NAME: _________SOLUTION___________________________

ELEE 6340
Fall 2003

EXAM I

INSTRUCTIONS:

This exam is open-book and open-notes. You may use any material or calculator that you wish. Please show *all of your work* and *write neatly* in order to receive credit. Put all of your answers in terms of the parameters given in the problems, unless otherwise noted. Include units with all answers in order to receive full credit.

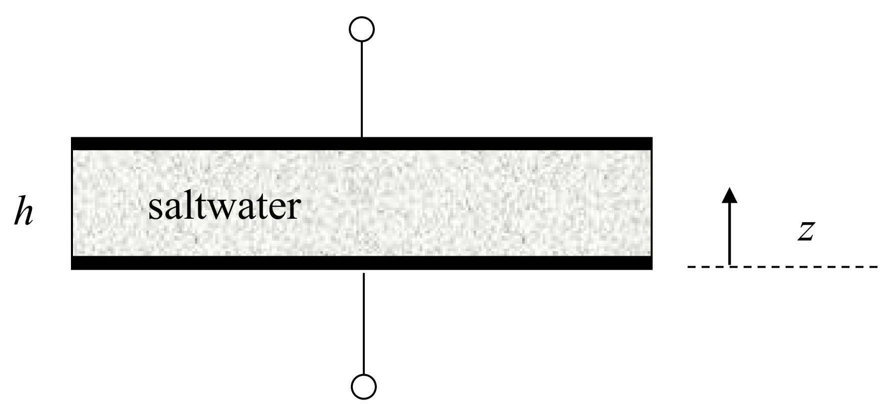
Please write all of your work on the sheets attached.

Problem 1 (30 pts)

A parallel-plate capacitor is filled with saltwater. The capacitor has a 1.0 [cm] separation between the plates. The area of each plate is 100 [cm^2]. At DC, the input resistance between the two plates is 0.5 [Ω]. At a frequency of 100 MHz, the input impedance is 0.475211-*j*(0.105455) [Ω]. Assume that the conductivity of the saltwater does not change with frequency.

Determine the following:

1. The complex effective relative permittivity ε_{rc} of the saltwater at 100 MHz.
2. The complex relative permittivity $\hat{\varepsilon}_r$ of the saltwater at 100 MHz.
3. The phasor charge Q on the top plate of the capacitor at 100 [MHz], assuming that the electric field inside the capacitor is $\mathbf{E} = \hat{\mathbf{z}}(10)$ [V/m].



ROOM FOR EXTRA WORK

(1)

The input impedance is related to the complex effective permittivity as

$$Z_{in} = \frac{1}{j\omega\left(\frac{A}{h}\right)\varepsilon_0\varepsilon_{rc}}.$$

Hence

$$\varepsilon_{rc} = \frac{1}{j\omega\left(\frac{A}{h}\right)\varepsilon_0 Z_{in}}.$$

Therefore, we have

$$\varepsilon_{rc} = 80 - j360.502.$$

(2)

The effective complex relative permittivity is related to the actual complex relative permittivity by

$$\varepsilon_{rc} = \hat{\varepsilon}_r - j\frac{\sigma}{\omega\varepsilon_0}.$$

Hence,

$$\hat{\varepsilon}_r = \varepsilon_{rc} + j\frac{\sigma}{\omega\varepsilon_0}.$$

We now need to calculate the conductivity. At DC,

$$Z_{in} = \frac{h}{\sigma A} = 0.5\,\Omega.$$

Therefore,

$\sigma = 2.0$ S/m .

Using this value, we then obtain

$$\hat{\varepsilon}_r = 80 - j1 .$$

(c)

The charge on the top plate is given by

$$Q = \rho_s A = A\left(\mathbf{D}\cdot\left(-\hat{\mathbf{z}}\right)\right) = -D_z A = -\varepsilon_0 \hat{\varepsilon}_r E_z A .$$

The value is then

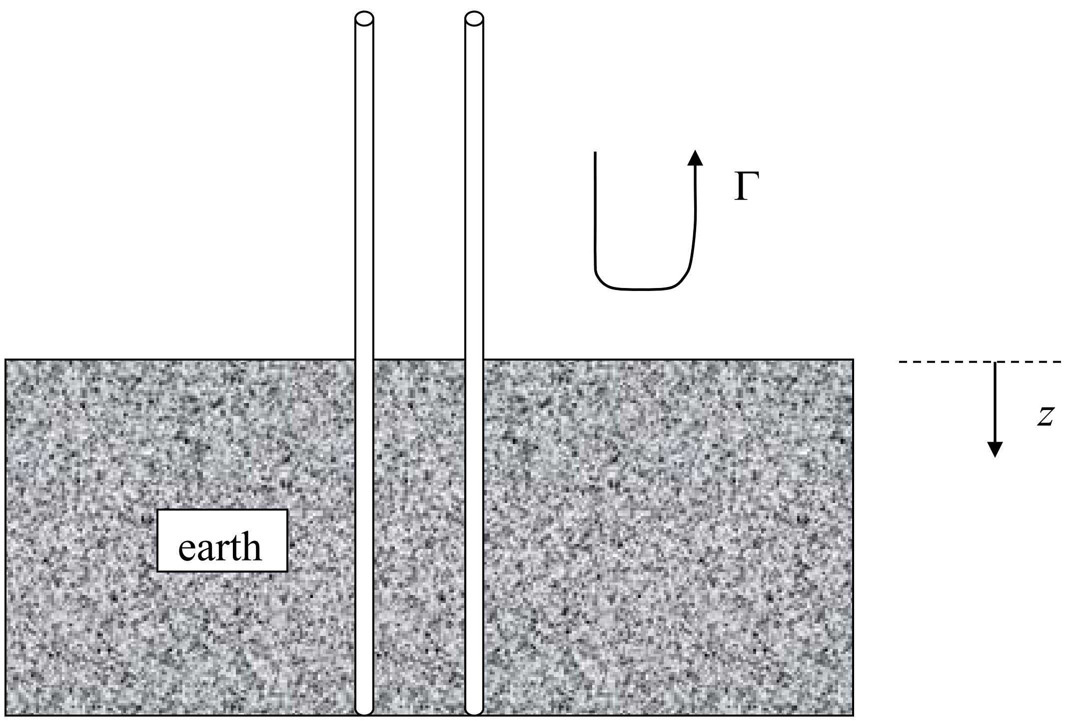
$$Q = -7.0833\times10^{-11} + j8.8562\times10^{-13} \text{ C} .$$

Problem 2 (30 pts)

A twin-line transmission line in air has a characteristic impedance of 300 [Ω]. The transmission line runs into the earth, which has a complex relative permittivity of $\varepsilon_{rc} = 4.0 - j1.0$. Assume that the wires are perfectly conducting, and that the earth (and the line inside of it) extends to $z = \infty$.

1. Determine the reflection coefficient Γ.

2. Determine the attenuation in dB of the signal 1 meter below the surface of the earth, relative to the signal just below the surface of the earth, at a frequency of 100 [MHz]



ROOM FOR EXTRA WORK

(1)

The reflection coefficient is given by

$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

where

Z_0 = impedance in air region = 300 Ω
Z_1 = impedance in earth region

We need to find Z_1. To do this, we can use

$$Z_1 = \sqrt{\frac{j\omega L}{G + j\omega C}} = \sqrt{\frac{j\omega L}{\omega C\left(\frac{\varepsilon''}{\varepsilon'}\right) + j\omega C}} = \sqrt{\frac{j\omega L_0}{\omega C_0\left(\frac{\varepsilon'}{\varepsilon_0}\right)\left(\frac{\varepsilon''}{\varepsilon'}\right) + j\omega C_0\left(\frac{\varepsilon'}{\varepsilon_0}\right)}}$$

where the zero subscript denotes the value for the air line.

Hence,

$$Z_1 = \sqrt{\frac{j\omega L_0}{\omega C_0 \varepsilon_r'' + j\omega C_0 \varepsilon_r'}} = \sqrt{\frac{j\omega L_0}{j\omega C_0\left(\varepsilon_r' - j\varepsilon''\right)}} = \sqrt{\frac{L_0}{C_0}}\frac{1}{\sqrt{\left(\varepsilon_r' - j\varepsilon''\right)}}$$

or

$$Z_1 = \frac{Z_0}{\sqrt{\varepsilon_{rc}}}.$$

Hence,

$$Z_1 = 146.637 + j18.052.$$

From this we obtain

$$\Gamma = -0.34118 + j0.05421.$$

(2)

The wavenumber in the earth is

$$k_z = k_0\sqrt{\varepsilon_{rc}} .$$

From this we obtain

$$k_z = 4.22382 - j0.51997 .$$

Hence,

$$\alpha = 0.51997 \text{ n/m} .$$

After propagating one meter into the earth, the wave is attenuated by a factor of $\exp(-\alpha z) = \exp(-\alpha)$.

Hence, the dB attenuation A_{dB} is

$$A_{dB} = 20\log_{10}\left(e^{-\alpha}\right).$$

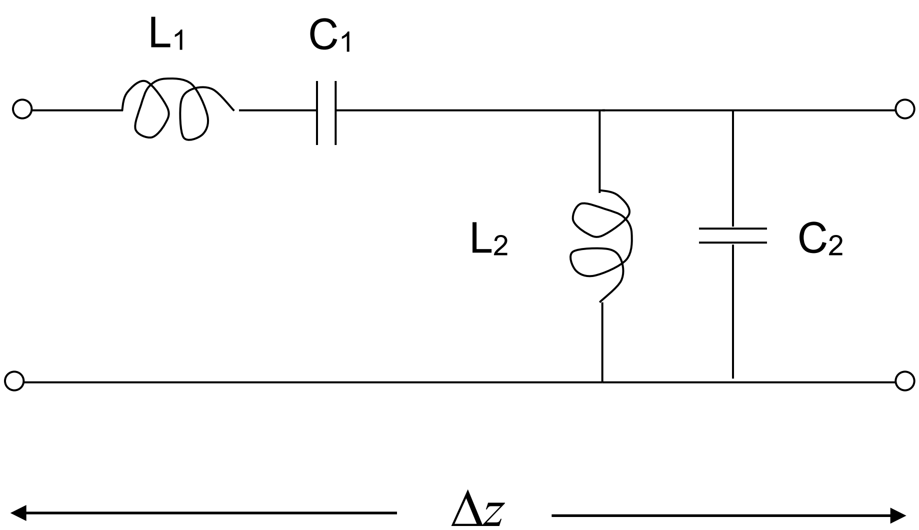
The result is

$$A_{dB} = -4.516$$

Problem 3 (40 pts)

An artificial transmission lines is made by cascading sections of the circuit shown below. The values L_1, C_1, L_2, C_2 are the actual element values in Henrys (for the inductors) or Farads (for the capacitors).

1. Find a formula for the propagation constant γ in terms of the element values and the unit-cell dimension Δz.
2. Find simple formulas for the phase constant β both at very low frequency and at very high frequency. To get the signs correct, assume that the group velocity is always a positive number at any frequency.
3. Explain why $\alpha = 0$ for both $\omega < \omega_1$ and $\omega > \omega_2$, where $\omega_1 = 1/\sqrt{L_1 C_1}$ and $\omega_2 = 1/\sqrt{L_2 C_2}$ (assume that $\omega_1 < \omega_2$). Explain why $\alpha > 0$ for $\omega_1 < \omega < \omega_2$.
4. Make a qualitative sketch of β and α versus frequency. Label the points ω_1 and ω_2 on your sketch.



ROOM FOR EXTRA WORK

(1)

The general formula for the propagation constant is

$$\gamma = (ZY)^{1/2}.$$

Hence we have

$$\gamma = \left[\left(j\omega L_1 + \frac{1}{j\omega C_1}\right)\left(j\omega C_2 + \frac{1}{j\omega L_2}\right)\right]^{1/2} \frac{1}{\Delta z}.$$

We can also write this as

$$\gamma = \left[\left(\frac{1}{j\omega C_1}\left(1-\left(\frac{\omega}{\omega_1}\right)^2\right)\right)\left(\frac{1}{j\omega L_2}\left(1-\left(\frac{\omega}{\omega_2}\right)^2\right)\right)\right]^{1/2} \frac{1}{\Delta z}$$

or

$$\gamma = \pm\frac{1}{j\omega\sqrt{L_2 C_1}}\left[\left(1-\left(\frac{\omega}{\omega_1}\right)^2\right)\left(1-\left(\frac{\omega}{\omega_2}\right)^2\right)\right]^{1/2} \frac{1}{\Delta z}.$$

(2)

At very low frequency we have

$$\gamma = \pm\frac{1}{j\omega\sqrt{L_2 C_1}}\frac{1}{\Delta z}$$

so

$$\beta = \mp\frac{1}{\omega\sqrt{L_2 C_1}}\frac{1}{\Delta z}$$

In order to obtain a positive group velocity, we have

$$\beta = -\frac{1}{\omega\sqrt{L_2 C_1}}\frac{1}{\Delta z}.$$

At very high frequency we have

$$\gamma = \pm j\omega\sqrt{L_1 C_2}\frac{1}{\Delta z}.$$

Hence we have

$$\beta = \pm\omega\sqrt{L_1 C_2}\frac{1}{\Delta z}.$$

In order to have a positive group velocity, we have

$$\beta = \omega\sqrt{L_1 C_2}\frac{1}{\Delta z}.$$

(3)

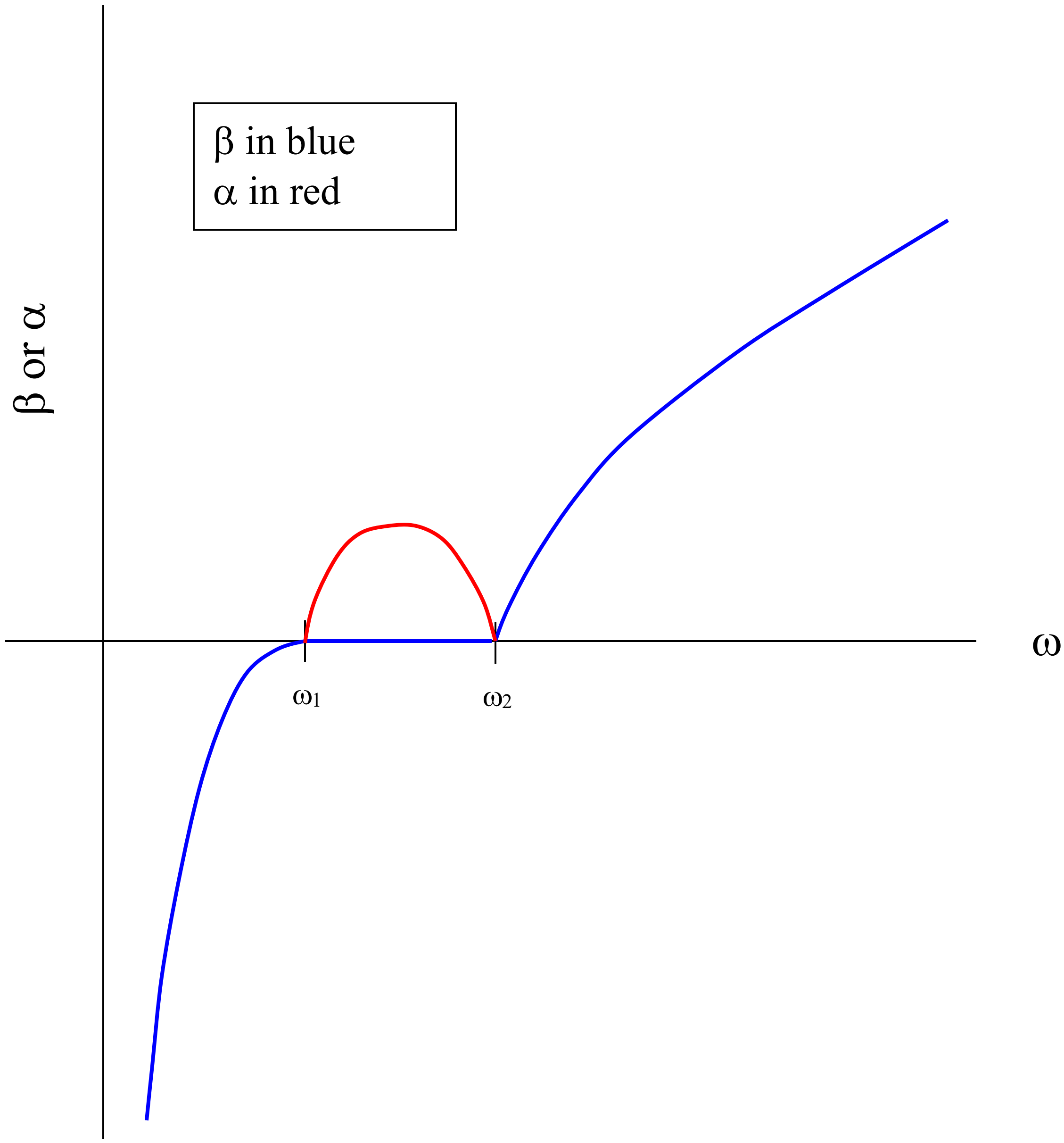
Consider the formula

$$\gamma = \left[\left(j\omega L_1 + \frac{1}{j\omega C_1}\right)\left(j\omega C_2 + \frac{1}{j\omega L_2}\right)\right]^{1/2}\frac{1}{\Delta z}.$$

For $\omega < \omega_1$ both of the terms in parenthesis are -*j* numbers. Hence we have the square root of a negative number, which is pure imaginary. Hence there is a β but no α. When $\omega > \omega_2$ both terms are a +*j* number, and the conclusion is the same. When $\omega_1 < \omega < \omega_2$, the first term is a +*j* number and the second term is a –*j* number. Hence we have the square root of a positive real number. This gives us an α but no β**.**

(4)

Please see the sketch below. Note that this artificial transmission line acts as a “stopband filter”, attenuating all frequencies in the stopband region $\omega_1 < \omega < \omega_2$.


β in blue
α in red
β or α
ω
ω_1
ω_2