ECE 6340
Intermediate EM Waves

Fall 2016

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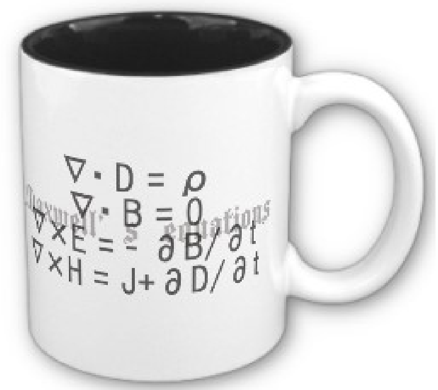
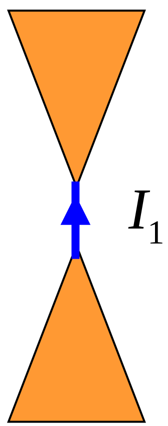
Dept. of ECE



Notes 29

Mutual Impedance

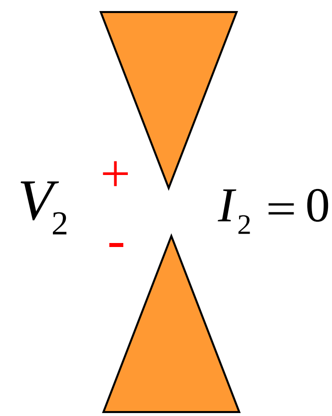
The mutual impedance Z_{21} between two antennas is calculated.



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

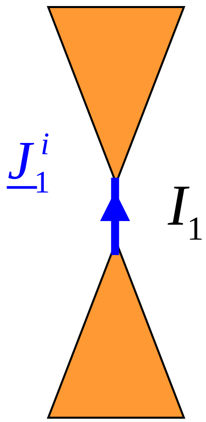
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2 = 0}$$



Mutual Impedance (cont.)

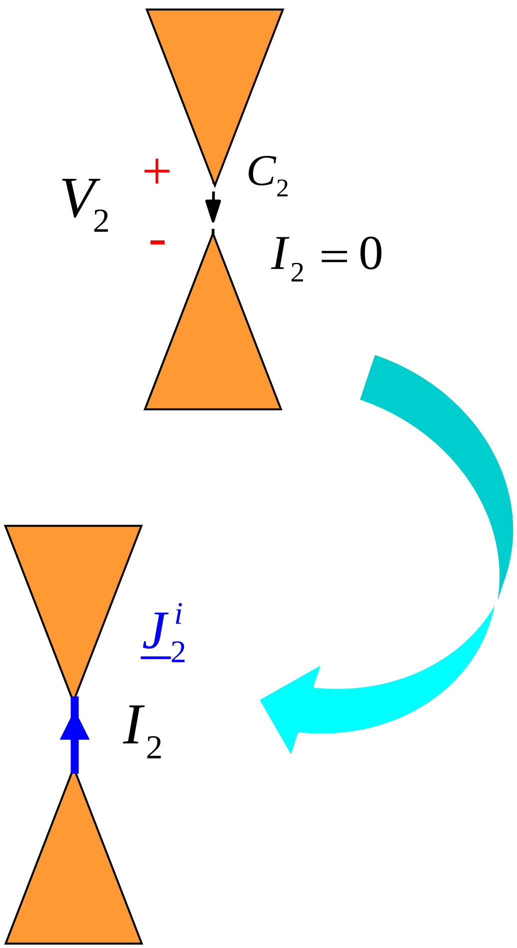
The open-circuit voltage V_2 is put in the form of a reaction.



$$V_2 = \int_{C_2} \underline{E}_1 \cdot d\underline{r}$$

$$= -\frac{1}{I_2} \int_V \underline{E}_1 \cdot \underline{J}_2^{\,i} \, dV$$

$$= -\frac{1}{I_2} \left\langle \underline{J}_1^{\,i}, \underline{J}_2^{\,i} \right\rangle$$



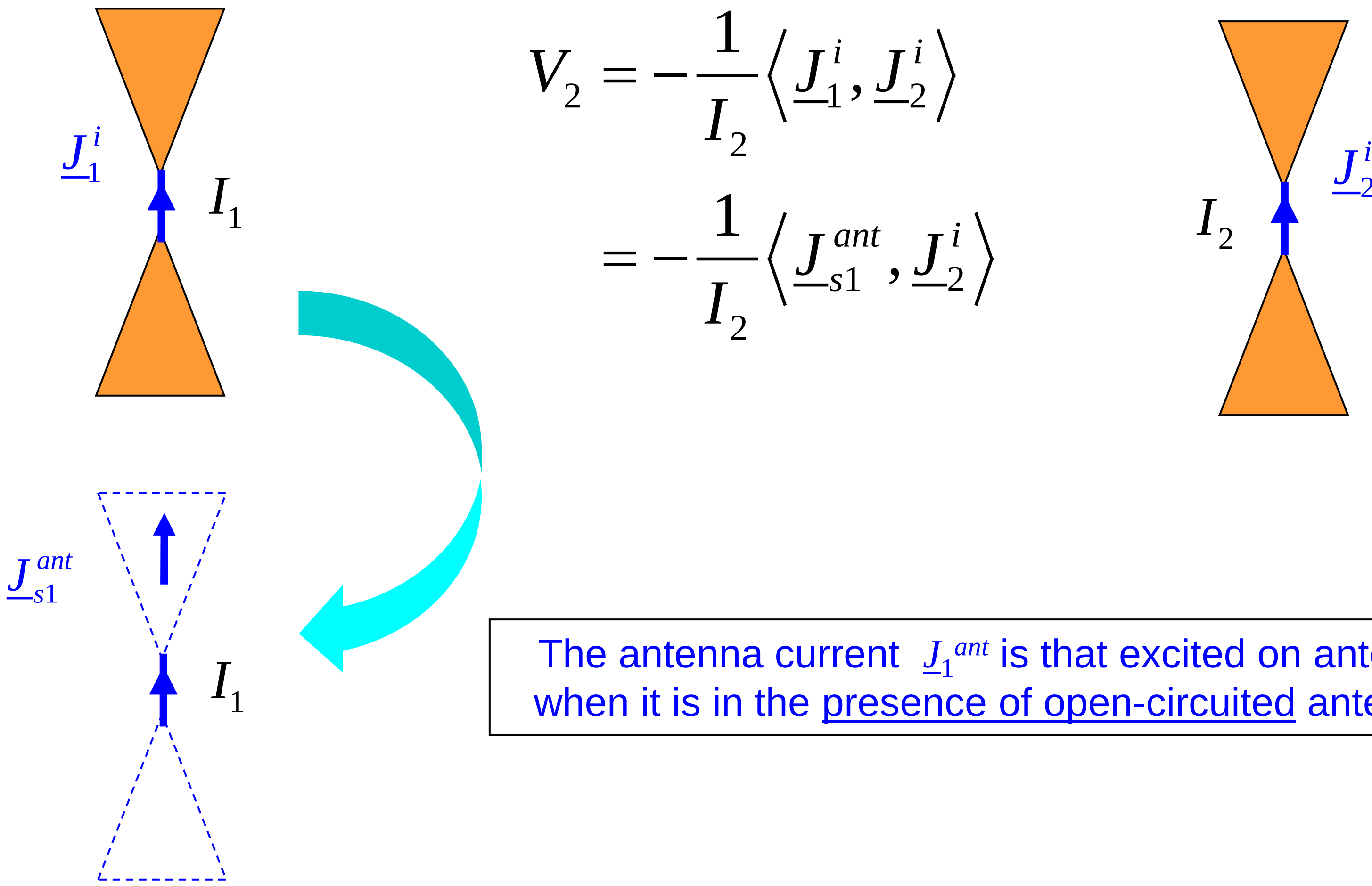
$\underline{E}_1$ = field radiated by $\underline{J}_1^{\,i}$ in the presence of <u>open-circuited</u> antenna 2.

Mutual Impedance (cont.)

The equivalence principle is used to replace antenna 1 with its surface current.

$$V_2 = -\frac{1}{I_2}\left\langle \underline{J}_1^{\,i}, \underline{J}_2^{\,i} \right\rangle$$

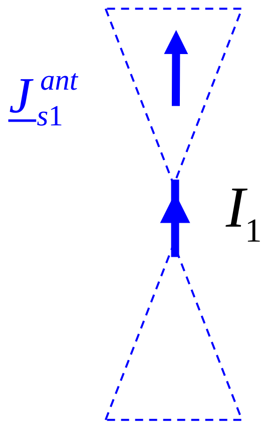
$$= -\frac{1}{I_2}\left\langle \underline{J}_{s1}^{\,ant}, \underline{J}_2^{\,i} \right\rangle$$



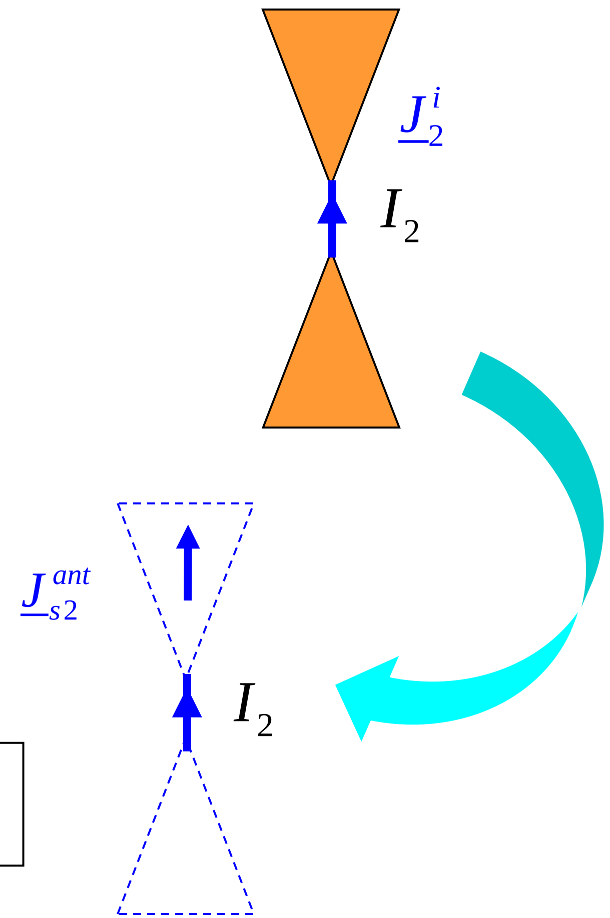
The antenna current $\underline{J}_1^{\,ant}$ is that excited on antenna 1 when it is in the presence of open-circuited antenna 2.

Mutual Impedance (cont.)

Reciprocity is invoked, and then the equivalence principle.



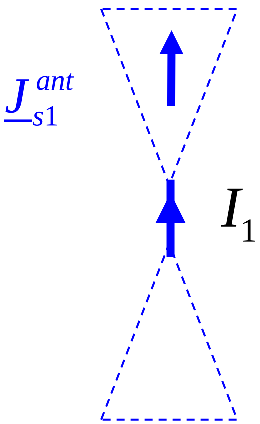
$$
\begin{aligned}
V_2 &= -\frac{1}{I_2}\left\langle \underline{J}_{s1}^{ant}, \underline{J}_2^{i} \right\rangle \\
&= -\frac{1}{I_2}\left\langle \underline{J}_2^{i}, \underline{J}_{s1}^{ant} \right\rangle \\
&= -\frac{1}{I_2}\left\langle \underline{J}_{s2}^{ant}, \underline{J}_{s1}^{ant} \right\rangle
\end{aligned}
$$



The antenna current $\underline{J}_2^{ant}$ is that excited by I_2 on antenna 2 when it is by itself (antenna 1 is absent).

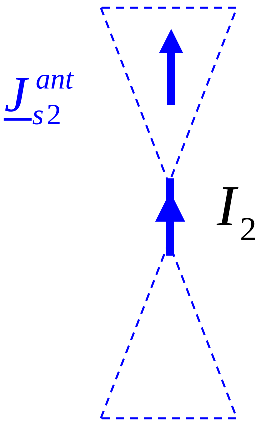
Mutual Impedance (cont.)

Reciprocity is invoked one more time.



$$V_2 = -\frac{1}{I_2}\left\langle \underline{J}_{s2}^{ant}, \underline{J}_{s1}^{ant} \right\rangle$$

$$= -\frac{1}{I_2}\left\langle \underline{J}_{s1}^{ant}, \underline{J}_{s2}^{ant} \right\rangle$$



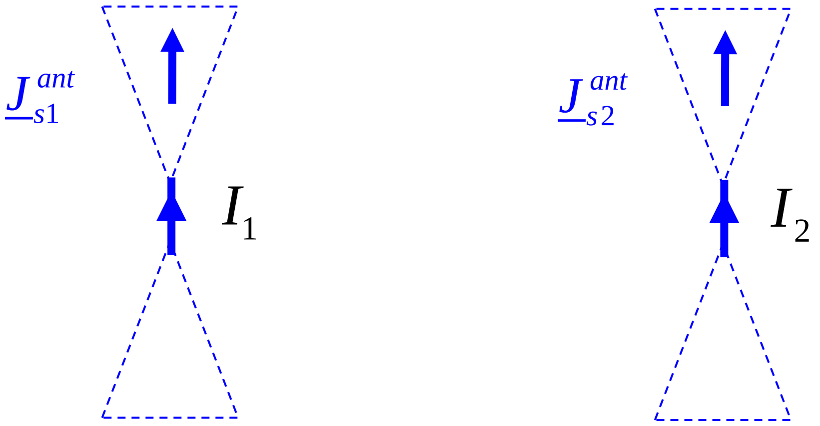
The mutual impedance is then

$$Z_{21} = -\frac{1}{I_1 I_2}\left\langle \underline{J}_{s1}^{ant}, \underline{J}_{s2}^{ant} \right\rangle$$

Mutual Impedance (cont.)

Summary

$$Z_{21} = -\frac{1}{I_1 I_2} \int_{S_2} \underline{E}\left[\underline{J}_{s1}^{ant}\right] \cdot \underline{J}_{s2}^{ant} \, dS$$



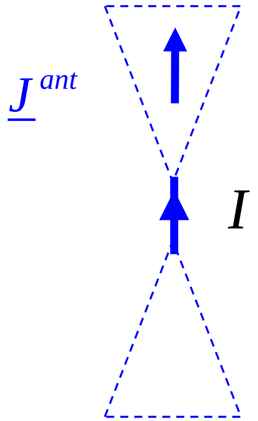
$\underline{J}_{s1}^{ant}$ = current on antenna 1 when it is excited by I_1 in the presence of open-circuited antenna 2.

$\underline{J}_{s2}^{ant}$ = current on antenna 2 when it is excited by I_2 in the absence of antenna 1.

Self Impedance

If the two currents come together to one surface, we have the self-impedance formula for a single antenna:

$$Z_{in} = -\frac{1}{I^2}\int_S \underline{E}\left[\underline{J}_s^{ant}\right]\cdot \underline{J}_s^{ant}\, dS$$



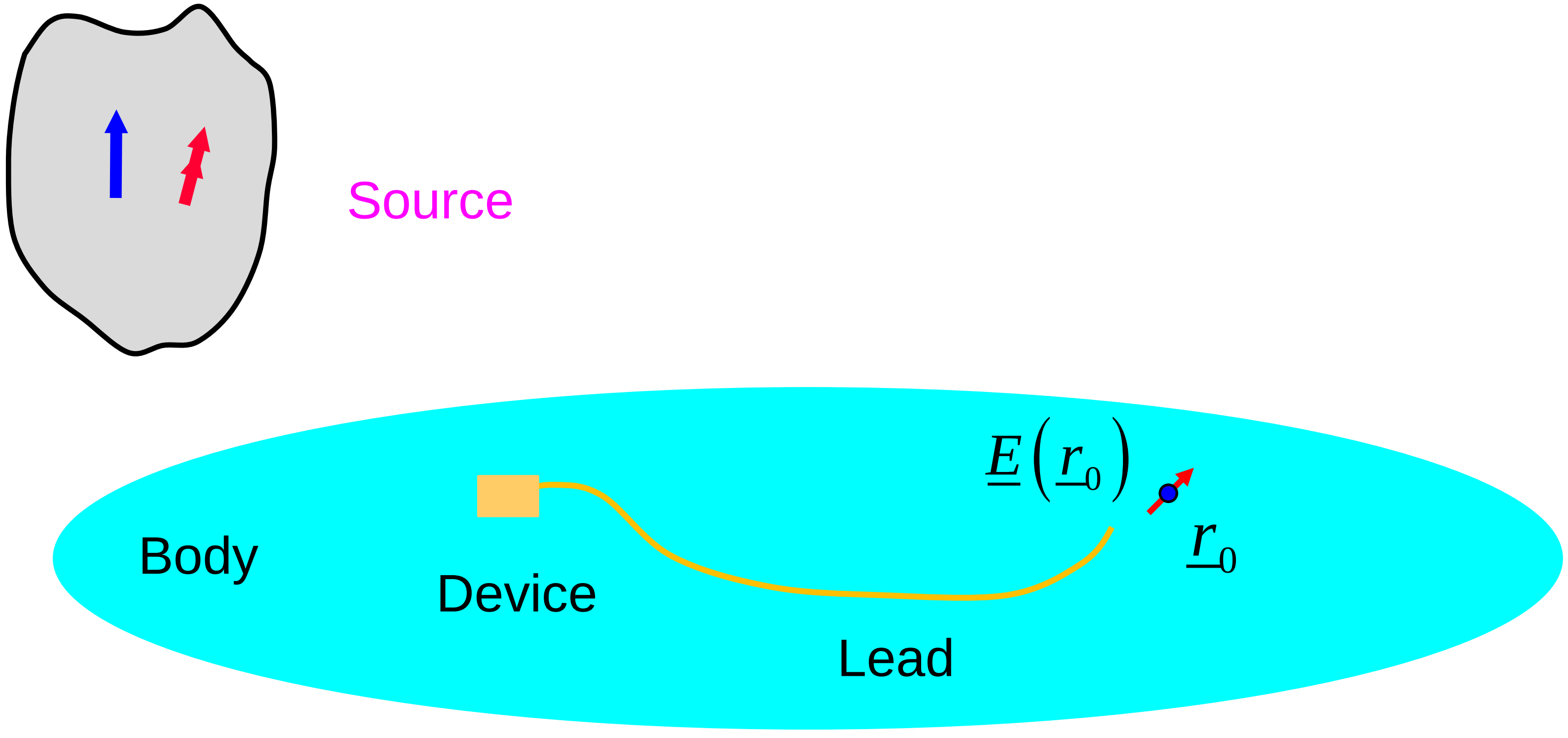
Note:
This is a variational expression for the input impedance of an antenna.

$\underline{J}^{ant}$ = current on the antenna when it is excited by I.

Transfer Function

The concept of a "transfer function" is very useful in biomedical problems, for calculating the electric field (and hence the heating) at a point inside a body.

The goal is to calculate the electric field inside the body at point $\underline{r}_0$, when an external source is radiating.

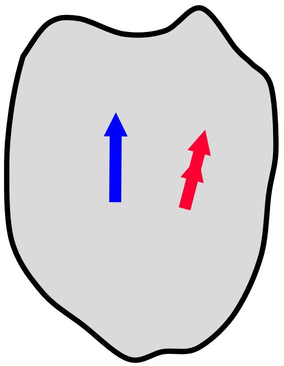


Transfer Function (cont.)

We use reciprocity, selecting the b source to be a <u>testing dipole</u> at the observation point, in the p direction.

$$E_p\left(\underline{r}_0\right) = \langle a, b \rangle$$

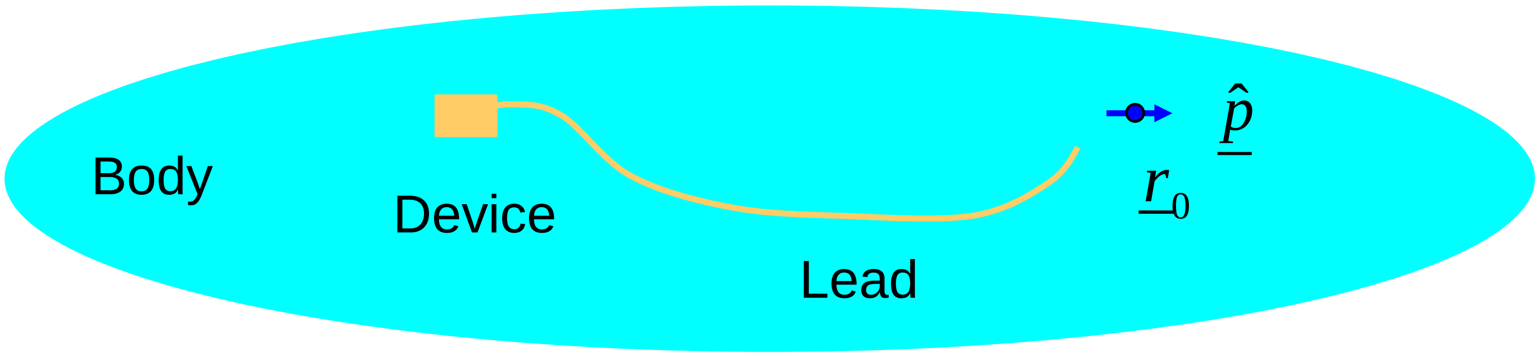
This is difficult to calculate numerically, since we have to discretize the body finely near the lead (the field from the a source is calculated with the body <u>and</u> the lead present).



a source

$$\underline{J}^{ib} = \underline{\hat{p}}\,\delta\left(\underline{r} - \underline{r}_0\right)$$

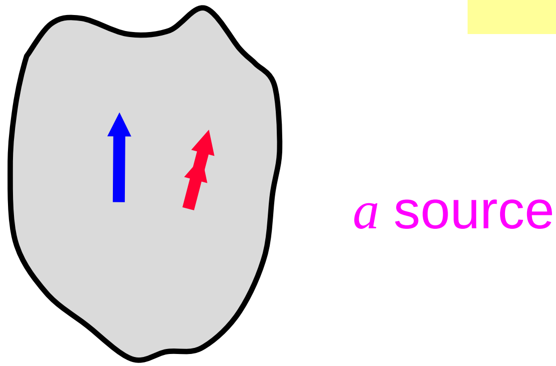
b source

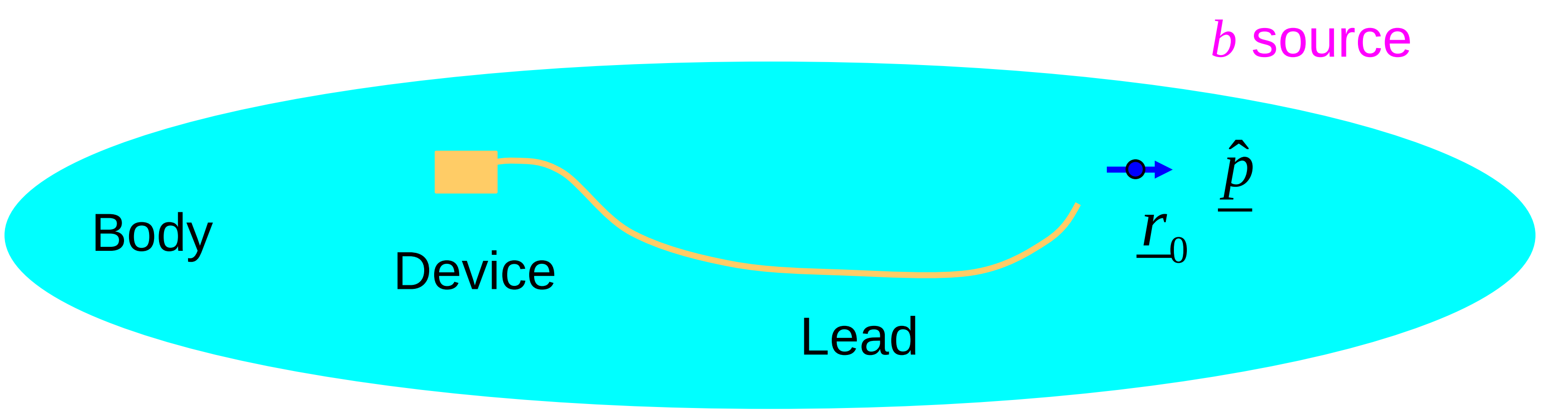


Transfer Function (cont.)

From reciprocity, we have

$$E_p\left(\underline{r}_0\right) = \langle b, a \rangle = \int_V \left(\underline{E}^b \cdot \underline{J}^{ia} - \underline{H}^b \cdot \underline{M}^{ia}\right) dV$$



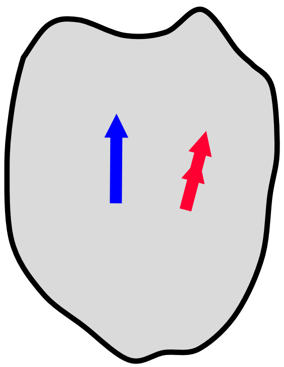


Transfer Function (cont.)

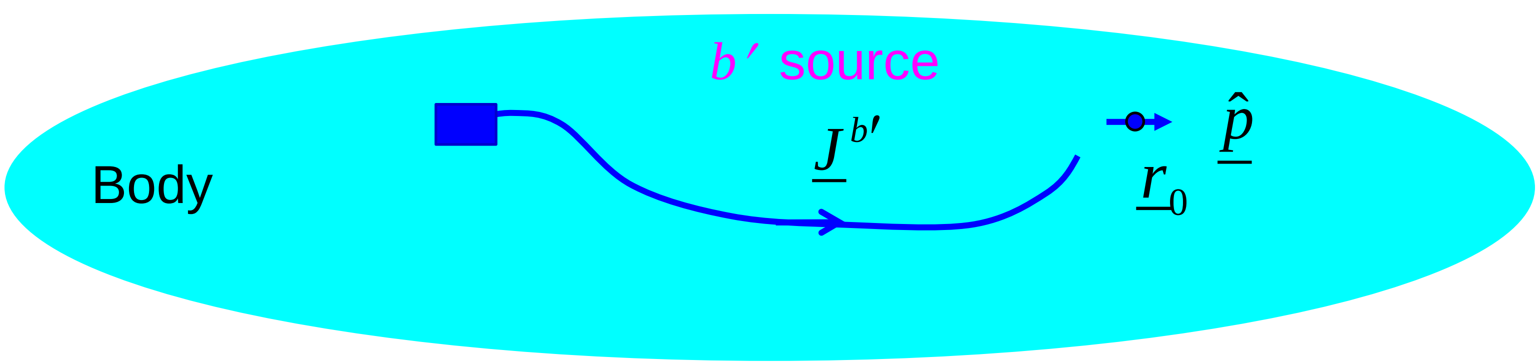
Next, apply the equivalence principle to <u>remove</u> the device and the lead, and keep the currents that are on them.

$$E_p\left(\underline{r}_0\right) = \langle b', a \rangle$$

The set of currents called b' consists of the dipole b plus the currents on the device and lead that are set up by the radiating dipole b (in the absence of source a).



a source



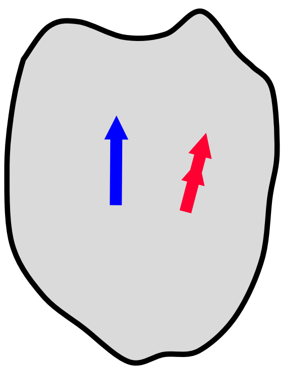
Transfer Function (cont.)

Next, apply reciprocity one more time.

$$E_p\left(\underline{r}_0\right) = \left\langle b', a \right\rangle = \left\langle a, b' \right\rangle = \int_V \left(\underline{E}^a \cdot \underline{J}^{ib'}\right) dV$$

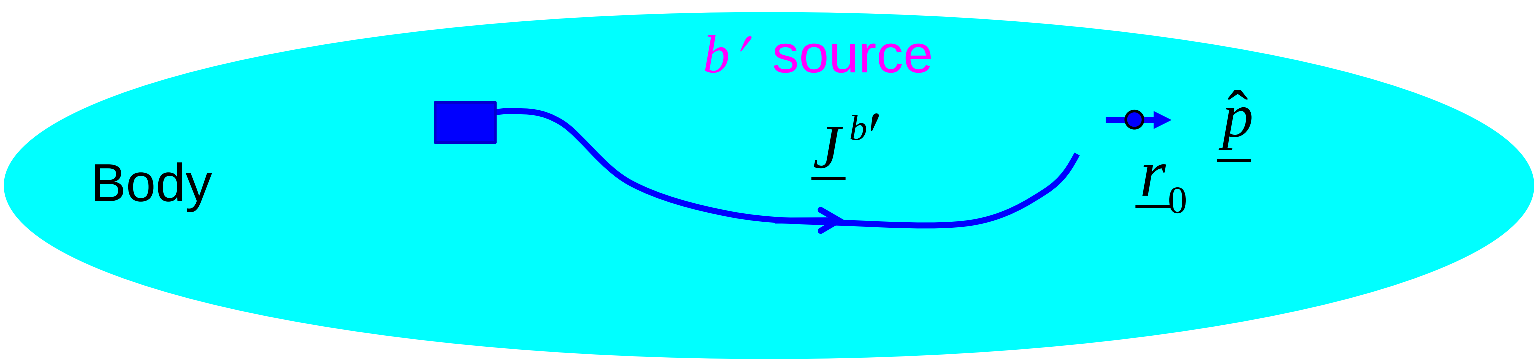
$$= \int_{V_L} \left(\underline{E}^a \cdot \underline{J}^{ib'}\right) dV + E_p^a\left(\underline{r}_0\right)$$

V_L is the volume containing the lead and the device.



a source

The field from the a source is now calculated with the body present but without the lead present (it is therefore the same as the “incident” field, which is defined as the field inside the body from the a source with the lead absent.



Transfer Function (cont.)

For the volume integral we have:

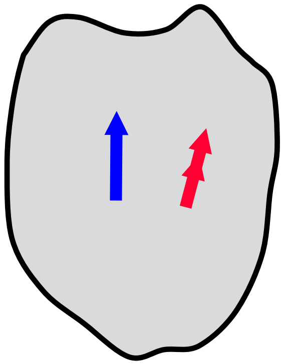
$$\int_{V_L} \left(\underline{E}^a \cdot \underline{J}^{ib'} \right) dV = \int_{C_L} \left(\underline{E}^a \cdot \underline{\hat{\ell}} \right) I_L(\ell)\, dl + \int_{S_D} \left(\underline{E}^a \cdot \underline{J}_{sD} \right) dS$$

C_L is the contour of the lead.

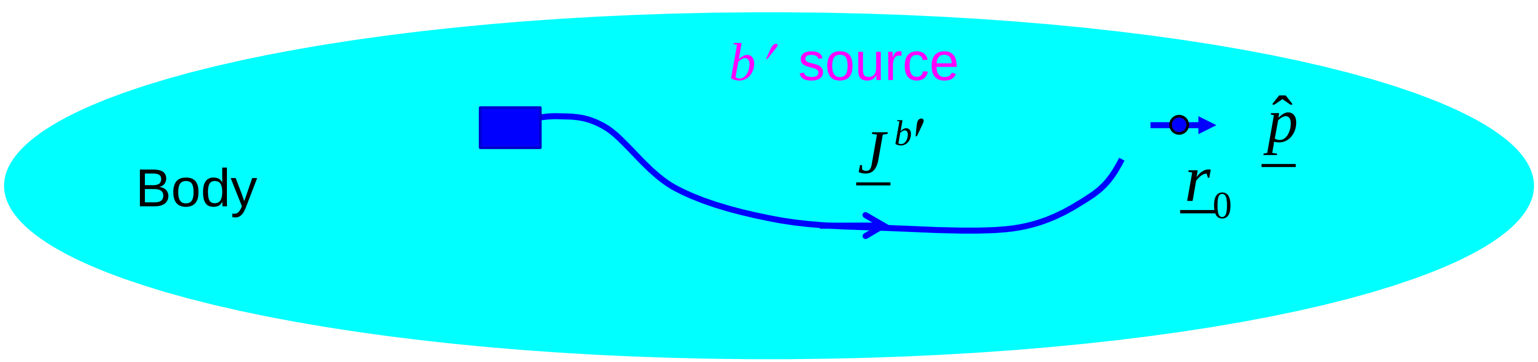
I_L is the current on the lead (in the direction of the contour) due to the unit-amplitude dipole at $\underline{r}_0$ (with the a source turned off).

S_D is the surface of the device.

$\underline{J}_{sD}$ is the current on the device due to the unit-amplitude dipole at $\underline{r}_0$ (with the a source turned off).



a source



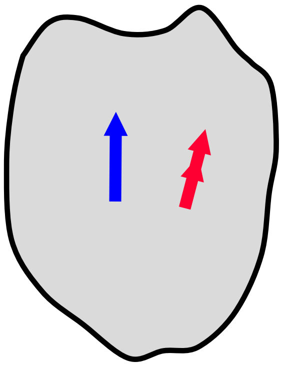
Transfer Function (cont.)

Summary

$$E_p\left(\underline{r}_0\right) = \int_{C_L} \left(\underline{E}^{inc} \cdot \underline{\hat{\ell}}\right) T\left(\ell\right) dl + \int_{S_D} \left(\underline{E}^{inc} \cdot \underline{J}_{sD}\right) dS + E_p^{inc}\left(\underline{r}_0\right)$$

$T\left(\ell\right) = I_L\left(\ell\right) \equiv$ transfer function

Source



$\underline{E}^{inc}$ = the field produced by the external source radiating in the presence of the body, but without the lead or the device present.