EXAM 1 Fall 2002

Prob. 1

1) $\epsilon_{rc} = \frac{1}{\epsilon_0}\left(\hat{\epsilon} - j\frac{\sigma}{\omega}\right)$

$= (60 - j35) - j\frac{\sigma}{\omega\epsilon_0}$

$= (60 - j35) - j(7.1902)$

$= 60 - j(42.1902)$ (unitless)

2) $\underline{E} = -\hat{\underline{z}}\,(100)$ [V/m]

3) $\underline{D} = \hat{\epsilon}\,\underline{E} = \epsilon_0(60 - j35)\,\underline{E}$

$= -\hat{\underline{z}}\,\epsilon_0(6000 - j3500)$ [C/m²]

4) $\underline{J}^c = \sigma\underline{E} = -\hat{\underline{z}}\,(400)$ [A/m²]

5) $\rho_s = \underline{D} \cdot \hat{\underline{n}}$

$= \underline{D} \cdot \hat{\underline{z}}$

$= -\epsilon_0 (6000 - j3500) \; [C/m^2]$

6) $\underline{J}^P = j\omega (\hat{\epsilon} - \epsilon_0) \underline{E}$

$= j\omega\epsilon_0 (\hat{\epsilon}_r - 1) \underline{E}$

$= j\omega\epsilon_0 [(60 - j35) - 1] \underline{E}$

$= j\omega\epsilon_0 (59 - j35) \underline{E}$

$= -\hat{\underline{z}} (j\omega\epsilon_0)(59 - j35)(100)$

$= \hat{\underline{z}} (-j)(55.63)(59 - j35)$

$= \hat{\underline{z}} (-1,947,05 - j\,3282,17)$ $[A/m^2]$

7) $\underline{J}^{eq} = j\omega (\epsilon_c - \epsilon_0) \underline{E}$

$= j\omega (\hat{\epsilon} - \epsilon_0 - j\frac{6}{\omega}) \underline{E}$

$= \underline{J}^P + 6\underline{E} = \underline{J}^P + \underline{J}^c$

$= \hat{\underline{z}} (-2347,05 - j\,3282,17) \left[\frac{A}{m^2}\right]$

Prob. 2

$$L = \frac{Z_0^A}{\eta_0} \mu_r \mu_0 = 1.6677 \times 10^{-7} \ [H/m]$$

$$C = \frac{\eta_0}{Z_0^A} \epsilon_r \epsilon_0 = 1.4678 \times 10^{-10} \ [F/m]$$

$$G = C\left(\frac{\omega \epsilon''}{\epsilon'}\right) = \omega C \tan\delta$$

$$= 9.222 \times 10^{-4} \ [S/m]$$

$$\gamma = [j\omega L (G + j\omega C)]^{1/2}$$

$$= [j(1.0478 \times 10^3)(9.222 \times 10^{-4} + j(0.9222))]^{1/2}$$

$$= [-966.33 + j(0.9663)]^{1/2}$$

$$= \underbrace{0.01555}_{\alpha} + j(31.086)$$

Prob. 3

$$\frac{R}{L} = \frac{G}{C}$$

$$R = \frac{L}{C} G = L \left(\frac{G}{C} \right)$$

$$= \left(\frac{Z_0^A}{\eta_0} \right) \mu \left(\frac{\omega \epsilon''}{\epsilon''} \right)$$

$$= \frac{Z_0^A}{\eta_0} \mu_0 \left(\omega \tan \delta \right)$$

$$= Z_0^A \left(\omega \sqrt{\mu_0 \epsilon_0} \right) \tan \delta$$

$$= Z_0^A k_0 \operatorname{Tan} \delta$$

Prob. 4

$$k_z = \left(k^2 - k_c^2\right)^{1/2}$$

$$= k_o \left(\epsilon_{rc} - \left(\frac{k_c}{k_o}\right)^2\right)^{1/2}$$

k_c is the same for an air-filled guide. In this case,

$$\frac{k_c}{k_o} = \frac{k_c^{Air}}{k_o} = \frac{\omega_{co}\sqrt{\mu_o\epsilon_o}}{\omega\sqrt{\mu_o\epsilon_o}} = \frac{f_{co}}{f}$$

Hence

$$k_z = k_o \left(\epsilon_{rc} - \left(\frac{f_{co}}{f}\right)^2\right)^{1/2}$$