NAME: ____________________________________

ECE 6340
Fall 2012

EXAM I

INSTRUCTIONS:

This exam is open-book and open-notes. You may use any material or calculator that you wish. Laptops or other devices that may be used to communicate are not allowed.

- Put all of your answers in terms of the parameters given in the problems, unless otherwise noted.
- Include units with all numerical answers.
- Please circle your final answers.
- Please write all of your work on the sheets attached (if you need more room, you may write on the backs of the pages).

Please show *all of your work* and *write neatly* in order to receive credit.

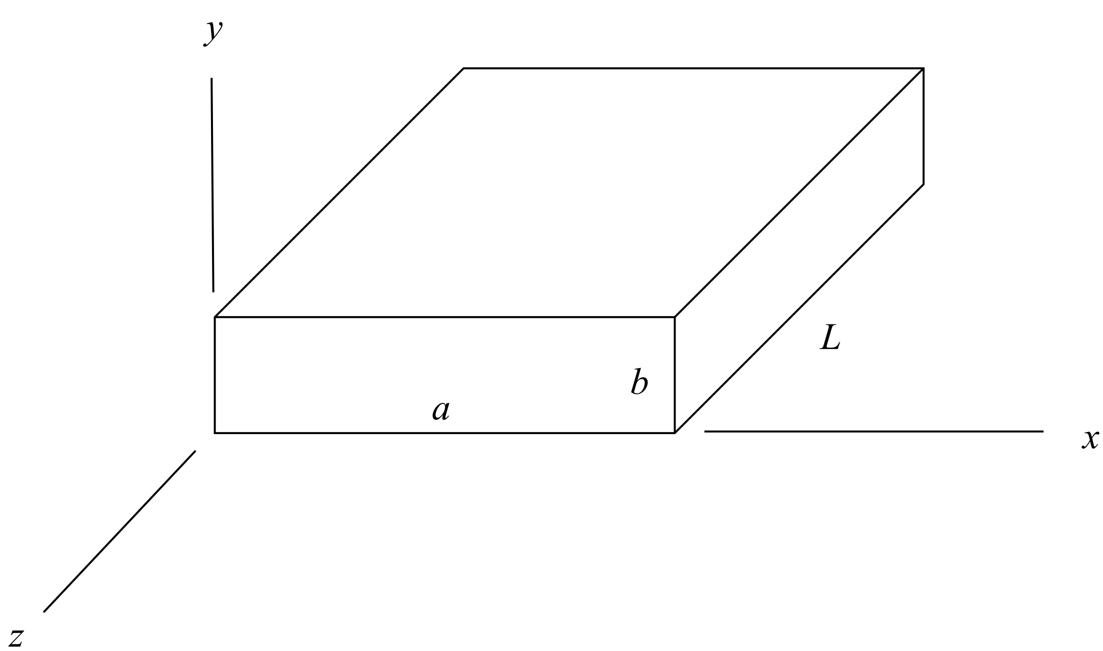
Problem 1 (25 pts)

The TE_{101} mode in a hollow perfectly conducting rectangular waveguide resonator (having dimensions $a \times b \times L$) has an electric field that is given by

$$\underline{E} = \underline{\hat{y}} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi z}{L}\right).$$

(There is also a magnetic field inside the resonator, but the formula for it is not given. Do not assume that the magnetic field is zero!)

Determine the total time-average force in the x direction $\langle \mathscr{F}_x \rangle$ on the right wall of the resonator at $x = a$.



ROOM FOR WORK

Problem 2 (30 pts)

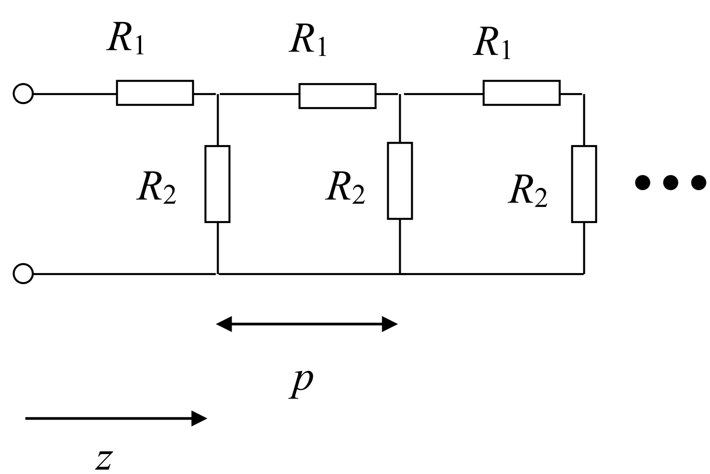
A semi-infinite cascade of resistors is shown below. Assume that the change in voltage across a unit cell of length p is small.

(a) Determine an approximate formula for the DC input impedance seen at $z = 0$ looking into the cascade of resistors, using transmission line theory (assuming that the length p is small).

(b) Assuming that a DC voltage of 1.0 Volt is applied at $z = 0$, determine an approximate formula for the voltage at $z = Np$ where N is an integer, using transmission line theory (assuming that the length p is small).

(c) Give a mathematical constraint that will ensure that the voltage variation across a unit cell is small, and hence ensure that your results are accurate. A good starting point would be to assume that the voltage magnitude changes by less than 10% as you go across a unit cell.

(d) Derive an exact expression for the DC input impedance seen looking into the cascade. Do this by taking advantage of the fact that the input impedance at $z = 0$ should be the same as the impedance seen looking to the right at $z = p$ (just to the right of the first resistor R_2), since the cascade is infinite.



ROOM FOR WORK

Problem 3 (20 pts)

A shallow dish of pure (distilled) water is placed in a microwave oven as shown below. The frequency of the oven is $f = 2.54$ [GHz]. The volume of the water is $V = 100$ [cm^3] = 10^{-4} [m^3]. Immediately above the water, in the air region, the electric field is perfectly vertical (in the z direction) and uniform, and is given by

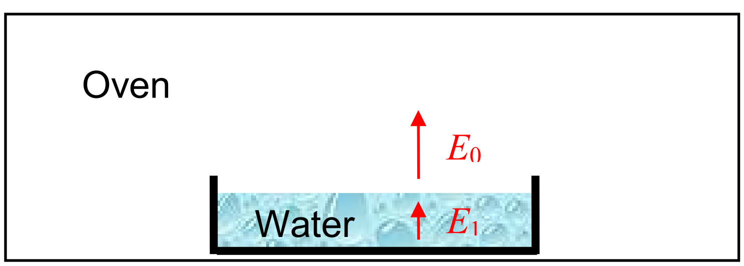
$$\underline{\mathscr{E}}_0 = \underline{\hat{z}}\, A_0 \cos(\omega t)\,,$$

where A_0 = 3000 [V/m]. Assume that the electric field $\underline{\mathscr{E}}_1$ inside the water is also uniform and vertical (but is not necessarily the same as the electric field $\underline{\mathscr{E}}_0$ above the water). The water has a complex permittivity given by

$$\varepsilon = \varepsilon' - j\varepsilon'' = \varepsilon_0\left(80 - j10\right).$$

(a) Determine the electric field $\underline{\mathscr{E}}_1(t)$ inside the water as a function of time.

(b) Determine the time-average power dissipated inside the water.



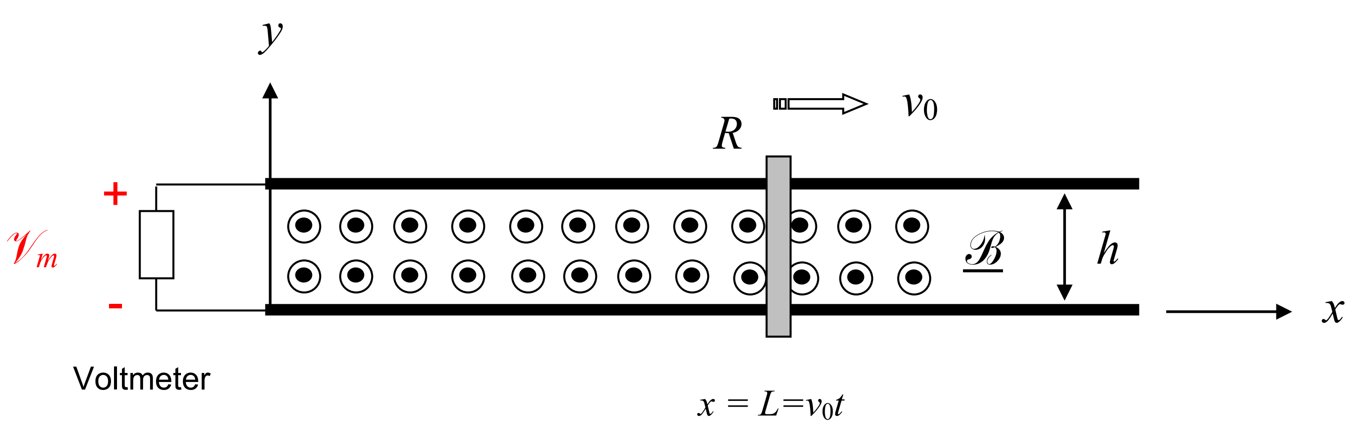
ROOM FOR WORK

Problem 4 (25 pts)

A voltmeter is connected to a set of perfectly conducting rails as shown below. The voltmeter has an internal resistance of R_m, and hence can be modeled as this resistance. Between the two rails is a sliding resistor with a resistance of R. At time t the resistor is located at $x = L = v_0 t$ and is moving to the right with a constant velocity v_0. Between the rails there is a magnetic field that is given by

$$\underline{\mathscr{B}}(x,t) = \underline{\hat{z}}\, e^{-x} \cos(\omega t).$$

Determine the voltage reading on the voltmeter $\mathscr{V}_m(t)$ as a function of time.



ROOM FOR WORK