ECE 6340
Intermediate EM Waves

Fall 2016

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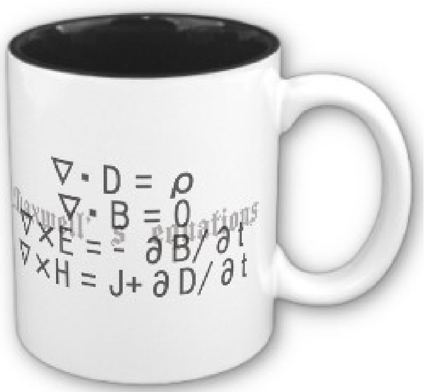
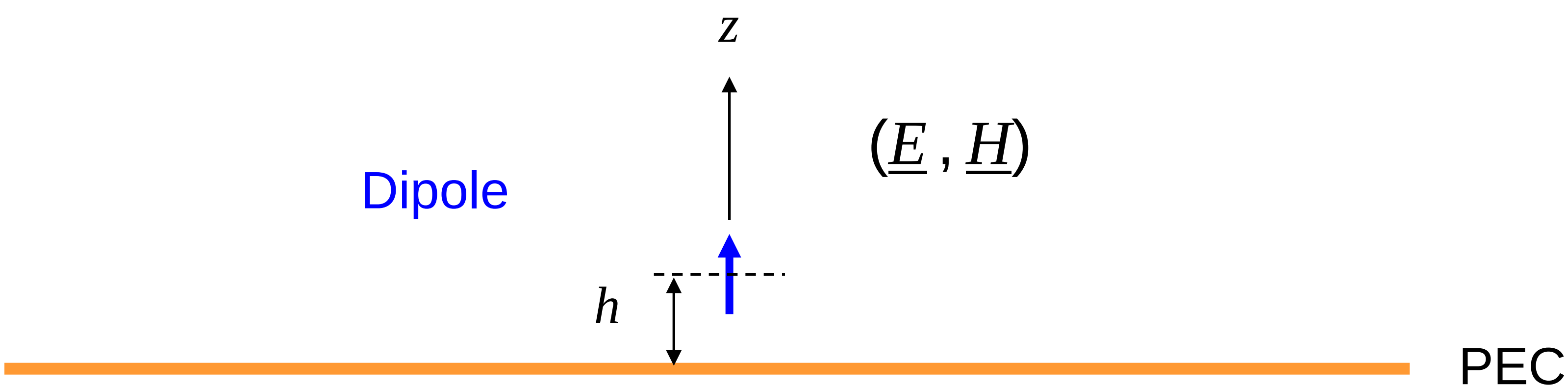
Dept. of ECE



Notes 25

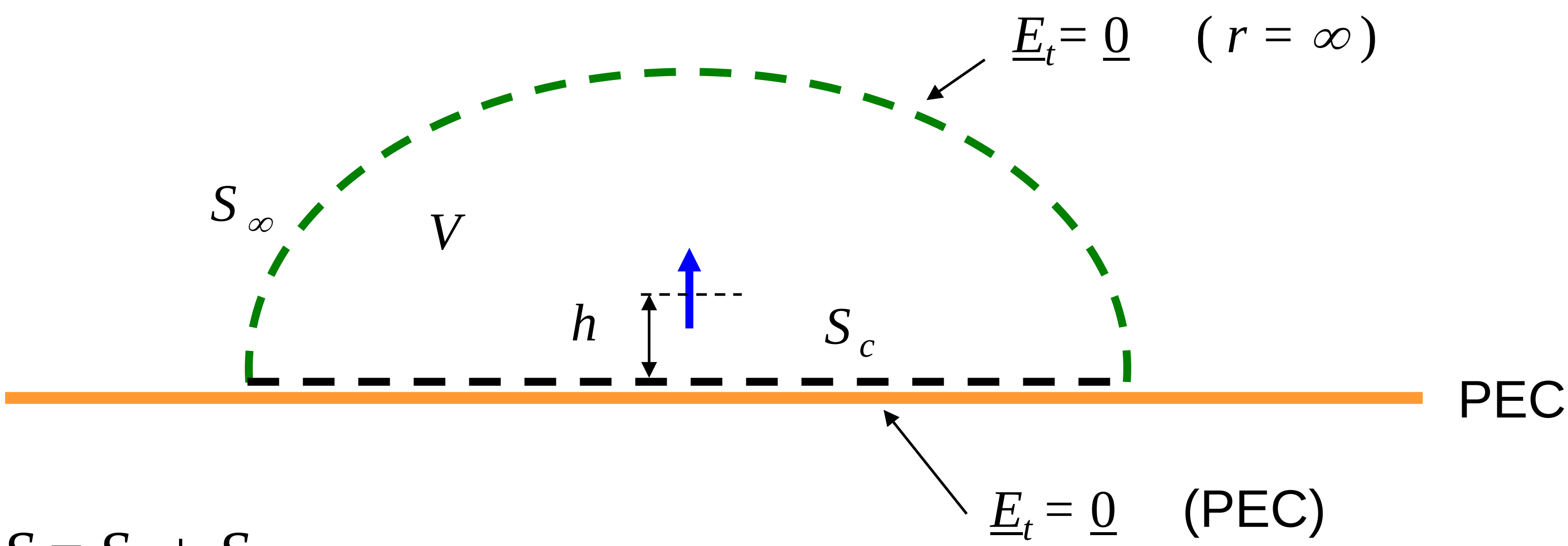
Image Theory

Vertical electric dipole (VED) over an infinite ground plane



$$\underline{E} = \underline{0} \quad \underline{H} = \underline{0}$$

Boundary Conditions



$$S = S_c + S_\infty$$

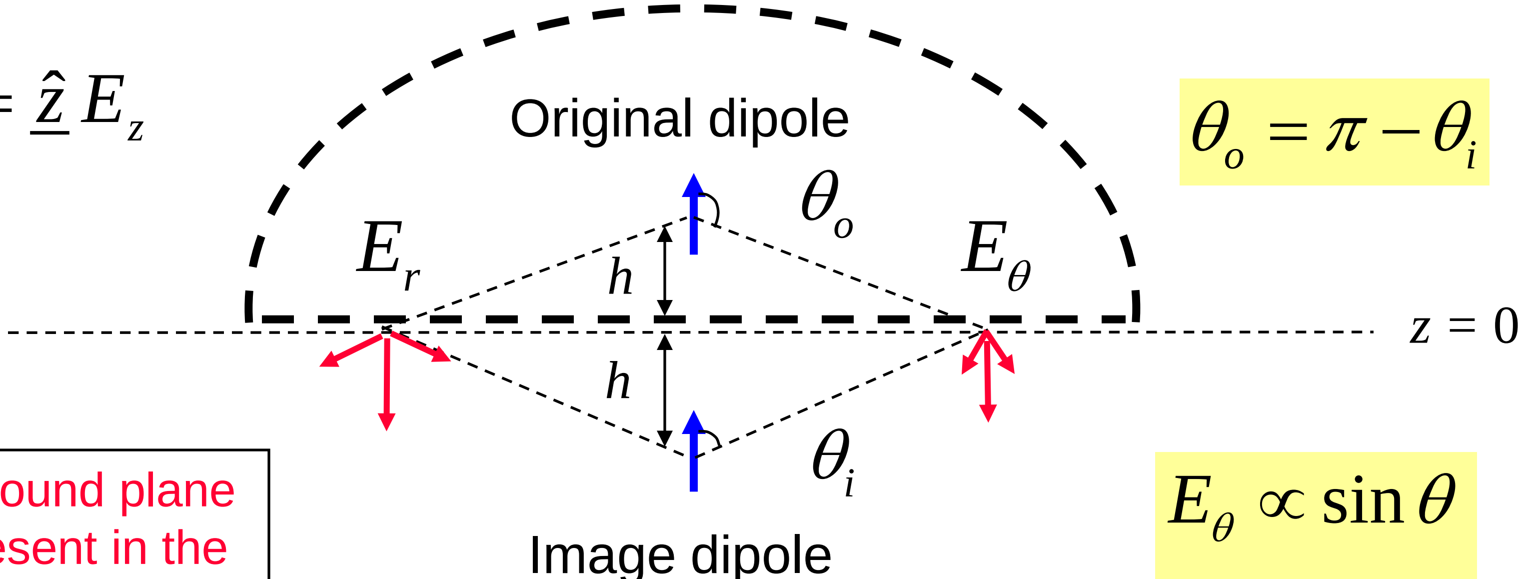
(1) Source condition inside S: A single VED exists inside S.

(2) Boundary condition on S: $\underline{E}_t = \underline{0}$ everywhere on S.

Any solution that satisfies these two conditions must be correct inside S.

Image Picture

$\underline{E} = \underline{\hat{z}}\, E_z$



$$\theta_o = \pi - \theta_i$$

No ground plane is present in the image problem.

$$E_\theta \propto \sin\theta$$

$$E_r \propto \cos\theta$$

$$E_\theta^i = E_\theta^o$$

$$E_r^i = -E_r^o$$

Hence, at $z = 0$, $\underline{E}_t = \underline{0}$ on S_c

Also, $\underline{E}_t = \underline{0}$ on S_∞

Image Picture (cont.)

Hence, the image solution for $z > 0$ is the same as for the original problem.

Note:

For $z < 0$ the image solution is not valid because the source in the image problem in this region is not the same as the original source in this region (which has zero sources).

Vertical Magnetic Dipole

For a single electric dipole:

$$\underline{H} = \underline{\hat{\phi}}\, H_{\phi} \qquad H_{\phi} \propto \sin\theta$$

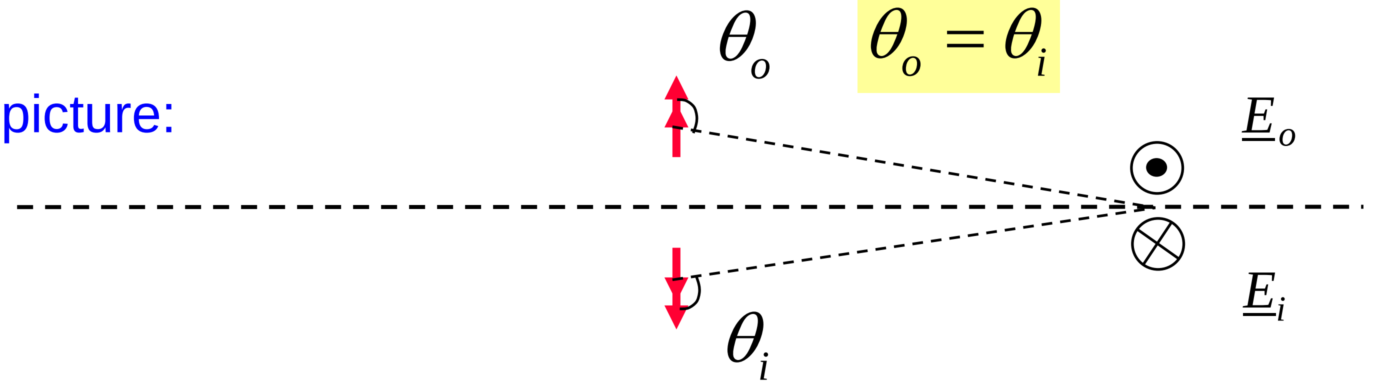
Duality:

For a single magnetic dipole:

$$\underline{E} = \underline{\hat{\phi}}\, E_{\phi} \qquad E_{\phi} \propto \sin\theta$$

Vertical Magnetic Dipole (cont.)

Image picture:



$$\theta_o = \theta_i$$

For a single magnetic dipole:

$$\underline{E} = \underline{\hat{\phi}}\, E_\phi \qquad E_\phi \propto \sin\theta$$

so $E_{\phi i} = E_{\phi o}$

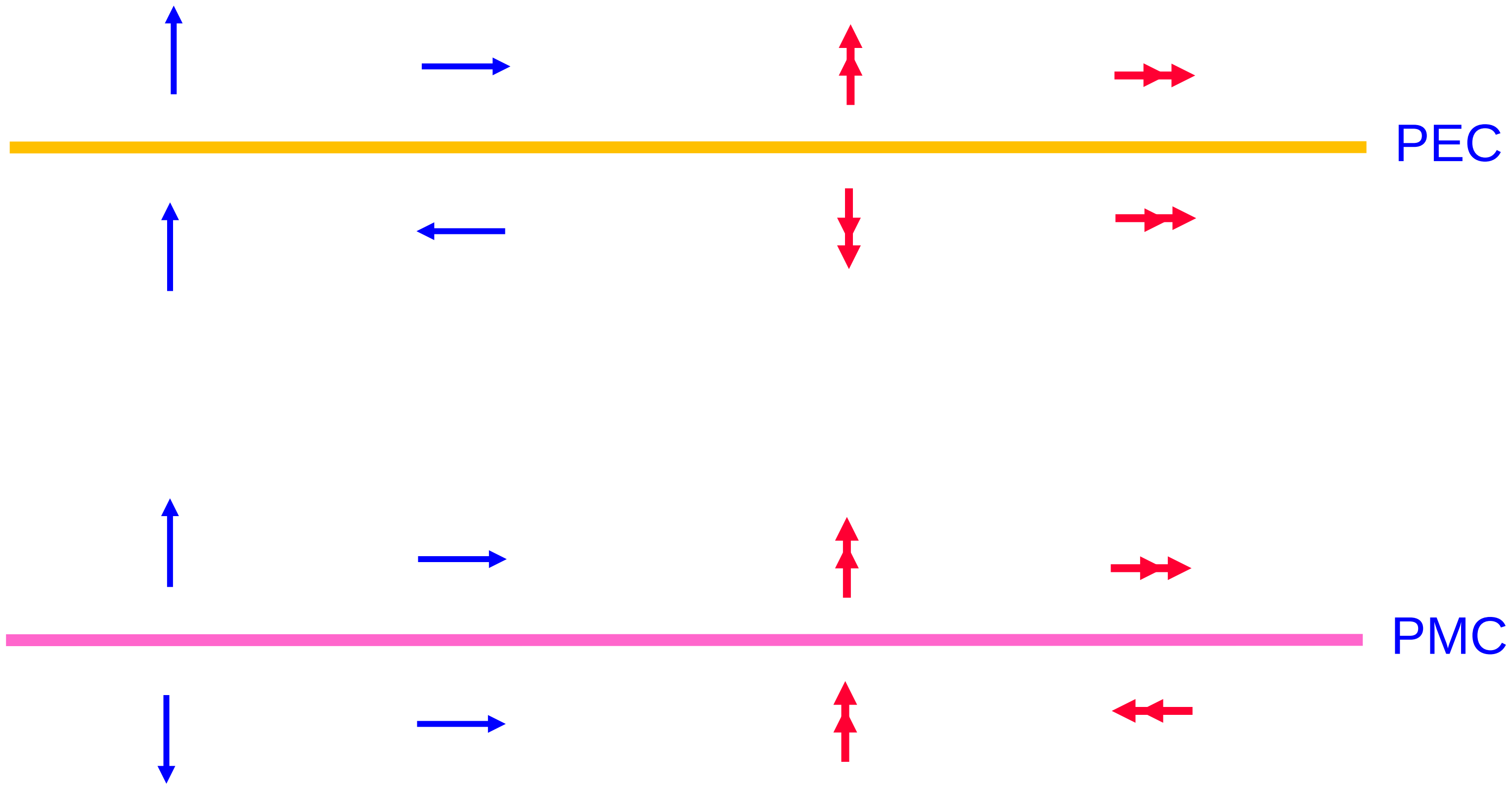
but $\underline{\hat{\phi}}_o = -\underline{\hat{\phi}}_i$

Therefore

At $z = 0$: $\underline{E}_i = -\underline{E}_o$

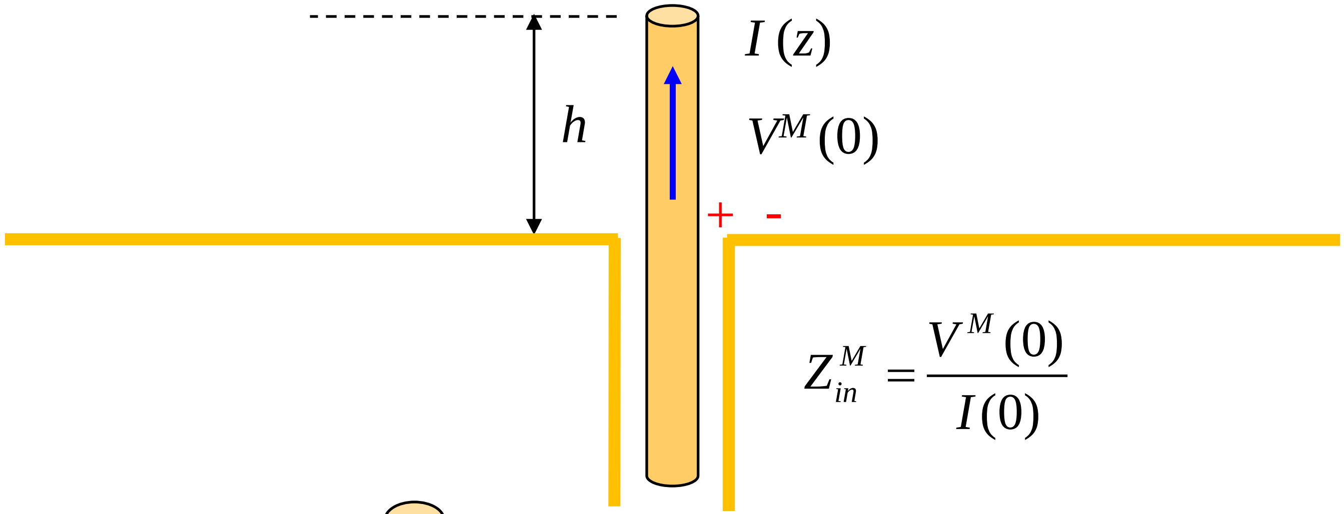
Hence $\underline{E}_t = \underline{0}$ at $z = 0$

All Possible Cases



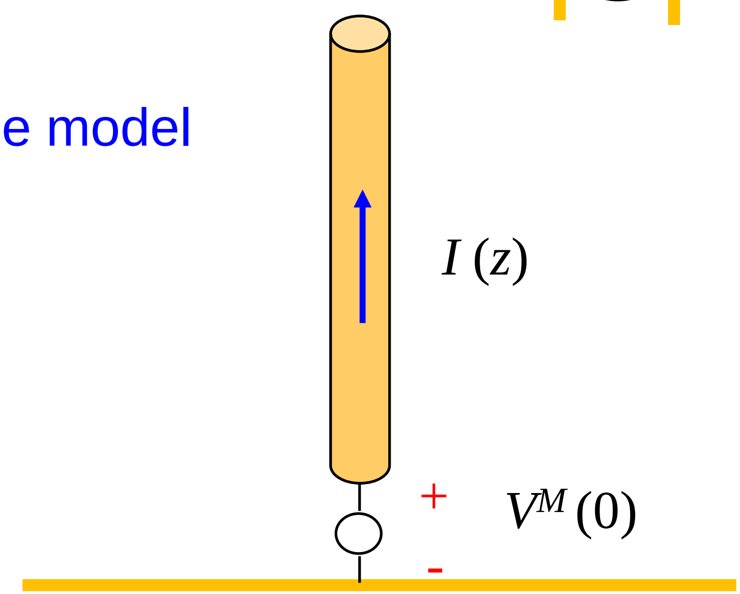
The PMC cases can be obtained from the PEC cases by duality.

Example: Monopole Antenna



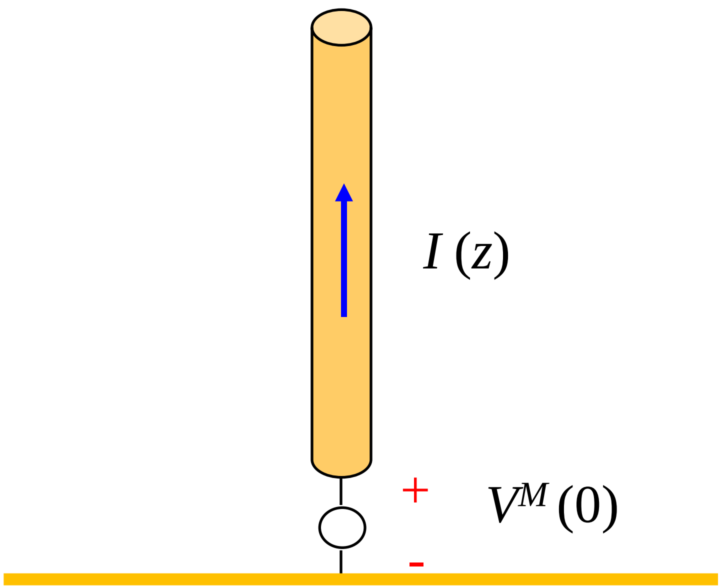
$$Z_{in}^{M} = \frac{V^{M}(0)}{I(0)}$$

Gap voltage model

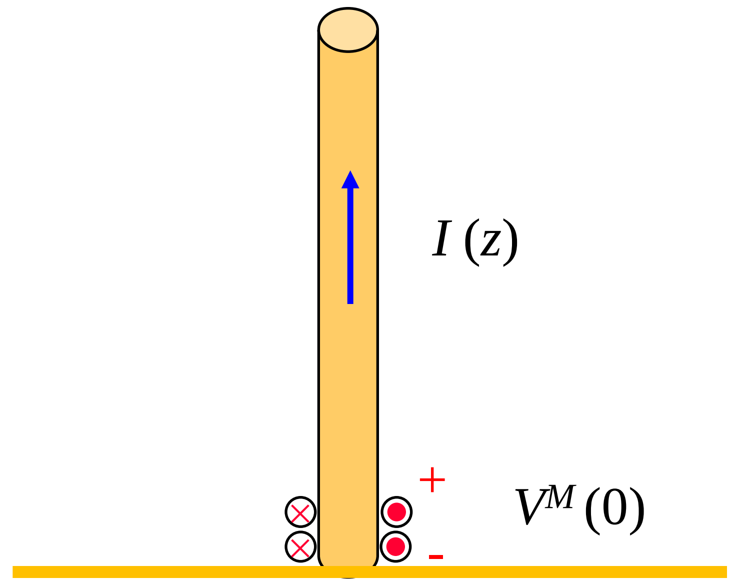


Example: Monopole Antenna

Gap voltage model



Magnetic frill model

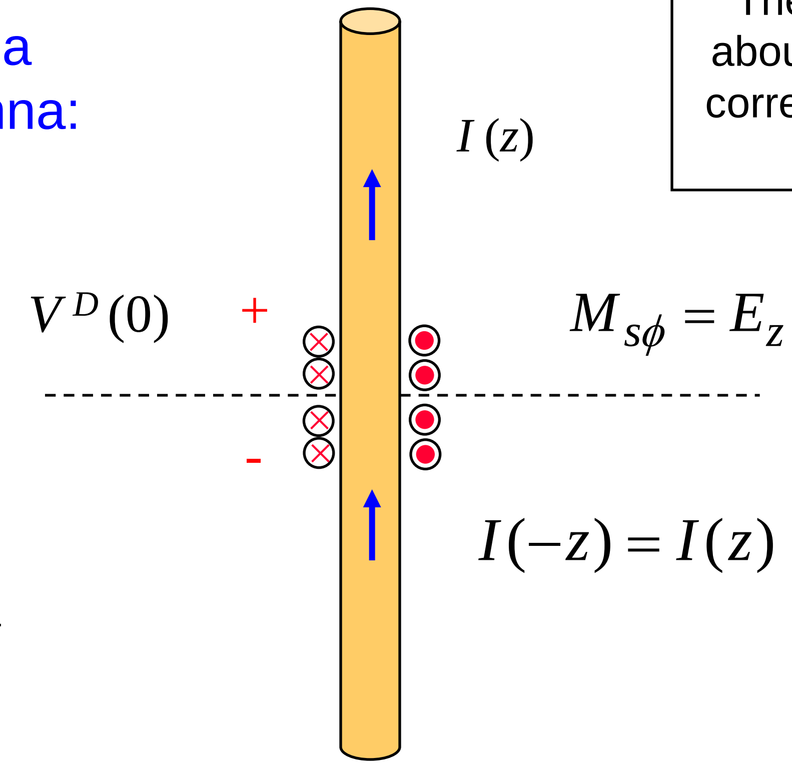


$$\underline{M}_s = -\underline{\hat{n}} \times \underline{E} = -\underline{\hat{\rho}} \times \underline{E}$$

$$M_{s\phi} = E_z$$

Monopole Antenna (cont.)

The image picture is a physical dipole antenna:



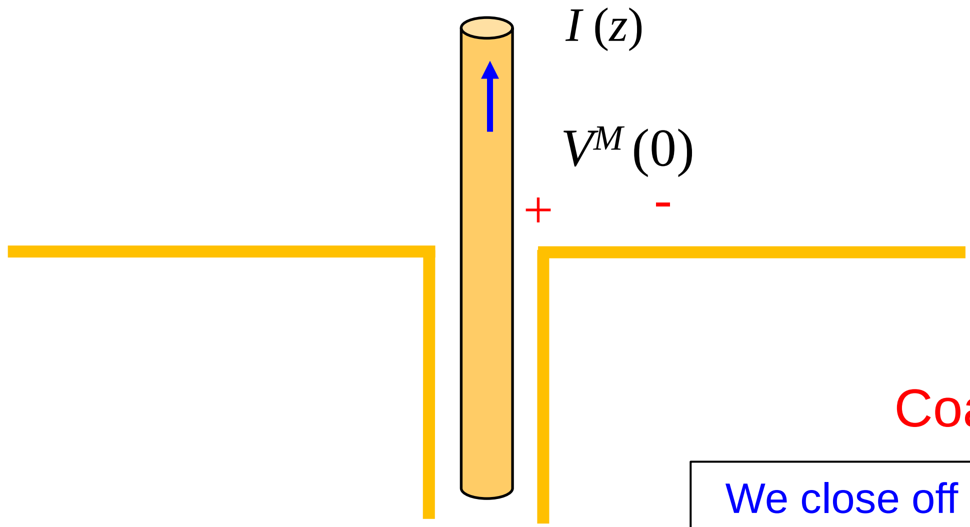
The fields are symmetric about $z = 0$, and satisfy the correct boundary conditions on the entire wire.

$$Z_{in}^{D} = \frac{V^{D}(0)}{I(0)} = \frac{2V^{M}(0)}{I(0)}$$

$$Z_{in}^{D} = 2Z_{in}^{M}$$

$$Z_{in}^{M} = 36.5\ [\Omega] \quad @\ h = \frac{\lambda_0}{4}$$

Monopole Antenna (cont.)

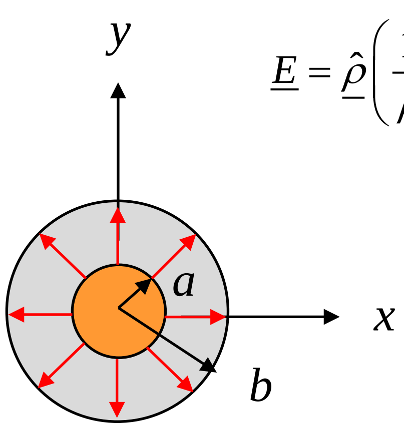


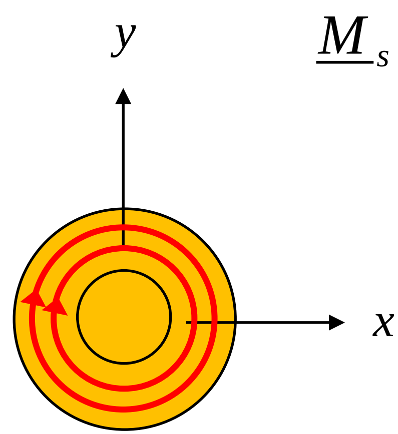
Coaxial frill model

We close off the ground plane and put a magnetic surface current ("frill") where the aperture used to be.

At $z = 0$:

$$\underline{E} = \hat{\underline{\rho}}\left(\frac{1}{\rho}\right)\left(\frac{V_0^M}{\ln\left(\frac{b}{a}\right)}\right)$$



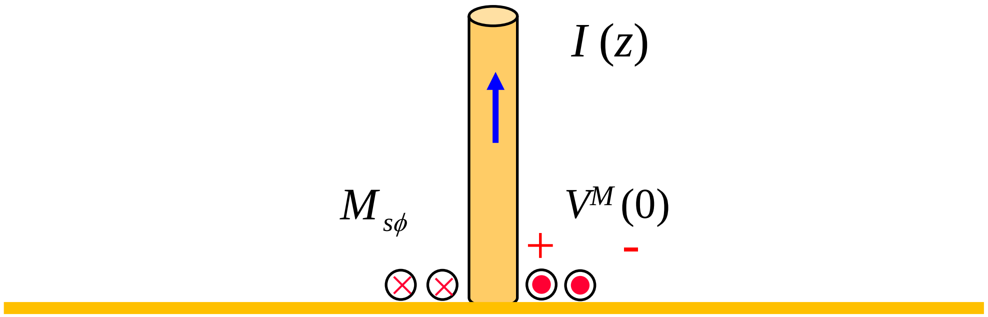


$$\underline{M}_s = -\hat{\underline{n}} \times \underline{E} = -\hat{\underline{z}} \times \underline{E}$$

$$M_{s\phi} = -E_\rho$$

Monopole Antenna (cont.)

Coaxial frill model

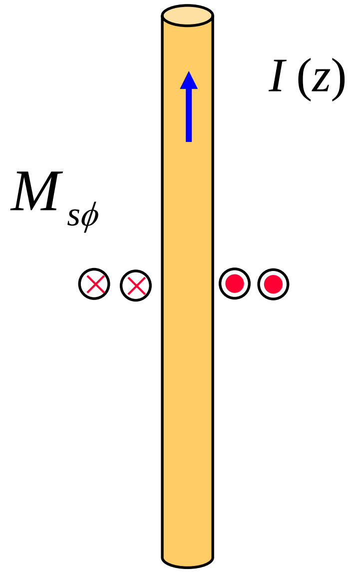


$$M_{s\phi} = -\left(\frac{1}{\rho}\right)\left(\frac{V_0^M}{\ln\left(\frac{b}{a}\right)}\right)$$

This is an accurate model of the coax feed for modeling purposes (often used in antenna theory).

Monopole Antenna (cont.)

After image theory:



Electric Field Integral Equation (EFIE):

$$\underline{E}_t\left[\underline{J}_s\right] = -\underline{E}_t\left[M_{s\phi}\right]$$

$$\underline{J}_s \approx \underline{\hat{z}}\left(\frac{I(z)}{2\pi a}\right)$$

$$M_{s\phi} = -2\left(\frac{1}{\rho}\right)\left(\frac{V_0^M}{\ln\left(\frac{b}{a}\right)}\right)$$

The factor of 2 is from image theory.

Corner Reflector

Corner reflector:

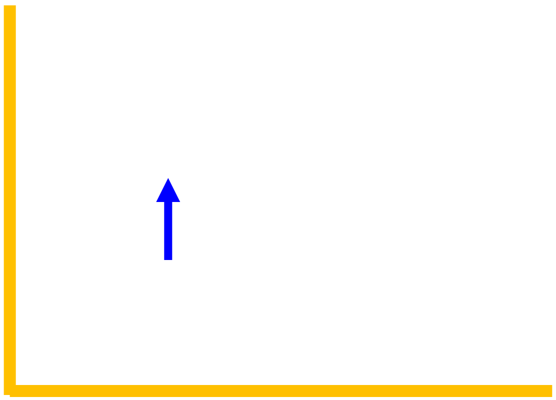
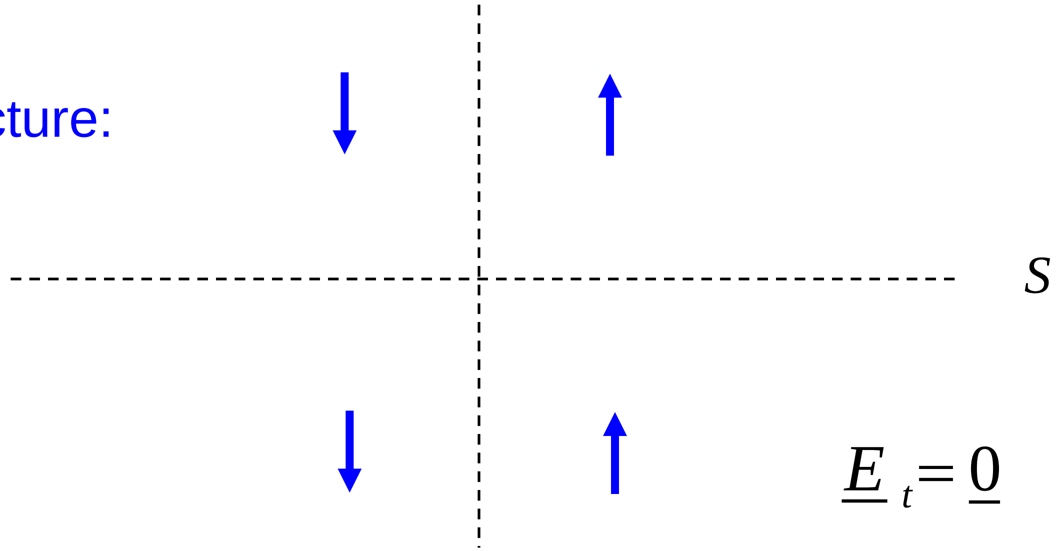


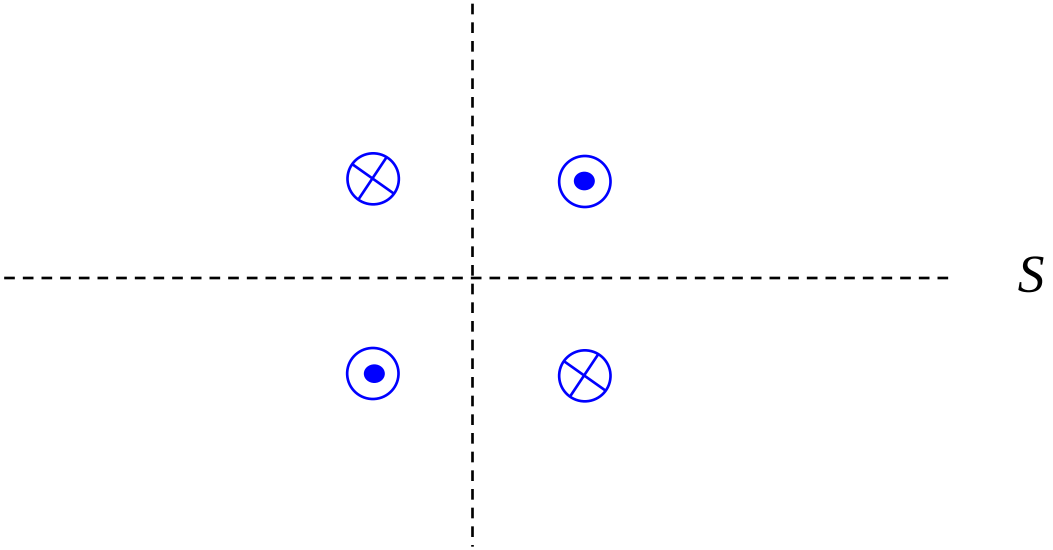
Image picture:



$$\underline{E}_t = \underline{0} \quad \text{on} \quad S$$

Corner Reflector (cont.)

Also, we can have this orientation:



Dipole in a Rectangular Waveguide