

NAME: \_\_\_\_\_

**ECE 6340**  
**Fall 2011**  
**EXAM I**

**INSTRUCTIONS:**

This exam is open-book and open-notes. You may use any material or calculator that you wish. Laptops or other devices that may be used to communicate are not allowed.

- Put all of your answers in terms of the parameters given in the problems, unless otherwise noted.
- Include units with all numerical answers in order to receive full credit.
- Please write all of your work on the sheets attached (if you need more room, you may write on the backs of the pages)

**Please show *all of your work* and *write neatly* in order to receive credit.**

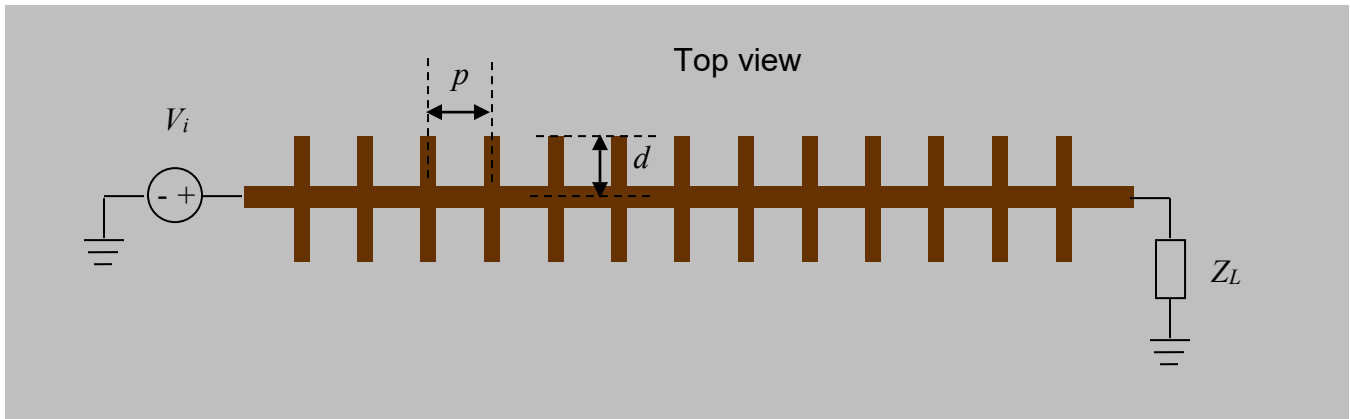
### Problem 1 (40 pts)

A microstrip line is on a grounded substrate as shown below. The line has a characteristic impedance  $Z_0$  and a wavenumber  $k_z$  that is described by

$$k_z = k_0 \sqrt{\epsilon_r^{eff}}.$$

Along the line there are closely spaced microstrip stubs, with two stubs connected to the line at each attachment point. Each stub has a length  $d$ , and each stub is open-circuited at the end. Each stub has a characteristic impedance  $Z_{0s}$  and a wavenumber  $k_{zs}$ . The spacing between the stubs is  $p$ , where  $p \ll \lambda_0$ . It may be assumed that the main line and the stubs are lossless. At the left end of the system there is a voltage source  $V_i$ . At the right end of the system there is a load impedance  $Z_L$ .

- Find the propagation constant  $\gamma$  on the stub-loaded line.
- Find the value of the load impedance that should be used to avoid any reflections at the end of the line.
- Make a qualitative sketch of how  $\beta / k_0$  and  $\alpha / k_0$  vary with frequency, plotting them against the normalized frequency variable  $k_{zs}d$ . In your sketch, plot up to  $k_{zs}d = \pi$ .



ROOM FOR WORK

## Problem 2 (20 pts)

A radiating source that is in free space produces the following fields on the surface of a sphere of radius  $a$ :

$$\begin{aligned}E_r &= \frac{1}{2\pi} \eta e^{-jka} \left( \frac{1}{a^2} \right) \left[ 1 + \frac{1}{jka} \right] \cos \theta \\E_\theta &= \frac{1}{4\pi} (j\omega\mu) e^{-jka} \left( \frac{1}{a} \right) \left[ 1 + \frac{1}{jka} + \frac{1}{(jka)^2} \right] \sin \theta \\H_\phi &= \frac{1}{4\pi} (jk) e^{-jka} \left( \frac{1}{a} \right) \left[ 1 + \frac{1}{jka} \right] \sin \theta\end{aligned}$$

where  $k$  is the (real-valued) wavenumber of free space and  $\eta$  is the (real-valued) intrinsic impedance of free space. The source is confined to the region  $r < a$ .

Calculate the VARS that are consumed by the region of space that is outside the sphere ( $r > a$ ).

(Note: The source is actually an infinitesimal unit-amplitude vertical electric dipole at the origin, but you do not need to know this, so do not assume it!)

ROOM FOR WORK

### Problem 3 (40 pts)

A perfectly conducting circular loop of radius  $a$  is connected to a load resistor  $R_L$  as shown below. The loop is immersed in an external (applied) magnetic field given by

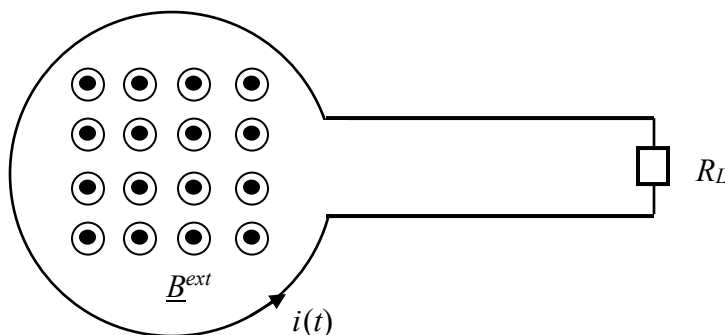
$$\underline{B}^{ext} = \underline{\hat{z}} f(t).$$

(The direction  $z$  is the direction coming out of the paper.) Assume that the external magnetic field is constant (with respect to space) and it does not affect the load resistor or the wires attached to it. The loop has an inductance  $L$ . This means that if a current  $i(t)$  flows through the loop as shown below, the loop will produce its own (self) magnetic flux through the loop, which is described by

$$\psi_{self} = Li(t),$$

where the flux  $\psi_{self}$  is the flux coming out of the loop in the positive  $z$  direction due to the current  $i(t)$ . The total flux through the loop is the sum of the flux from the external magnetic field and the self flux from the current flowing through the loop.

- Derive a differential equation for the current  $i(t)$  flowing through the loop, assuming that we know the applied external field function  $f(t)$ .
- Assume that  $f(t) = B_0 \cos(\omega t)$ . Determine the phasor current  $I$  flowing through the loop, using your answer from part (a).
- As a continuation of part (b), determine the open-circuit voltage across the load resistor (if  $R_L$  becomes an open circuit) and the short-circuit current through the load resistor (If  $R_L$  becomes a short circuit), and thus determine the Thévenin equivalent circuit for the loop being excited by the external magnetic field.
- How would the answer to part (a) change if the loop radius  $a$  were changing as a function of time, so that  $a = a(t)$ , and hence  $L = L(t)$ ?



ROOM FOR WORK