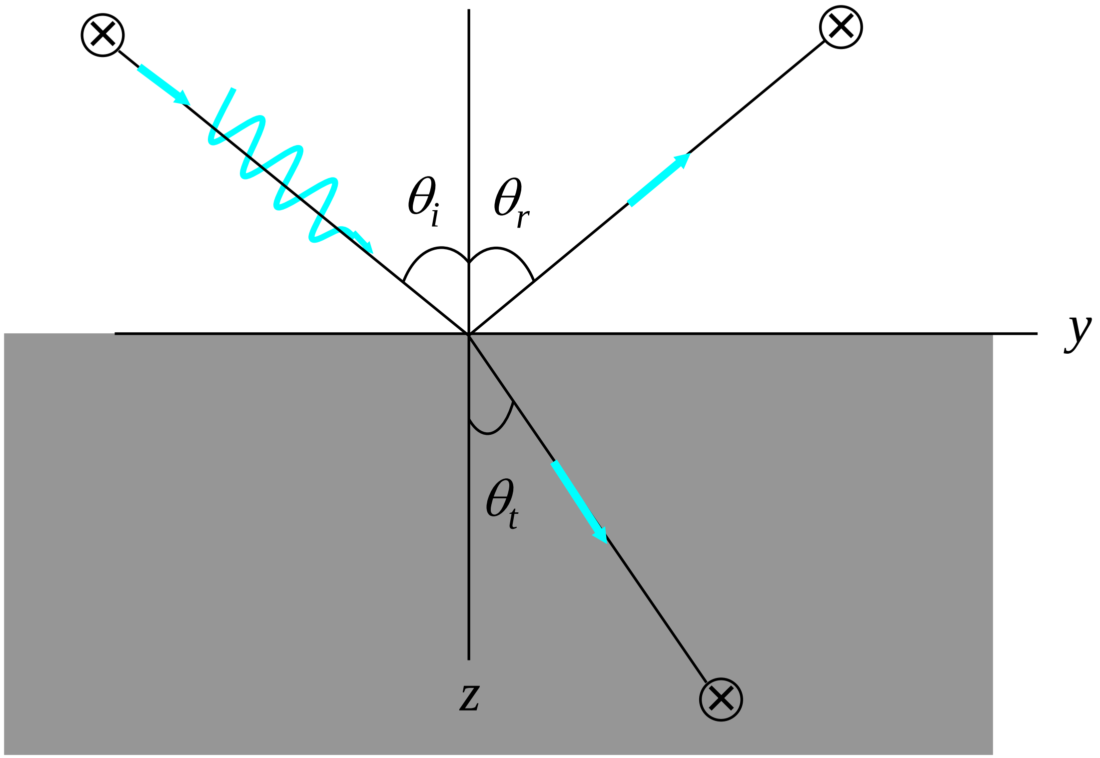
View from origin

$$\underline{E}^t = [\,\underline{\hat{x}}\,E_3 + \underline{\hat{r}}\,E_4\,]\,e^{-jk_{z2}z}e^{-jk_y y}$$

Example (cont.)

$$\underline{E}^i = [\hat{\underline{x}} E_0 + \hat{\underline{p}} E_0(-j)] e^{-j(k_{z1}z + k_y y)}$$

$$\underline{E}^t = [\hat{\underline{x}} E_3 + \hat{\underline{r}} E_4] e^{-jk_{z2}z} e^{-jk_y y}$$



TE$_z$ part:

$$E_x^t = T^{TE} E_x^i$$

$$E_3 = T^{TE} E_0$$

$$= (1 + \Gamma^{TE}) E_0$$

$$= (0.7913) E_0$$

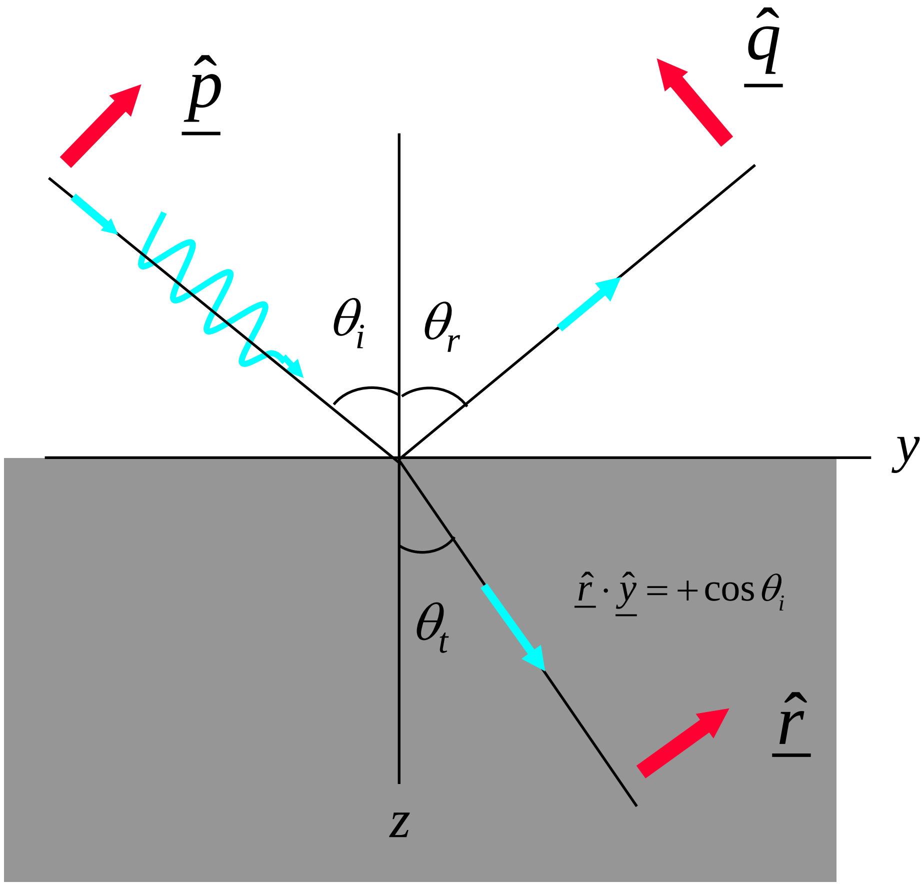
so

$$E_3 = (0.7913) E_0$$

Example (cont.)

$$\underline{E}^i = [\hat{\underline{x}}\, E_0 + \hat{\underline{p}}\, E_0(-j)]e^{-j(k_{z1}z + k_y y)}$$

$$\underline{E}^t = [\hat{\underline{x}}\, E_3 + \hat{\underline{r}}\, E_4]e^{-jk_{z2}z}e^{-jk_y y}$$



TM$_z$ part:

$$E_y^t = T^{TM} E_y^i$$

$$E_4 \cos\theta_t = T^{TM}(-jE_0)\cos\theta_i$$
$$= (1 + \Gamma^{TM})(-jE_0)\cos\theta_i$$

Hence

$$E_4 = -jE_0(1 + \Gamma^{TM})\cos\theta_i \sec\theta_t$$

so

$$E_4 = -jE_0(0.8018)$$

Example (cont.)

$$\underline{E}^{t} = [\,\underline{\hat{x}}\,E_3 + \underline{\hat{r}}\,E_4\,]e^{-jk_{z2}z}e^{-jk_y y}$$

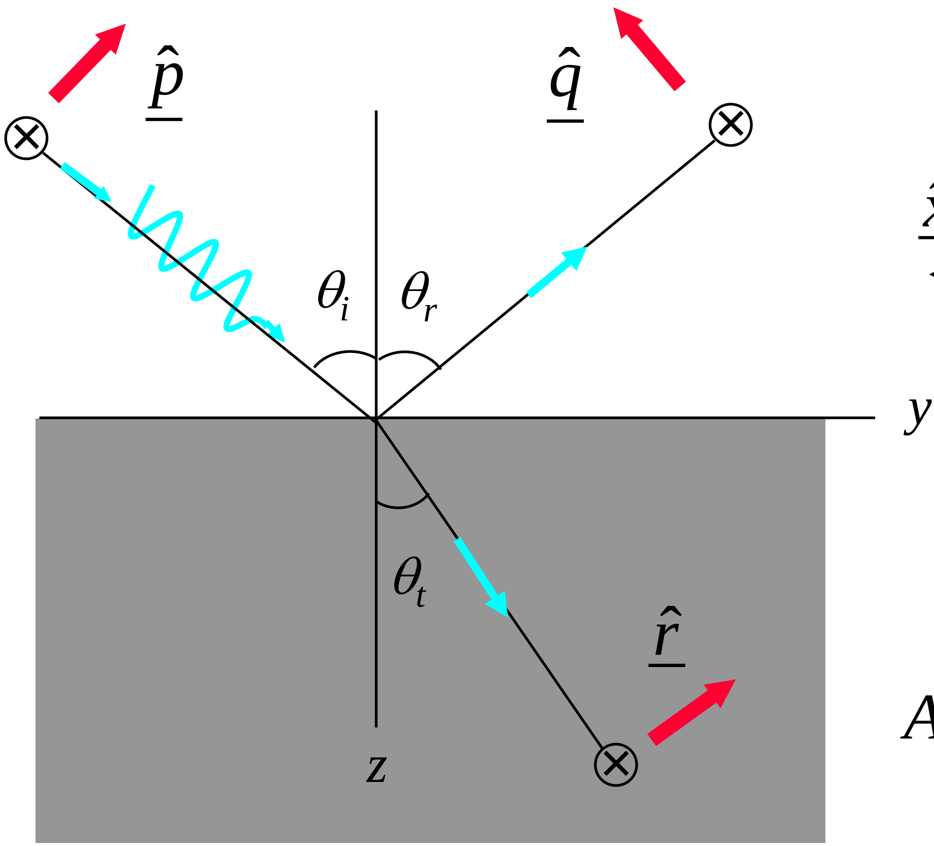
Hence

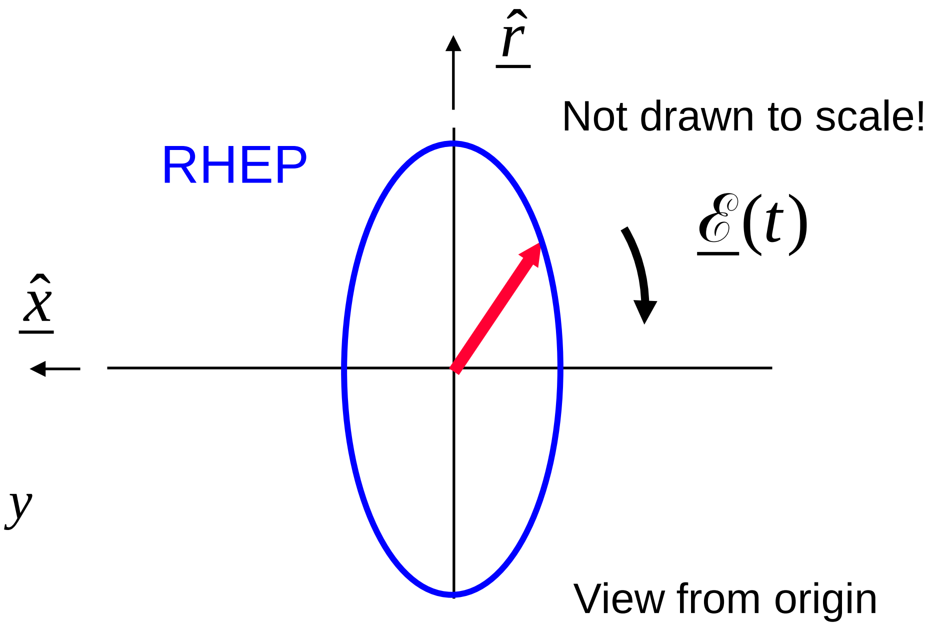
$$\underline{E}^{t} = E_0[\underline{\hat{x}}(0.7913) + \underline{\hat{r}}(-j\,0.8018)]e^{-jk_{z2}z}\,e^{-jk_y y}$$

Example (cont.)

Axial Ratio of Transmitted Wave

$$\underline{E}^t = E_0[\hat{\underline{x}}(0.7913) + \hat{\underline{r}}(-j0.8018)]e^{-jk_{z2}z}e^{-jk_y y}$$





$$AR^t = \frac{0.8018}{0.7913}$$

$$AR^t = 1.0133$$