ECE 6340
Intermediate EM Waves

Fall 2016

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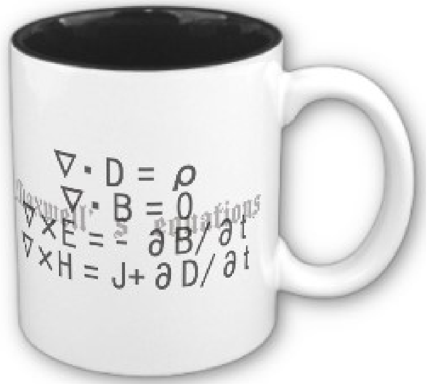
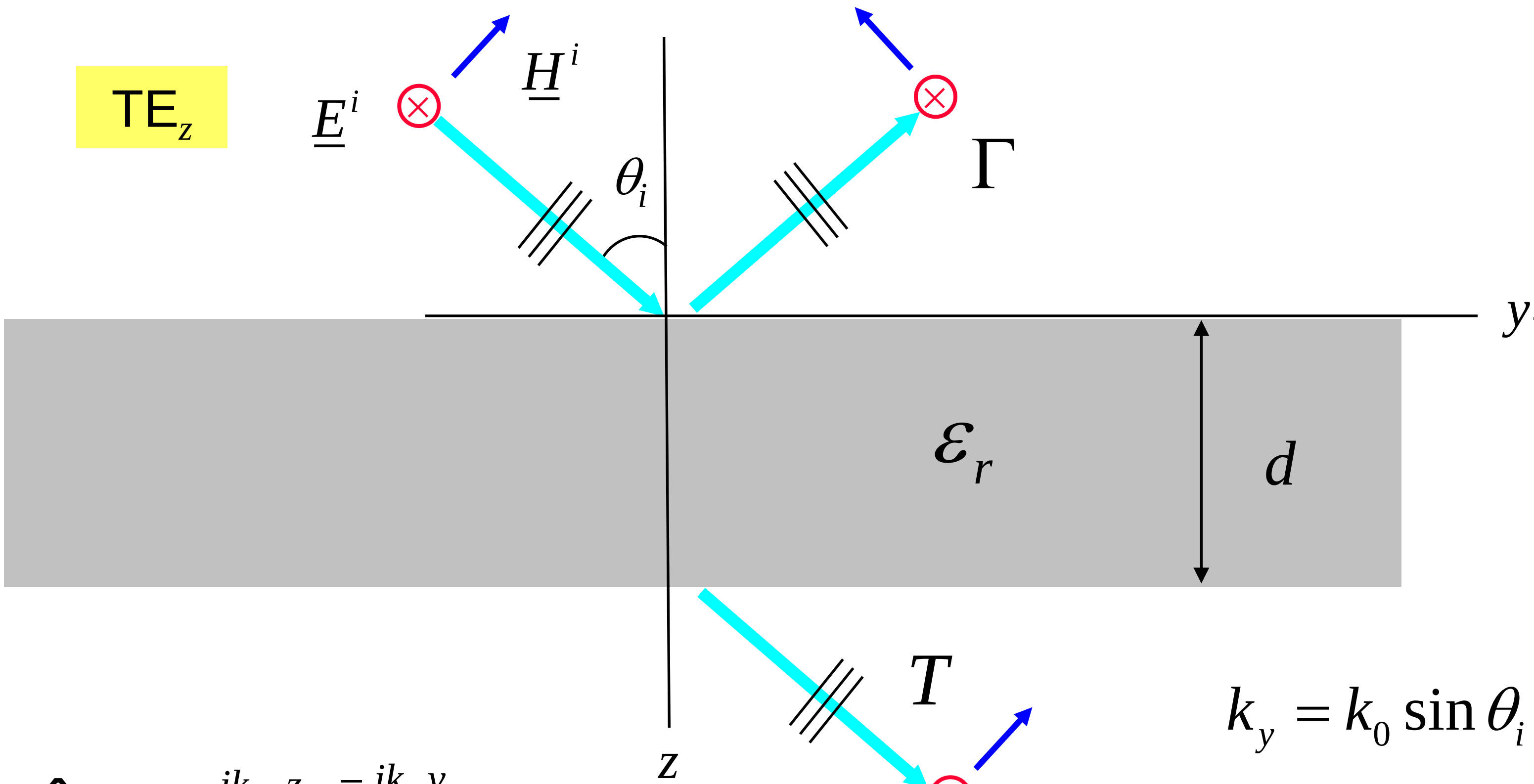
Dept. of ECE



Notes 21

Reflection from Slab



$$k_y = k_0 \sin\theta_i$$

$$k_{z0} = k_0 \cos\theta_i$$

$$\underline{E}^i = \underline{\hat{x}}\, E_0 e^{-jk_{z0}z} e^{-jk_y y}$$

$$\underline{E}^r = \underline{\hat{x}}\, E_0\, \Gamma\, e^{+jk_{z0}z} e^{-jk_y y}$$

$$\underline{E}^t = \underline{\hat{x}}\, E_0\, T\, e^{-jk_{z0}z} e^{-jk_y y}$$

Notes:

(1) $T \neq 1 + \Gamma$

(2) The origin is the reference plane for T.

Reflection from Slab (cont.)

Find the reflection coefficient Γ.

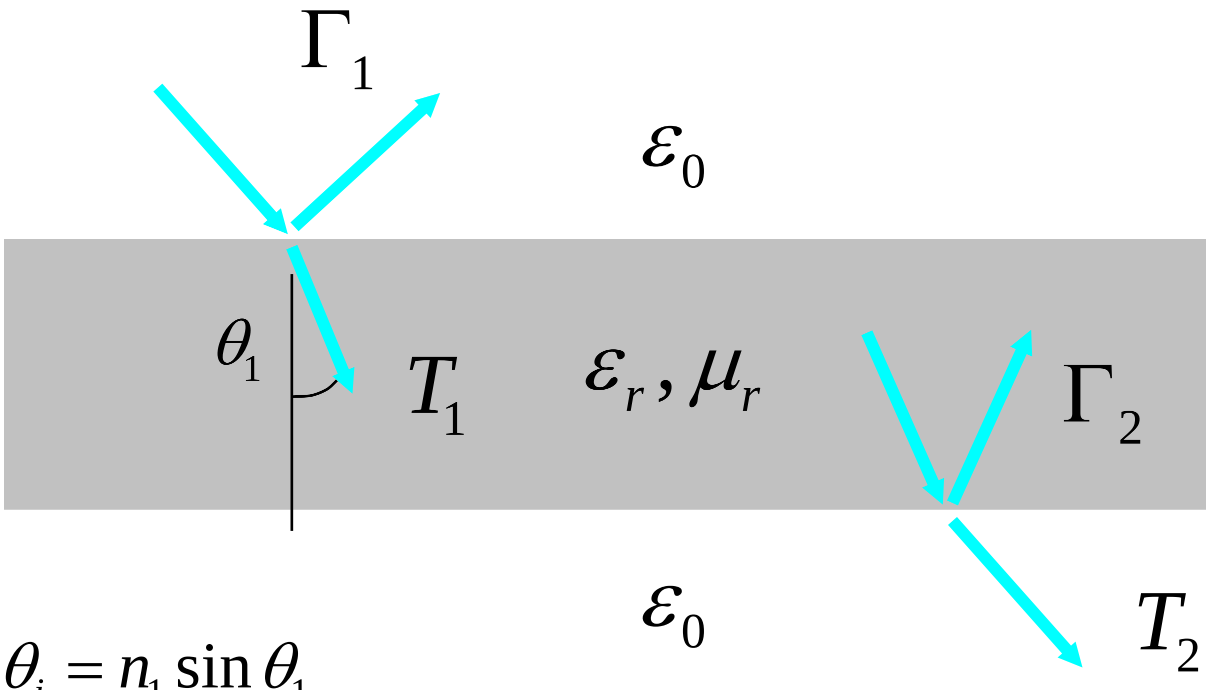
Three methods:

1) Plane-wave bounce method (interface reflections)
2) Steady-state wave representation
3) Transverse equivalent network (TEN)

Method #1

Plane-wave bounce method (interface reflections)

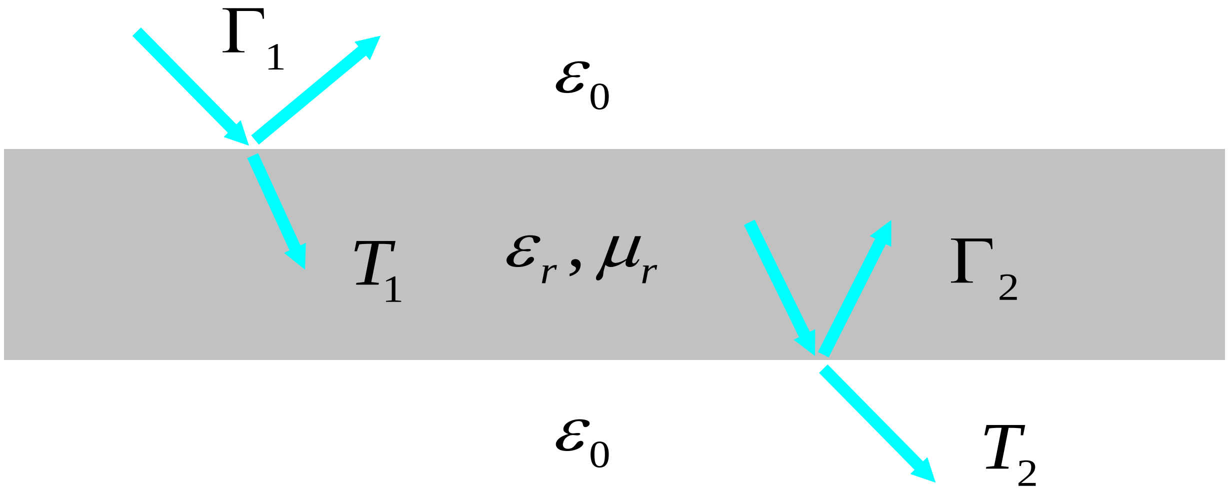
Define interface plane-wave reflection and transmission coefficients:



$$\sin\theta_i = n_1 \sin\theta_1$$

$$n_1 = \sqrt{\mu_r \varepsilon_r}$$

Interface Reflections (cont.)



$$\Gamma_1 = \frac{Z_{01}^{TE} - Z_{00}^{TE}}{Z_{01}^{TE} + Z_{00}^{TE}}$$

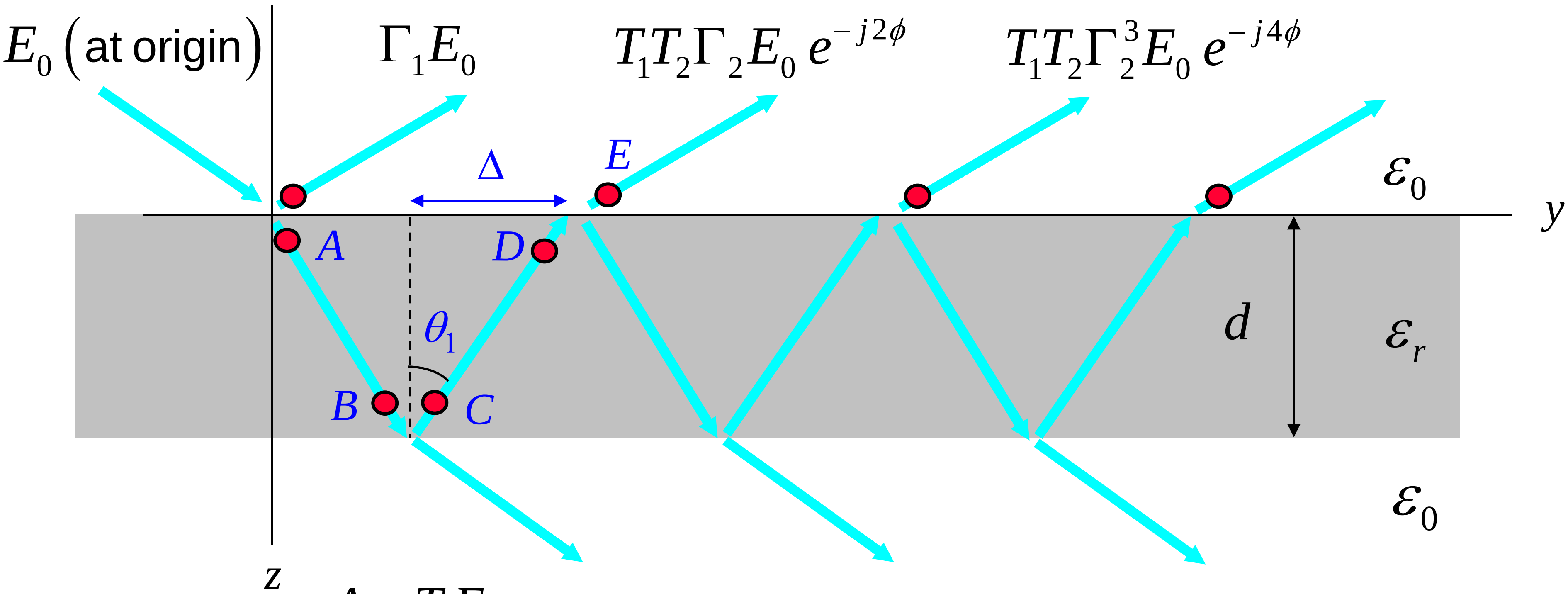
$$\Gamma_2 = \frac{Z_{00}^{TE} - Z_{01}^{TE}}{Z_{00}^{TE} + Z_{01}^{TE}}$$

$$\Gamma_2 = -\Gamma_1$$

$$T_1 = 1 + \Gamma_1$$

$$T_2 = 1 + \Gamma_2$$

Plane-Wave Bounce Diagram



$A: \; T_1 E_0$

$B: \; T_1 E_0 e^{-j\phi}$

$C: \; \Gamma_2 T_1 E_0 e^{-j\phi}$

$D: \; \Gamma_2 T_1 E_0 e^{-j2\phi}$

$E: \; T_2 \Gamma_2 T_1 E_0 e^{-j2\phi}$

$$\phi = k_{z1} d + k_y \Delta = \phi_z + \phi_y$$

$$k_{z1} = k_1 \cos\theta_1 \qquad k_y = k_0 \sin\theta_i$$

$$\Delta = d \tan\theta_1$$

Bounce Diagram (cont.)

At $z = 0$:

$$E_x^r = \Gamma_1 E_0 e^{-jk_y y} + T_1 T_2 \Gamma_2 E_0 e^{-j2\phi} e^{-jk_y(y-2\Delta)}$$
$$+ T_1 T_2 \Gamma_2^3 E_0 e^{-j4\phi} e^{-jk_y(y-4\Delta)} + \ldots\ldots$$

Note that (for p an integer)

$$e^{-j2p\phi} e^{-jk_y(y-2p\Delta)} = e^{-j2p\left(\phi_z+\phi_y\right)} e^{-jk_y y}\, e^{j2p\phi_y} = e^{-jk_y y} e^{-j2p\phi_z}$$

So we have

$$E_x^r = E_0 e^{-jk_y y} \underbrace{\left\{\Gamma_1 + T_1 T_2 \Gamma_2 e^{-j2\phi_z} + T_1 T_2 \Gamma_2^3 e^{-j4\phi_z} + \ldots\ldots\right\}}_{\Gamma}$$

Bounce Diagram (cont.)

Hence

$$\Gamma = \Gamma_1 + T_1T_2\Gamma_2 e^{-j2\phi_z} + T_1T_2\Gamma_2^3 e^{-j4\phi_z} + \ldots\ldots$$

$$= \Gamma_1 - T_1T_2\Gamma_1 e^{-j2\phi_z} - T_1T_2\Gamma_1\Gamma_2^2 e^{-j4\phi_z} + \ldots\ldots$$

$$= \Gamma_1(1 - T_1T_2 e^{-j2\phi_z} - T_1T_2\Gamma_2^2 e^{-j4\phi_z} + \ldots\ldots)$$

$$= \Gamma_1\left[1 - T_1T_2 e^{-j2\phi_z}\left(1 + \Gamma_2^2 e^{-j2\phi_z} + \Gamma_2^4 e^{-j4\phi_z} + \ldots\ldots\right)\right]$$

$$= \Gamma_1\left[1 - T_1T_2 e^{-j2\phi_z}\sum_{n=0}^{\infty}\left(\Gamma_2^{2n} e^{-j2\phi_z n}\right)\right]$$

Recall : $\Gamma_2 = -\Gamma_1$

Bounce Diagram (cont.)

or

$$\Gamma = \Gamma_1 \left[1 - T_1 T_2 e^{-j2\phi_z} \sum_{n=0}^{\infty} \left(\Gamma_1^2 e^{-j2\phi_z} \right)^n \right]$$

Next, use

$$\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}, \qquad |z| < 1,$$

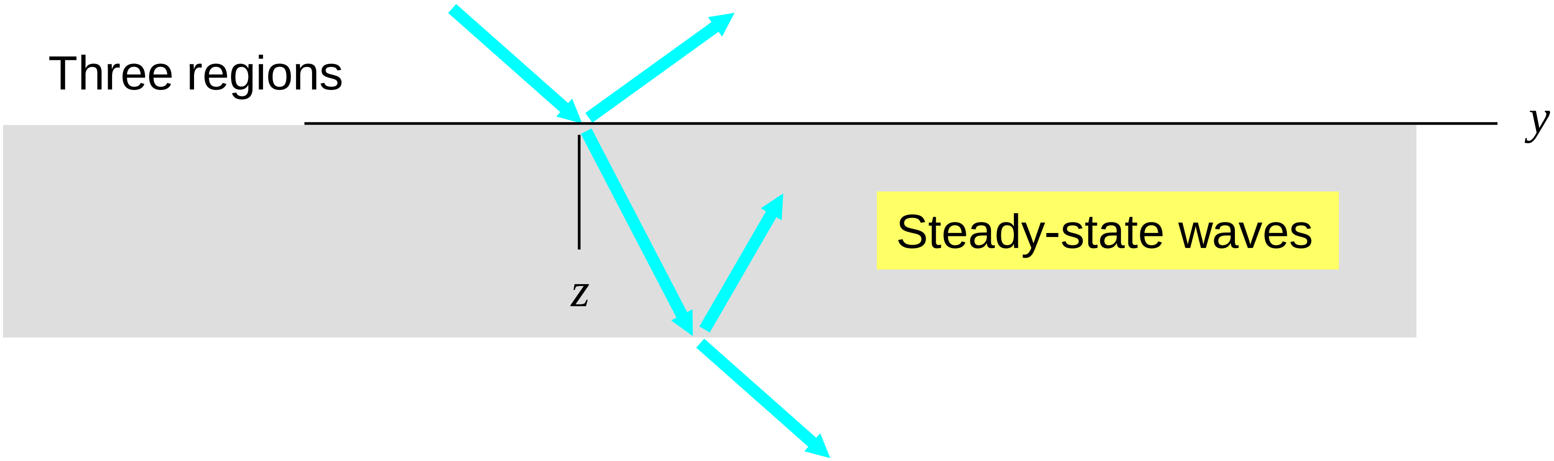
Hence

$$\Gamma = \Gamma_1 \left[1 - T_1 T_2 \, e^{-j2\phi_z} \left(\frac{1}{1 - \Gamma_1^2 e^{-j2\phi_z}} \right) \right]$$

Method # 2

Steady-State Wave Representation + B.C.s

#1 Three regions



#2

#3

(1) $$E_x = \left(E_0 e^{-jk_{z0}z} + E_0 \Gamma e^{+jk_{z0}z}\right) e^{-jk_y y}$$

(2) $$E_x = \left(A e^{-jk_{z1}z} + B e^{+jk_{z1}z}\right) e^{-jk_y y}$$

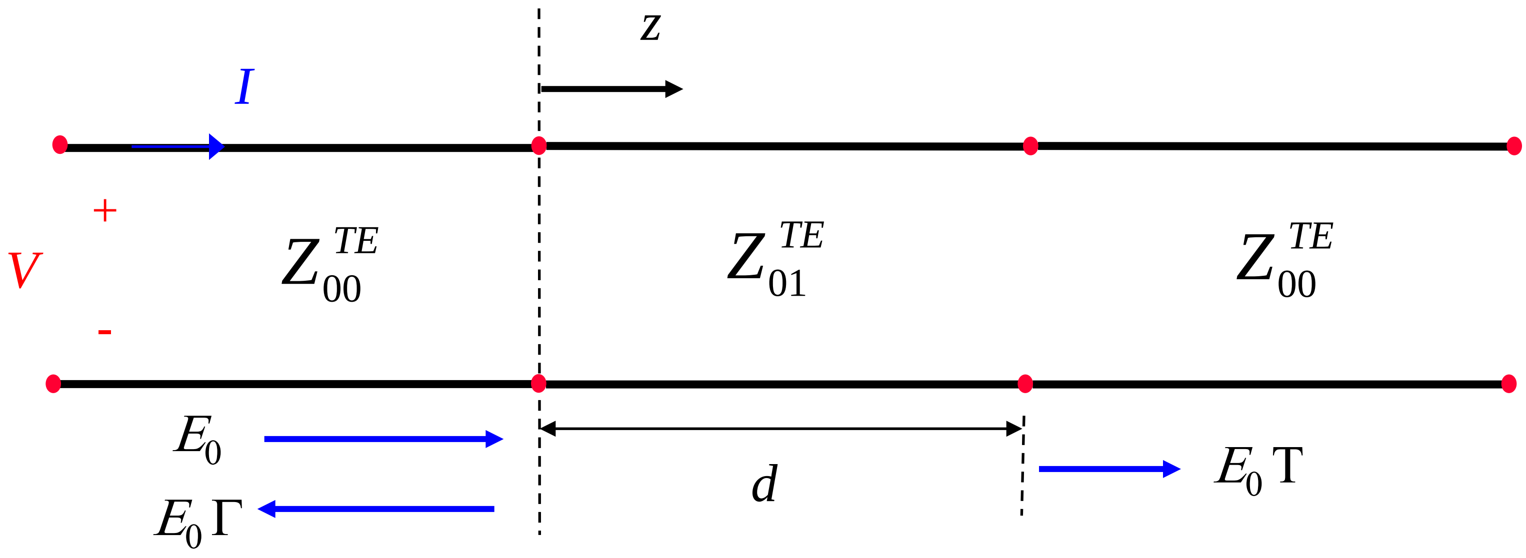
(3) $$E_x = T\, E_0\, e^{-jk_{z0}z} e^{-jk_y y}$$

4 unknowns: Γ, T, A, B

4 equations: E_x and H_y must match at both interfaces.

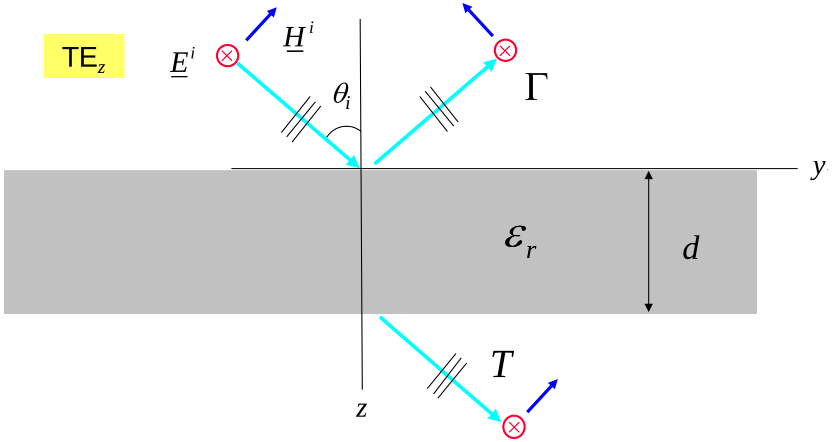
Method # 3

Transverse Equivalent Network (TEN)

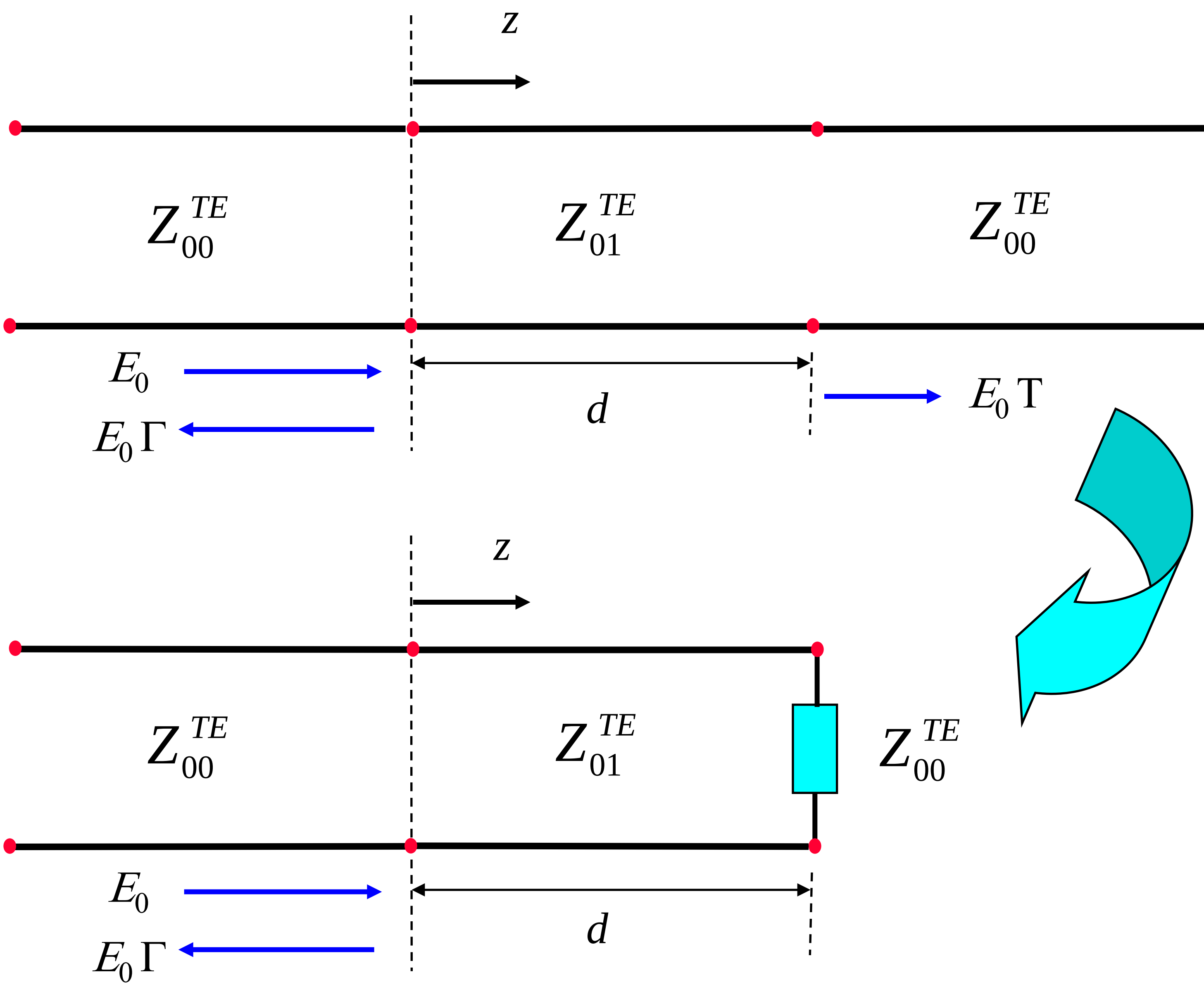


$$E_x(x,z) = V(z)e^{-jk_y y}$$

$$H_y(x,z) = I(z)e^{-jk_y y}$$

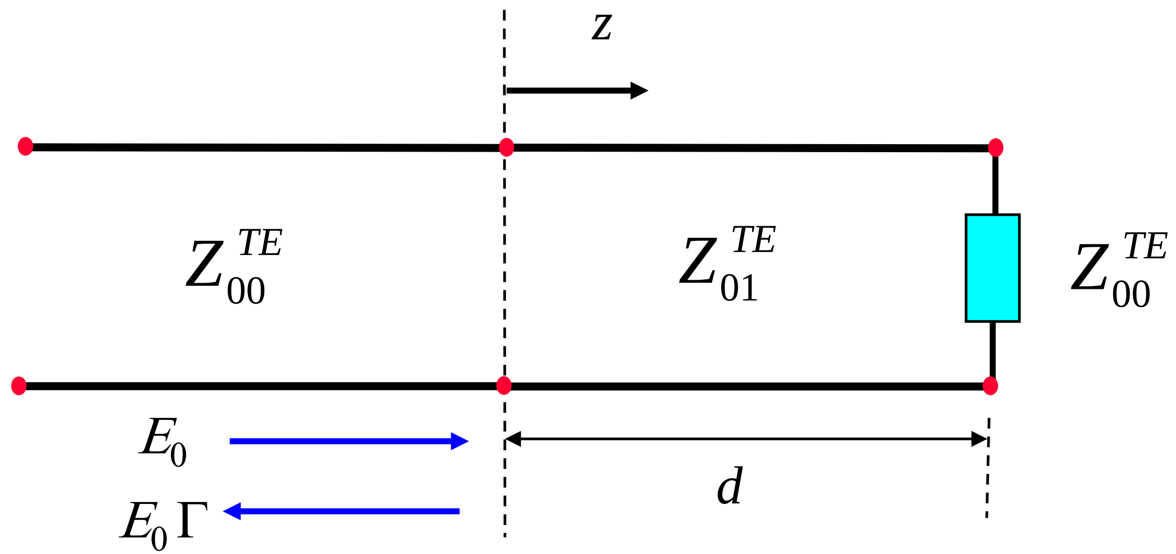


TEN (cont.)

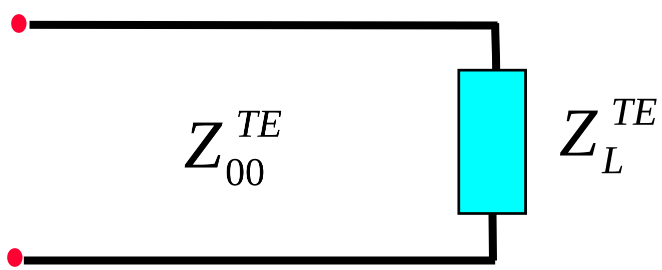


z
Z_{00}^{TE}
Z_{01}^{TE}
Z_{00}^{TE}
E_0
$E_0\Gamma$
d
$E_0 T$
z
Z_{00}^{TE}
Z_{01}^{TE}
Z_{00}^{TE}
E_0
$E_0\Gamma$
d

TEN (cont.)

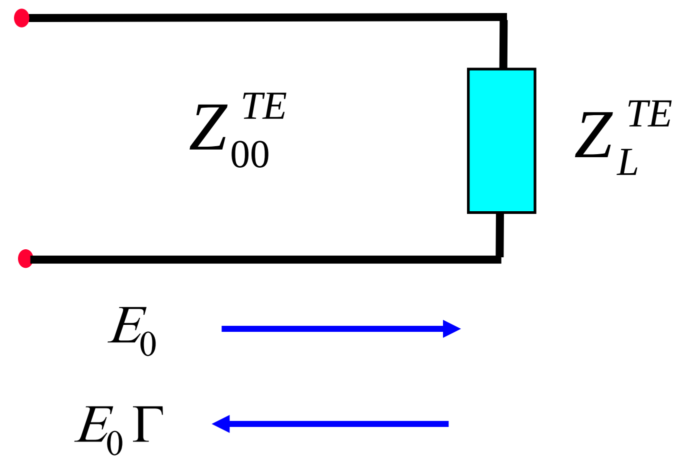


Equivalent circuit:



$$Z_L^{TE} = Z_{01}^{TE} \left[\frac{Z_{00}^{TE} + jZ_{01}^{TE} \tan(k_{z1} d)}{Z_{01}^{TE} + jZ_{00}^{TE} \tan(k_{z1} d)} \right]$$

TEN (cont.)

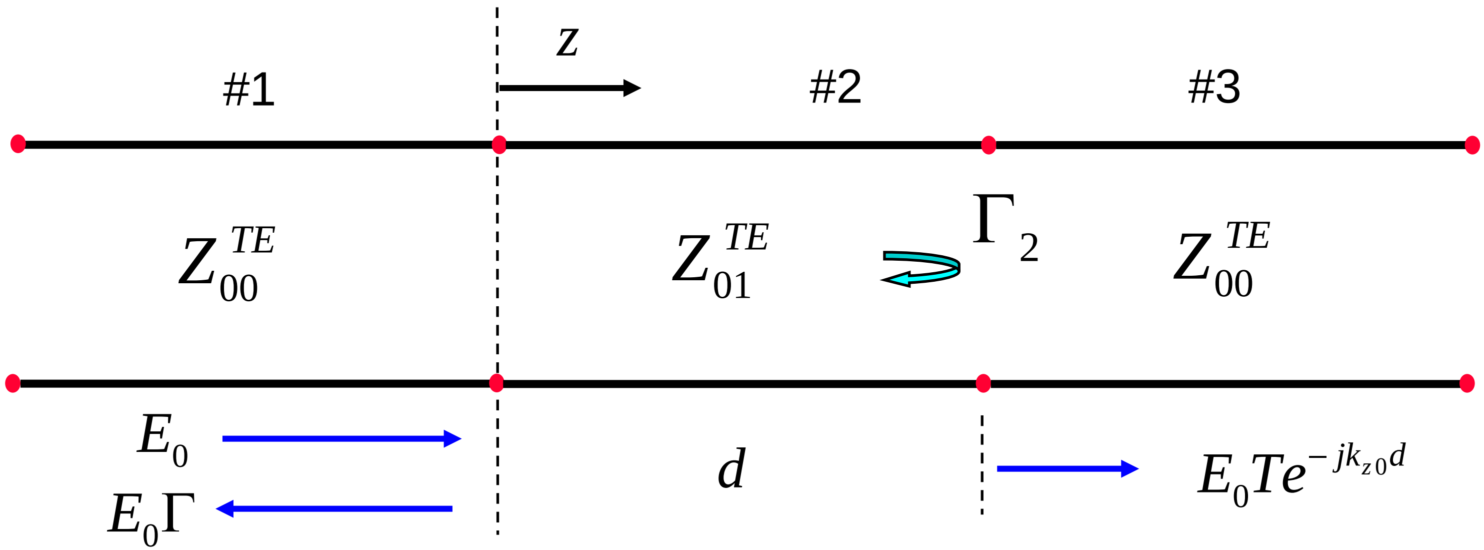


We then have

$$\Gamma = \frac{Z_L^{TE} - Z_{00}^{TE}}{Z_L^{TE} + Z_{00}^{TE}}$$

TEN (cont.)

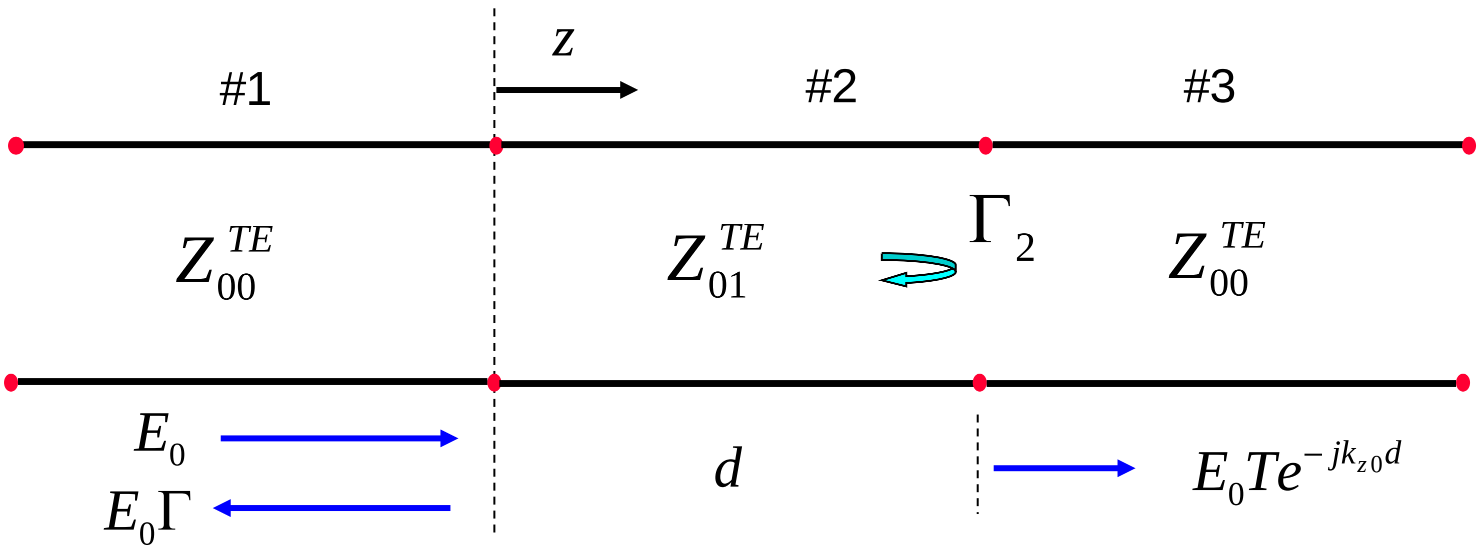
Find the transmission coefficient T.



Region 2: $$V_2(z) = \left(A e^{-jk_{z1}(z-d)} + A\Gamma_2 \, e^{+jk_{z1}(z-d)} \right)$$
$$= A\left(e^{-jk_{z1}(z-d)} + \Gamma_2 \, e^{+jk_{z1}(z-d)} \right)$$

Note: The phase reference point for the transmission coefficient is $z = 0$.

TEN (cont.)



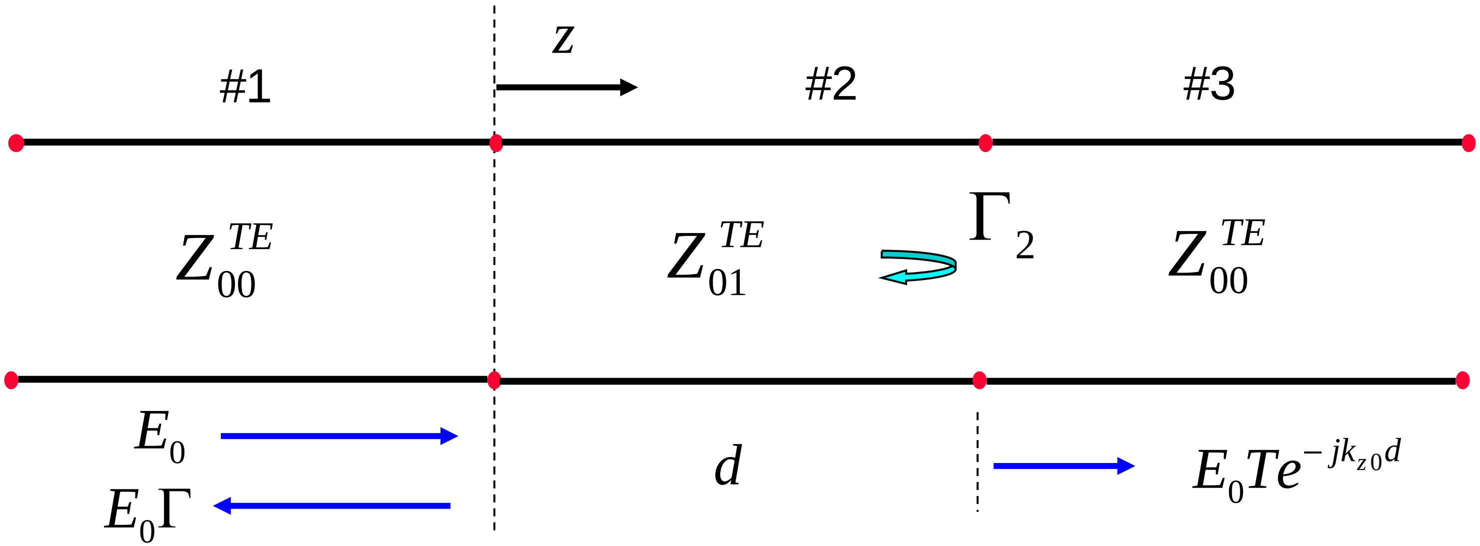
$$V_2(z) = A\left(e^{-jk_{z1}(z-d)} + \Gamma_2\, e^{+jk_{z1}(z-d)}\right)$$

At $z = 0$: $V_2(0^+) = V_1(0^-) \Rightarrow A\left(e^{+jk_{z1}d} + \Gamma_2 e^{-jk_{z1}d}\right) = E_0(1+\Gamma)$

Hence

$$A = E_0\left(\frac{1+\Gamma}{\left(e^{+jk_{z1}d} + \Gamma_2 e^{-jk_{z1}d}\right)}\right)$$

TEN (cont.)

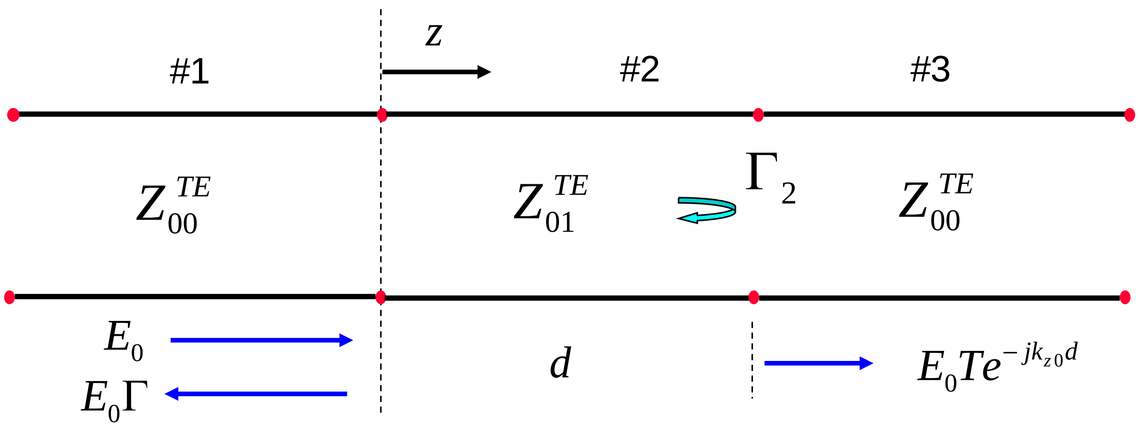


$$V_2(z) = A\left(e^{-jk_{z1}(z-d)} + \Gamma_2 \, e^{+jk_{z1}(z-d)}\right)$$ (now known)

We then have, on the output side:

$$V\left(d^+\right) = V\left(d^-\right) = V_2\left(d\right) = A\left(1 + \Gamma_2\right)$$

TEN (cont.)



Region 3: $V_3(z) = V(d)e^{-jk_{z0}(z-d)}$

Also $V_3(z) = E_0 T\, e^{-jk_{z0}z}$

where

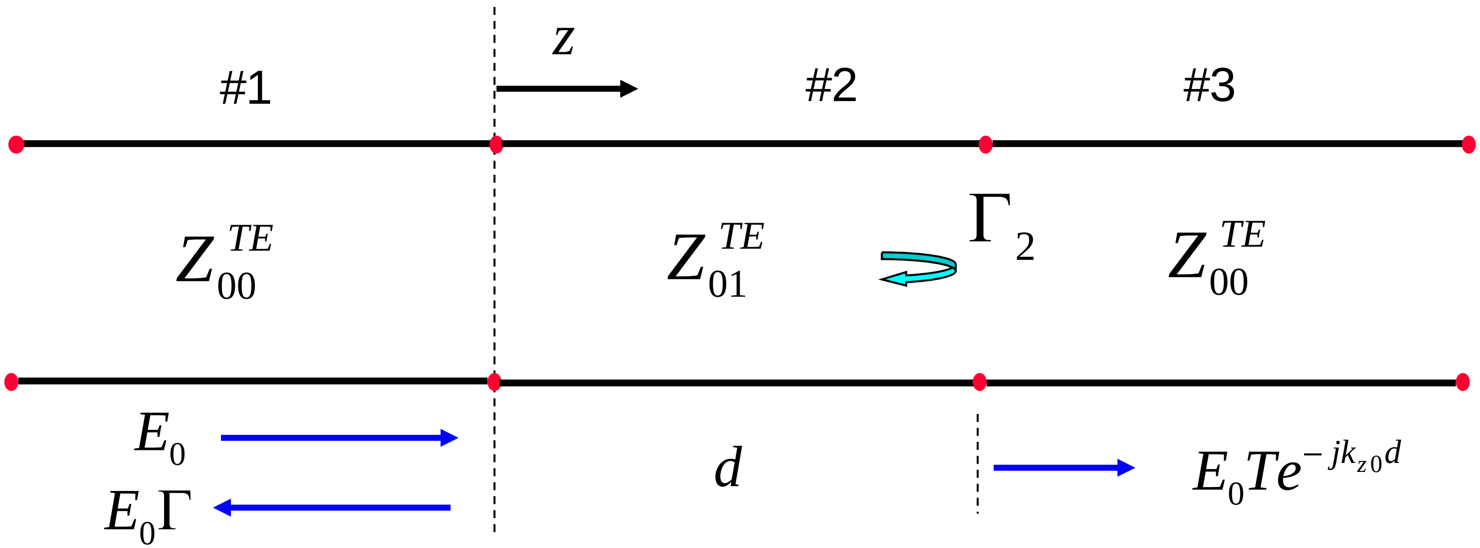
$$V(d) = V_2(d) = A(1+\Gamma_2)$$

$$A = E_0\left(\frac{1+\Gamma}{\left(e^{+jk_{z1}d} + \Gamma_2 e^{-jk_{z1}d}\right)}\right)$$

Hence

$$T = V(d)\left(\frac{1}{E_0}e^{+jk_{z0}d}\right)$$

TEN (cont.)



Final result:

$$T = \left(\frac{1+\Gamma}{\left(e^{+jk_{z1}d} + \Gamma_2 e^{-jk_{z1}d} \right)} \right) \left(1+\Gamma_2\right) e^{+jk_{z0}d}$$

Note: The phase reference point for the transmission coefficient is $z = 0$.