ECE 6340
Intermediate EM Waves

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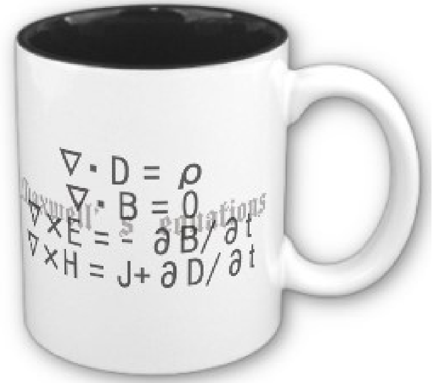
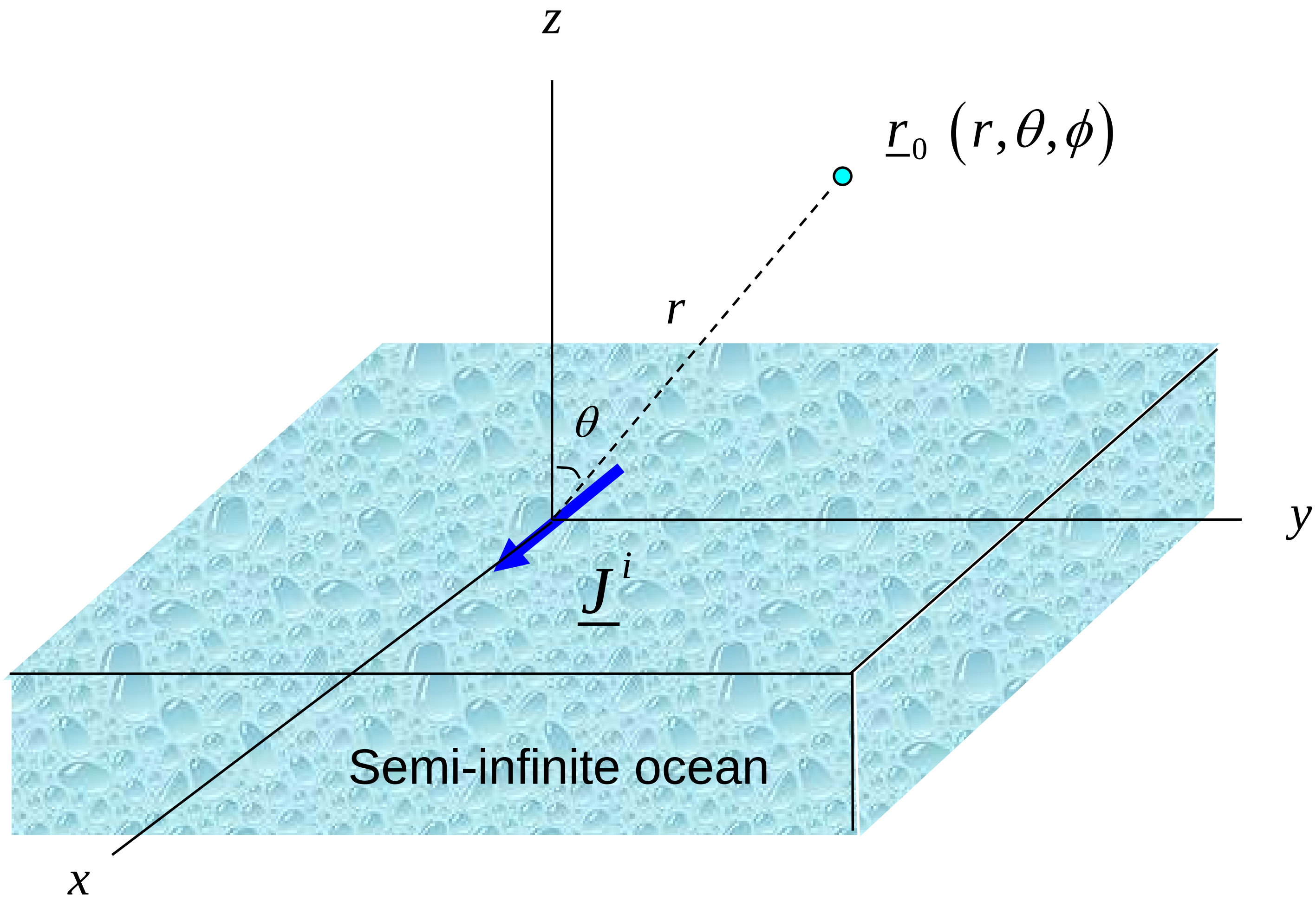


Notes 28

Dipole on Ocean

A dipole is located at the origin on the surface of the ocean.

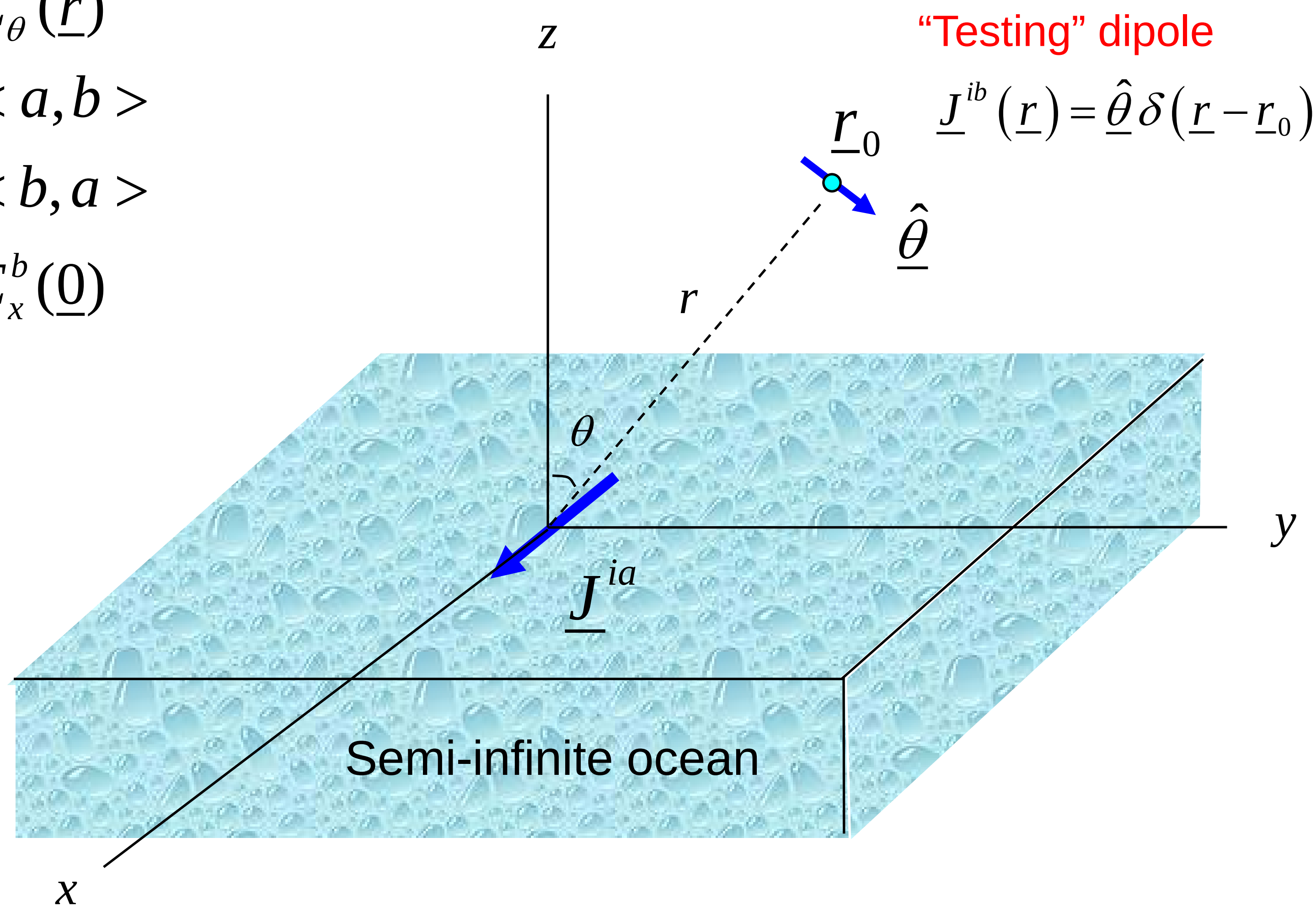
Goal: Calculate the far field of the dipole.



Dipole on Ocean (cont.)

The method is illustrated for E_θ.

$$
\begin{aligned}
E_\theta(\underline{r}) &= E_\theta^a(\underline{r}) \\
&= \langle a, b \rangle \\
&= \langle b, a \rangle \\
&= E_x^b(\underline{0})
\end{aligned}
$$

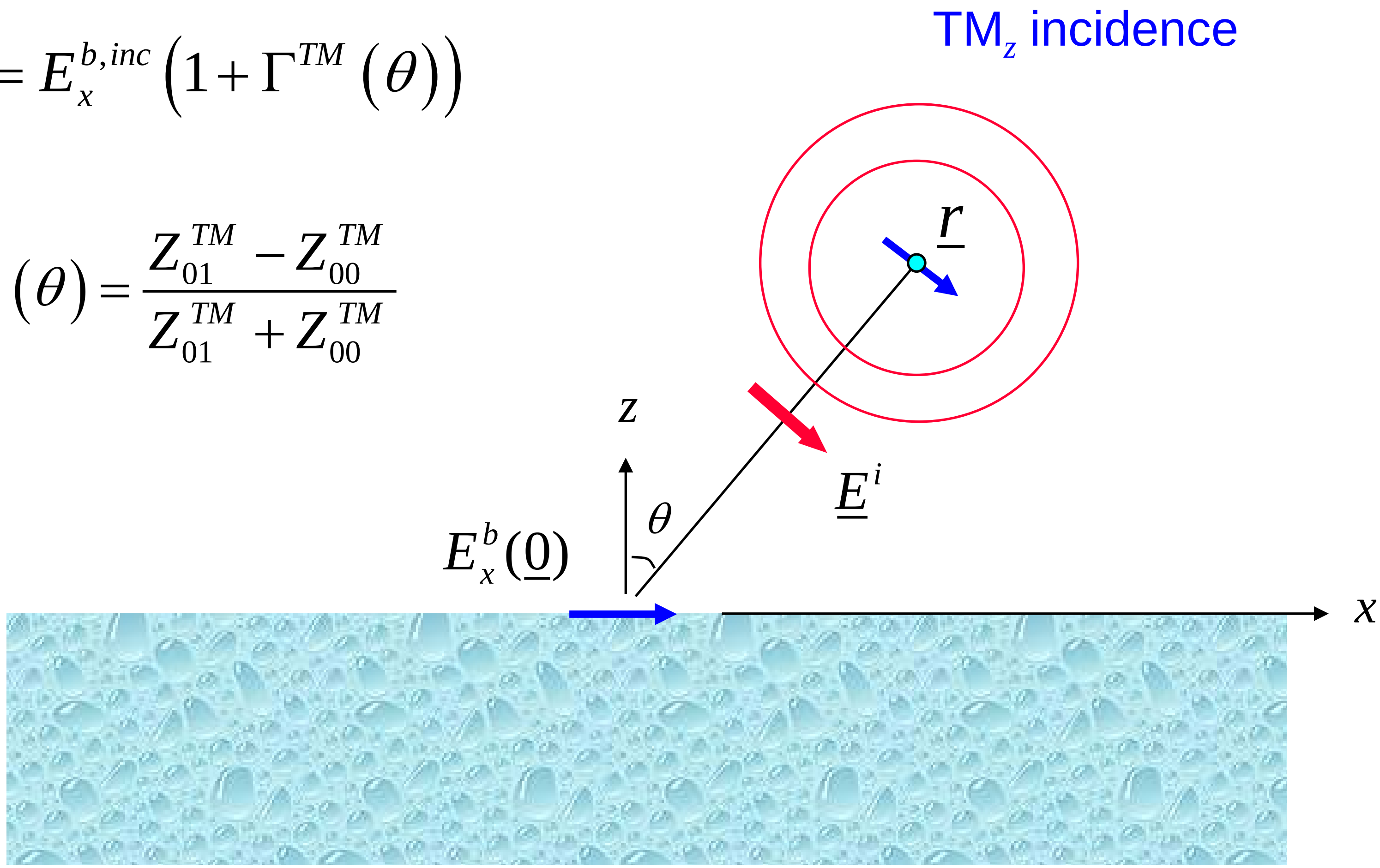


Dipole on Ocean (cont.)

From plane-wave theory we have:

$$E_x^b(\underline{0}) = E_x^{b,inc}\left(1+\Gamma^{TM}(\theta)\right)$$

$$\Gamma^{TM}(\theta) = \frac{Z_{01}^{TM} - Z_{00}^{TM}}{Z_{01}^{TM} + Z_{00}^{TM}}$$



Dipole on Ocean (cont.)

Incident field:

$$\underline{E}^{b,inc}\left(\underline{0}\right) = \underline{\hat{p}}\,E_0 = \underline{\hat{\theta}}\,E_0$$

where

$$E_0 = \left(\frac{-j\omega\mu_0}{4\pi r}\right)e^{-jk_0 r}$$

We have

$$\underline{\hat{\theta}}\cdot\underline{\hat{x}} = \cos\theta\cos\phi$$

Hence,

$$E_x^{b,inc}\left(\underline{0}\right) = E_0\cos\theta\cos\phi$$

Dipole on Ocean (cont.)

Hence:

$$E_\theta = E_0 \cos\theta \cos\phi \left(1 + \Gamma^{TM}(\theta)\right)$$

Note: The reflection coefficient depends only on θ and not ϕ.

A similar calculation can be done to determine the ϕ component of the radiated far field:

$$E_\phi = E_0 \left(-\sin\phi\right)\left(1 + \Gamma^{TE}(\theta)\right)$$

Note : $\underline{\hat{\phi}} \cdot \underline{\hat{x}} = -\sin\phi$

Dipole on Ocean (cont.)

$$\Gamma^{TM/TE} = \frac{Z_{01}^{TM/TE} - Z_{00}^{TM/TE}}{Z_{01}^{TM/TE} + Z_{00}^{TM/TE}}$$

where

$$Z_{01}^{TM} = \frac{k_{z1}}{\omega\varepsilon_1} = \frac{\sqrt{k_0^2\varepsilon_{rc}\mu_r - k_0^2\sin^2\theta}}{\omega\varepsilon_0\varepsilon_{rc}} = \eta_0\frac{\sqrt{\varepsilon_{rc}\mu_r - \sin^2\theta}}{\varepsilon_{rc}}$$

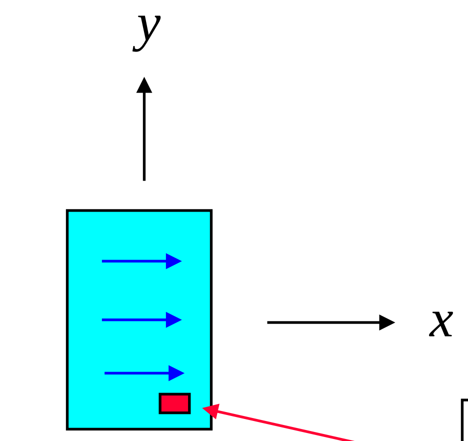
$$Z_{00}^{TM} = \eta_0\cos\theta$$

$$Z_{01}^{TE} = \frac{\omega\mu_1}{k_{z1}} = \frac{\omega\mu_0\mu_r}{\sqrt{k_0^2\varepsilon_{rc}\mu_r - k_0^2\sin^2\theta}} = \frac{\eta_0\mu_r}{\sqrt{\varepsilon_{rc}\mu_r - \sin^2\theta}}$$

$$Z_{00}^{TE} = \eta_0\sec\theta$$

Arbitrary Surface Current

$$E_0 = \left(\frac{-j\omega\mu_0}{4\pi R}\right)e^{-jk_0R} \approx \left(\frac{-j\omega\mu_0}{4\pi r}\right)e^{-jk_0r}e^{j\left(k_x x' + k_y y'\right)}$$



$J_{sx}\left(x', y'\right)$

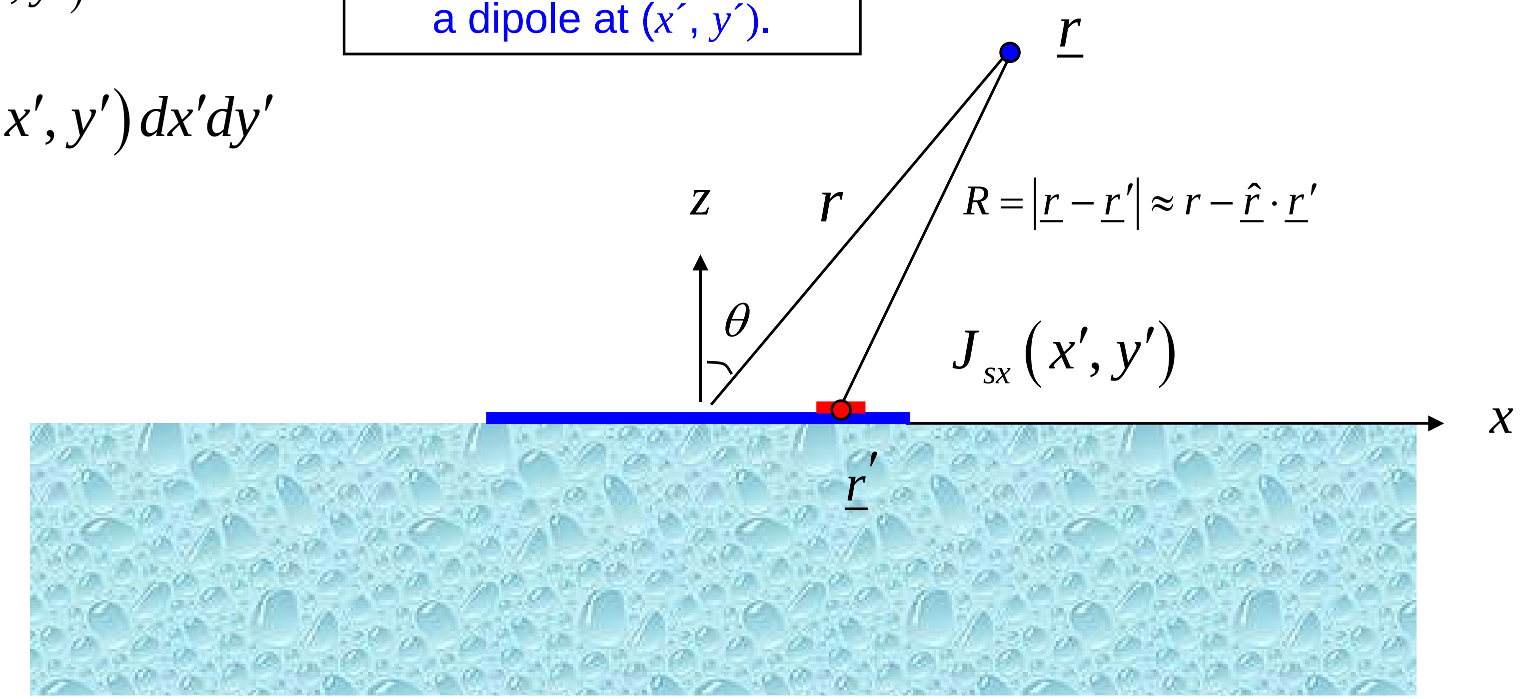
Each piece of the surface current acts like a dipole at (x', y').

$$Il = J_{sx}\left(x', y'\right)dx'dy'$$

$$e^{jk_0\left(\underline{r}'\cdot\underline{\hat{r}}\right)} = e^{j\left(k_x x' + k_y y'\right)}$$

$$k_x = k_0 \sin\theta\cos\phi$$

$$k_y = k_0 \sin\theta\sin\phi$$



$$R = \left|\underline{r} - \underline{r}'\right| \approx r - \underline{\hat{r}}\cdot\underline{r}'$$

$J_{sx}\left(x', y'\right)$

Arbitrary Surface Current (cont.)

$$E_\theta = E_0 \cos\theta \cos\phi \left(1 + \Gamma^{TM}(\theta)\right) \int_S J_{sx}(x', y') e^{j(k_x x' + k_y y')} dx' dy'$$

or

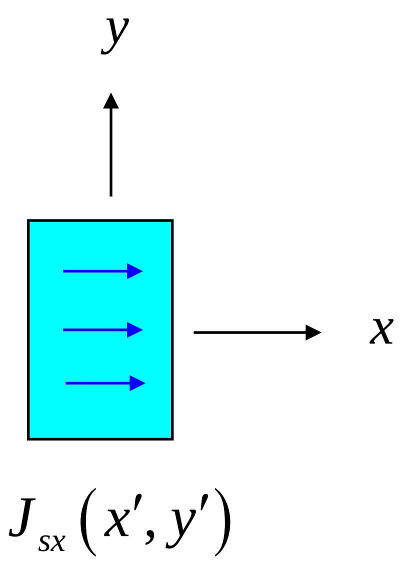
$$E_\theta = E_0 \cos\theta \cos\phi \left(1 + \Gamma^{TM}(\theta)\right) \tilde{J}_{sx}(k_x, k_y)$$

$$k_x = k_0 \sin\theta \cos\phi$$
$$k_y = k_0 \sin\theta \sin\phi$$

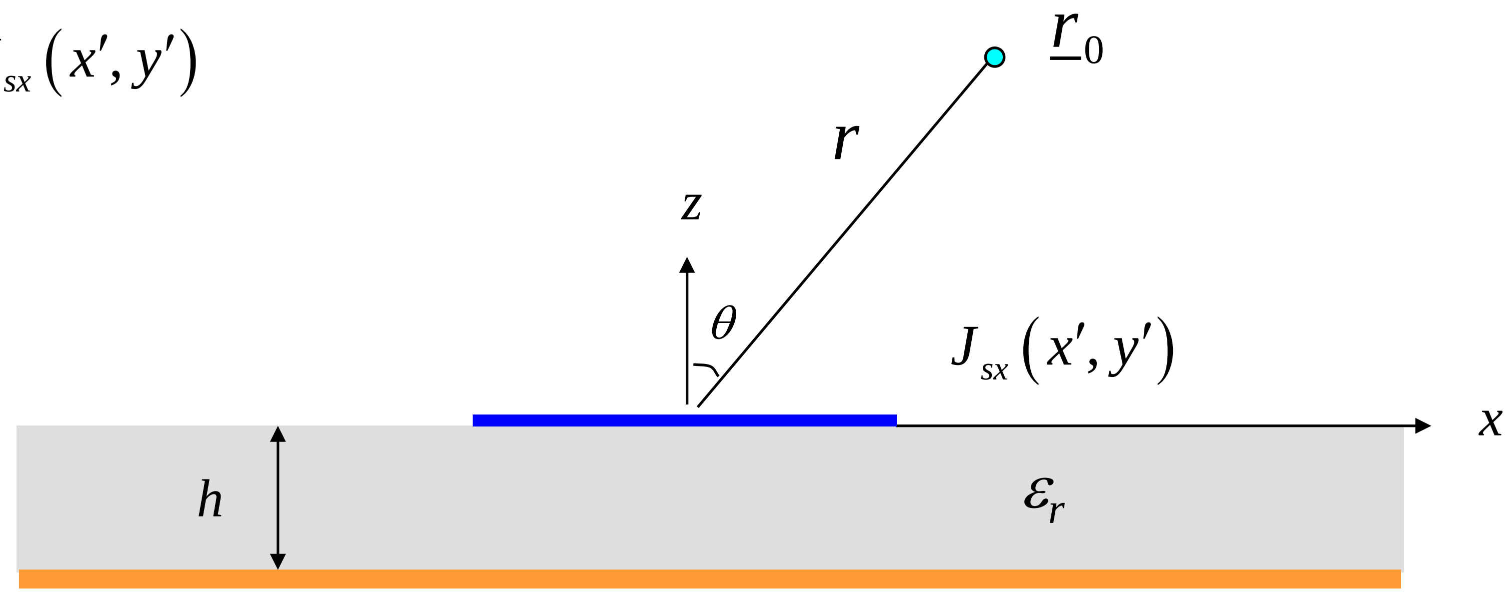
Similarly,

$$E_\phi = E_0 (-\sin\phi)\left(1 + \Gamma^{TE}(\theta)\right) \tilde{J}_{sx}(k_x, k_y)$$

Microstrip Antenna



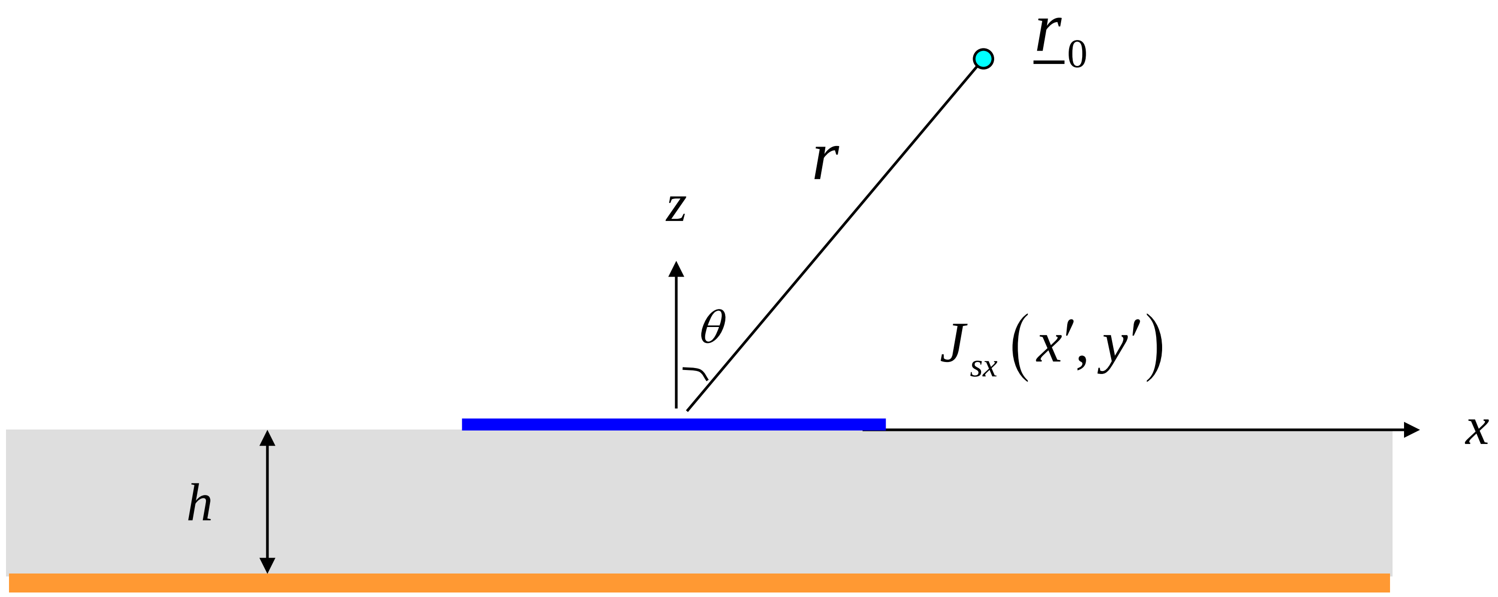
A microstrip patch antenna current is on top of a grounded dielectric substrate.



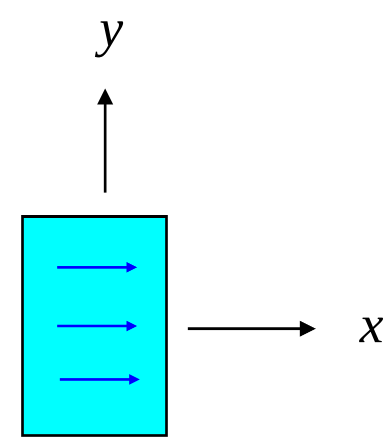
Microstrip Antenna (cont.)

$$E_{\theta} = E_0 \left(\cos\theta \cos\phi\right)\left(1+\Gamma^{TM}\left(\theta\right)\right) \tilde{J}_{sx}\left(k_x, k_y\right)$$

$$E_{\phi} = E_0 \left(-\sin\phi\right)\left(1+\Gamma^{TE}\left(\theta\right)\right) \tilde{J}_{sx}\left(k_x, k_y\right)$$



Microstrip Antenna (cont.)



$$J_{sx}\left(x',y'\right)$$

Dominant TM_{10} mode of the rectangular patch:

$$J_{sx}(x',y') = \cos\left(\frac{\pi x'}{L}\right)$$

$$\tilde{J}_{sx}(k_x,k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} J_{sx}(x',y')\, e^{+j(k_x x' + k_y y')}\, dx'\, dy'$$

$$= \int_{-W/2}^{W/2}\int_{-L/2}^{L/2} \cos\left(\frac{\pi x'}{L}\right) e^{+j(k_x x' + k_y y')}\, dx'\, dy'$$

$$= \int_{-L/2}^{L/2} \cos\left(\frac{\pi x'}{L}\right) e^{+j(k_x x')}\, dx' \int_{-W/2}^{W/2} e^{+j(k_y y')}\, dy'$$

Microstrip Antenna (cont.)

$$\tilde{J}_{sx}(k_x,k_y)=\int_{-L/2}^{L/2}\cos\left(\frac{\pi x'}{L}\right)e^{+j(k_x x')}dx'\int_{-W/2}^{W/2}e^{+j(k_y y')}dy'$$

Performing the integrations, we have

$$\tilde{J}_{sx}(k_x,k_y)=\left(\frac{\left(\frac{\pi L}{2}\right)\cos\left(k_x\frac{L}{2}\right)}{\left(\frac{\pi}{2}\right)^2-\left(\frac{k_x L}{2}\right)^2}\right)\left(W\,\mathrm{sinc}\left(\frac{k_y W}{2}\right)\right)$$

$$k_x=k_0\sin\theta\cos\phi$$
$$k_y=k_0\sin\theta\sin\phi$$

Microstrip Antenna (cont.)

Summary:

$$E_\theta = E_0 \left(\cos\theta\cos\phi\right)\left(1+\Gamma^{TM}\left(\theta\right)\right)\tilde{J}_{sx}\left(k_x,k_y\right)$$

$$E_\phi = E_0 \left(-\sin\phi\right)\left(1+\Gamma^{TE}\left(\theta\right)\right)\tilde{J}_{sx}\left(k_x,k_y\right)$$

$$\tilde{J}_{sx}(k_x,k_y) = \left(\frac{\left(\frac{\pi L}{2}\right)\cos\left(k_x\frac{L}{2}\right)}{\left(\frac{\pi}{2}\right)^2 - \left(\frac{k_x L}{2}\right)^2} \right)\left(W\,\mathrm{sinc}\left(\frac{k_y W}{2}\right)\right)$$

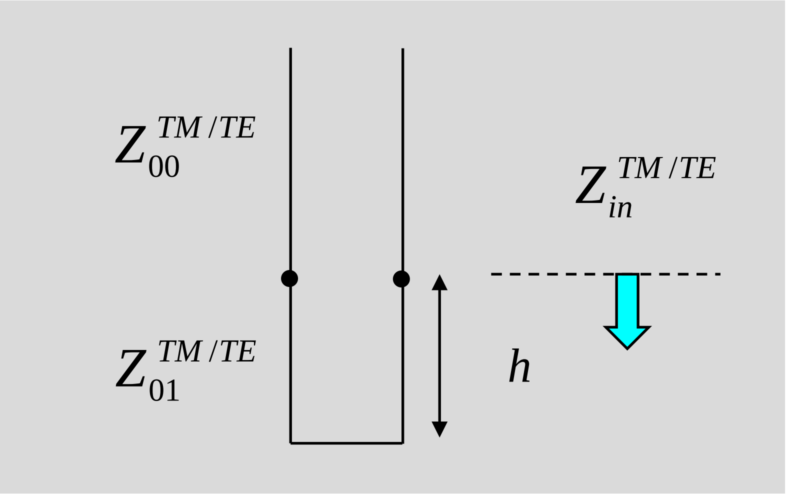
$$k_x = k_0\sin\theta\cos\phi$$

$$k_y = k_0\sin\theta\sin\phi$$

Microstrip Antenna (cont.)

Summary (cont.):

$$\Gamma^{TM/TE}(\theta) = \frac{Z_{in}^{TM/TE} - Z_{00}^{TM/TE}}{Z_{in}^{TM/TE} + Z_{00}^{TM/TE}}$$



$$Z_{in}^{TM/TE} = jZ_{01}^{TM/TE} \tan(k_{z1} h)$$

$$k_{z1} = k_0 \sqrt{\varepsilon_{rc}\mu_r - \sin^2\theta}$$

$$Z_{01}^{TM} = \eta_0 \frac{\sqrt{\varepsilon_{rc}\mu_r - \sin^2\theta}}{\varepsilon_{rc}}$$

$$Z_{01}^{TE} = \frac{\eta_0 \mu_r}{\sqrt{\varepsilon_{rc}\mu_r - \sin^2\theta}}$$

$$Z_{00}^{TM} = \eta_0 \cos\theta$$

$$Z_{00}^{TE} = \eta_0 \sec\theta$$