NAME: ____________**SOLUTION**_____________________

ECE 6340
Fall 2009

EXAM I

INSTRUCTIONS:

This exam is open-book and open-notes. You may use any material or calculator that you wish. Laptops or other devices that may be used to communicate are not allowed.

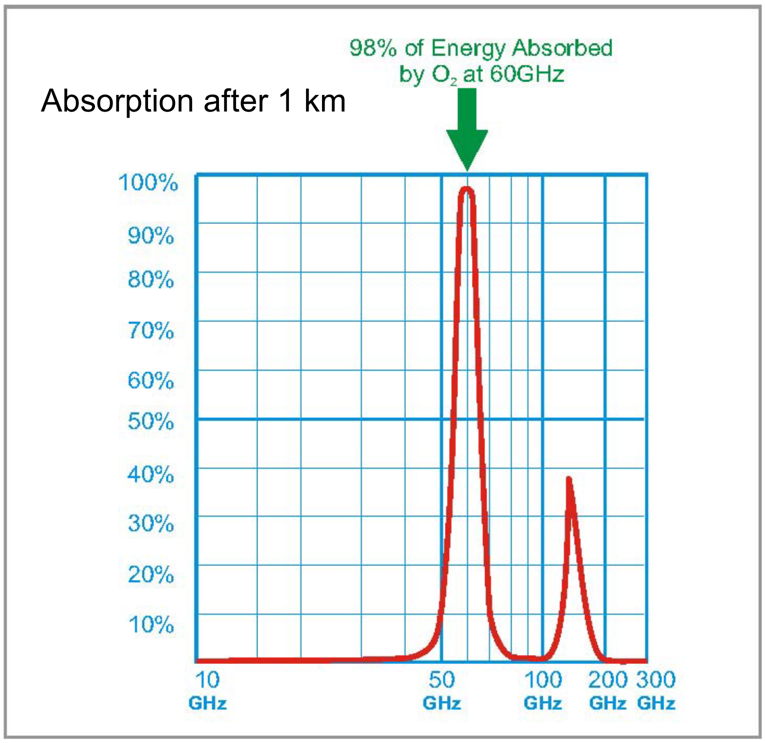
- Put all of your answers in terms of the parameters given in the problems, unless otherwise noted.
- Include units with all answers in order to receive full credit.
- Please write all of your work on the sheets attached.

Please show *all of your work* and *write neatly* in order to receive credit.

Problem 1 (20 pts)

A plane wave in air has a maximum attenuation of 17 dB/km at 60 GHz due to the absorption of the oxygen molecule. At a frequency of 55 GHz the attenuation is 4 dB/km. (This information was extracted from the plot shown below, though you don't need the plot for this problem.)

Give a formula that predicts the attenuation in dB/km at any frequency f_{GHz} near 60 GHz, where f_{GHz} is the frequency in GHz. All constants that appear in the formula should be evaluated.



ROOM FOR WORK

Using the Lorentz model, we have

$$A_{\mathrm{dB}} = A_{dB}^{\max}\left[\frac{\bar{\omega}}{\bar{\omega}^2 + p^2\left(1-\bar{\omega}^2\right)^2}\right],$$

where

$$\bar{\omega} = \frac{\omega}{\omega_0} = \frac{f}{f_0} = \frac{f_{GHz}}{f_0^{GHz}} = \frac{f_{GHz}}{60}$$

and

$$A_{dB}^{\max} = 17.$$

At 55 GHz we have

$$4 = 17\left[\frac{0.9167}{\left(0.9167\right)^2 + p^2\left(0.02551\right)}\right].$$

Solving for p, we have

$$p = 10.944.$$

Problem 2 (30 pts)

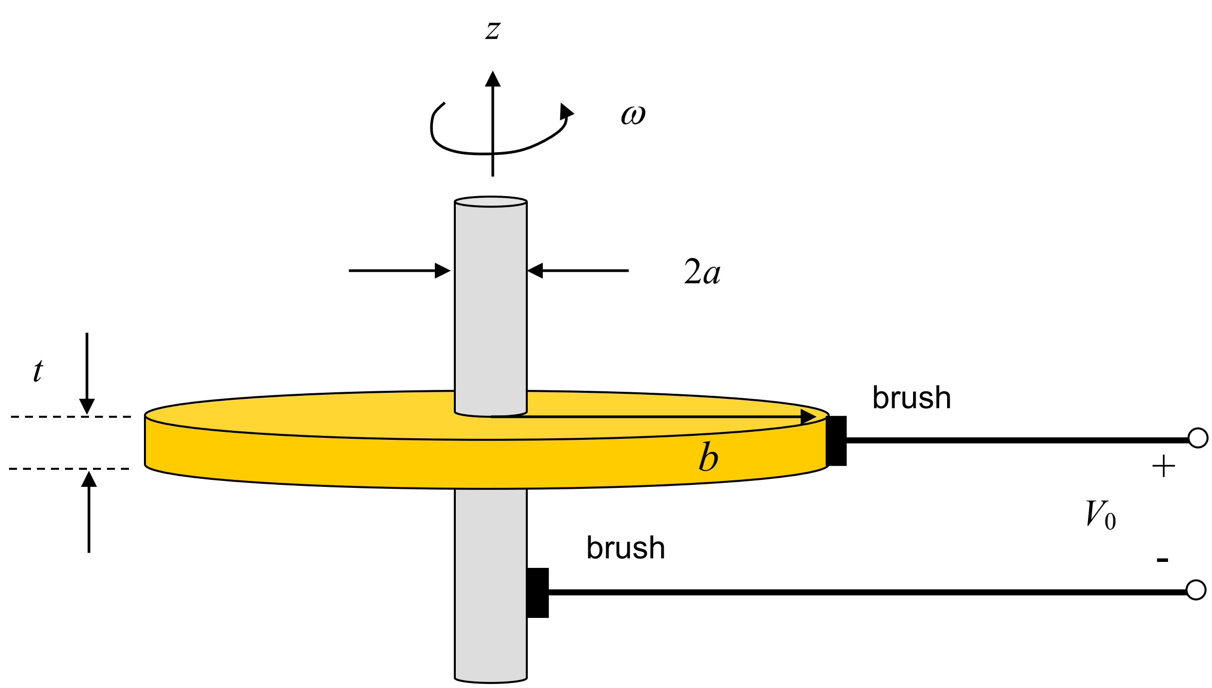
A DC generator is made from a conducting disk of inner radius a, outer radius b, and thickness t, spinning on a shaft at an angular velocity of ω radians/second as shown below. The disk has a conductivity σ. The disk is spinning in the presence of a uniform magnetic field, described by.

$$\underline{\mathscr{B}} = \hat{\underline{z}}\, B_0 \,.$$

The output leads are connected to the shaft and the disk by conducting brushes (sliding contacts).

(a) Assume that the output terminals of the generator are open-circuited, so that no current flows through the system. Derive an expression for the output voltage V_0.

(b) Next, find the DC internal resistance of the generator by calculating the resistance seen between the inner and outer edges of the disk. Assume that the current density inside the disk is in the radial direction and only depends on the radial distance ρ.



ROOM FOR WORK

Construct a closed path that goes from the "+" terminal to the "–" terminal of the output terminals, then through the output leads and the disk (going from the inner radius to the outer radius, in a clockwise direction.

Part (a)

Since there is no changing magnetic field, Faraday's law says that

$$\oint_C \underline{\mathscr{E}} \cdot d\underline{r} = 0 .$$

Hence we have

$$V_0 + \int_a^b \mathscr{E}_\rho d\rho = 0 .$$

Inside the disk, there is no current flow, and hence we have we have

$$\underline{\mathscr{J}} = \sigma\left(\underline{\mathscr{E}} + \underline{v} \times \underline{\mathscr{B}}\right) = \underline{0}$$

and therefore

$$\underline{\mathscr{E}} = -\underline{v} \times \underline{\mathscr{B}} .$$

Hence, inside the disk there is an electric field given by

$$\underline{\mathscr{E}} = -\left(\underline{\hat{\phi}} \times \underline{\hat{z}}\right) v_\phi \mathscr{B}_z .$$

Hence, we have

$$\mathscr{E}_\rho = -v_\phi \mathscr{B}_z = -\rho\omega\mathscr{B}_z = -\rho\omega B_0 .$$

The output voltage is then

$$V_0 = -\int_a^b \mathscr{E}_\rho d\rho = \omega B_0 \int_a^b \rho d\rho = \omega B_0 \frac{1}{2}\rho^2 \Big|_a^b .$$

Hence we have

ROOM FOR WORK

$$V_0 = \frac{\omega B_0}{2}\left(b^2 - a^2\right).$$

Part (b)

The internal resistance of the disk is

$$R = \frac{V}{I}.$$

For a given current I that enters the inside rim of the disk, we have at any radius ρ

$$J_\rho = \frac{I}{2\pi\rho t}.$$

Hence, the electric field is

$$\mathcal{E}_\rho = \frac{I}{2\pi\rho t\sigma}.$$

The voltage is then

$$V = \int_a^b \mathcal{E}_\rho \, d\rho = \int_a^b \left(\frac{I}{2\pi\rho t\sigma}\right) d\rho = \frac{I}{2\pi t\sigma}\int_a^b \frac{1}{\rho}\, d\rho = \left(\frac{I}{2\pi t\sigma}\right)\ln\left(\frac{b}{a}\right).$$

The internal resistance is then

$$R = \left(\frac{1}{2\pi t\sigma}\right)\ln\left(\frac{b}{a}\right) \quad [\Omega].$$

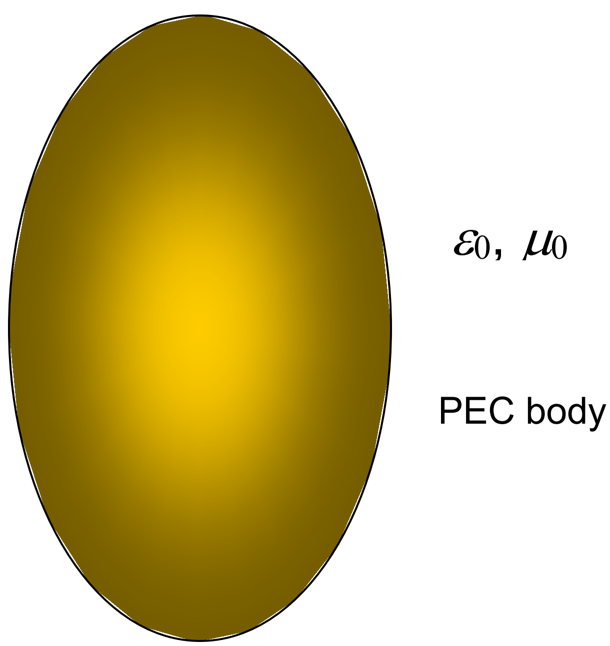
Problem 3 (20 pts)

Consider a perfectly conducting object that is in air, as shown below. Show that at any point on the surface of the object, the time-average inward pressure (force per unit area) on the object due to a time-harmonic (sinusoidally varying) electromagnetic field is given by

$$\langle \mathscr{P} \rangle = \frac{1}{4}\mu_0 \left|\underline{H}\right|^2 - \frac{1}{4}\varepsilon_0 \left|\underline{E}\right|^2 ,$$

where the electric and magnetic fields (in the phasor domain) on the right-hand side of the equation are those at the surface of the body in the air region.

Note: You may assume that the direction normal to the body (at a given point on the body) is labeled as z, if it is convenient to do so.



ROOM FOR WORK

Using the Maxwell stress tensor, we have

$$T_{ij} = \varepsilon_0\, \mathscr{E}_i \mathscr{E}_j + \frac{1}{\mu_0} \mathscr{B}_i \mathscr{B}_j - \delta_{ij}\left(\frac{1}{2}\varepsilon_0 \left|\underline{\mathscr{E}}\right|^2 + \frac{1}{2\mu_0}\left|\underline{\mathscr{B}}\right|^2\right).$$

Using z as the outward normal direction, the inward pressure is

$$\mathscr{P} = -T_{zz}\,.$$

Hence, we have

$$-\mathscr{P} = \varepsilon_0\, \mathscr{E}_z \mathscr{E}_z + \mu_0\, \mathscr{H}_z \mathscr{H}_z - \left(\frac{1}{2}\varepsilon_0 \left|\underline{\mathscr{E}}\right|^2 + \frac{1}{2}\mu_0 \left|\underline{\mathscr{H}}\right|^2\right).$$

Since the object is a perfect conductor, the normal (z) component of the magnetic field at the surface is zero. Hence, we can write

$$-\mathscr{P} = \varepsilon_0\, \mathscr{E}_z \mathscr{E}_z - \left(\frac{1}{2}\varepsilon_0 \left|\underline{\mathscr{E}}\right|^2 + \frac{1}{2}\mu_0 \left|\underline{\mathscr{H}}\right|^2\right).$$

Also, the electric field vector at the surface of the perfect conductor only has a z component, so we can write

$$-\mathscr{P} = \varepsilon_0 \left|\underline{\mathscr{E}}\right|^2 - \left(\frac{1}{2}\varepsilon_0 \left|\underline{\mathscr{E}}\right|^2 + \frac{1}{2}\mu_0 \left|\underline{\mathscr{H}}\right|^2\right).$$

Hence, we have

$$\mathscr{P} = \frac{1}{2}\mu_0 \left|\underline{\mathscr{H}}\right|^2 - \frac{1}{2}\varepsilon_0 \left|\underline{\mathscr{E}}\right|^2\,.$$

According to the time-average theorem, we then have

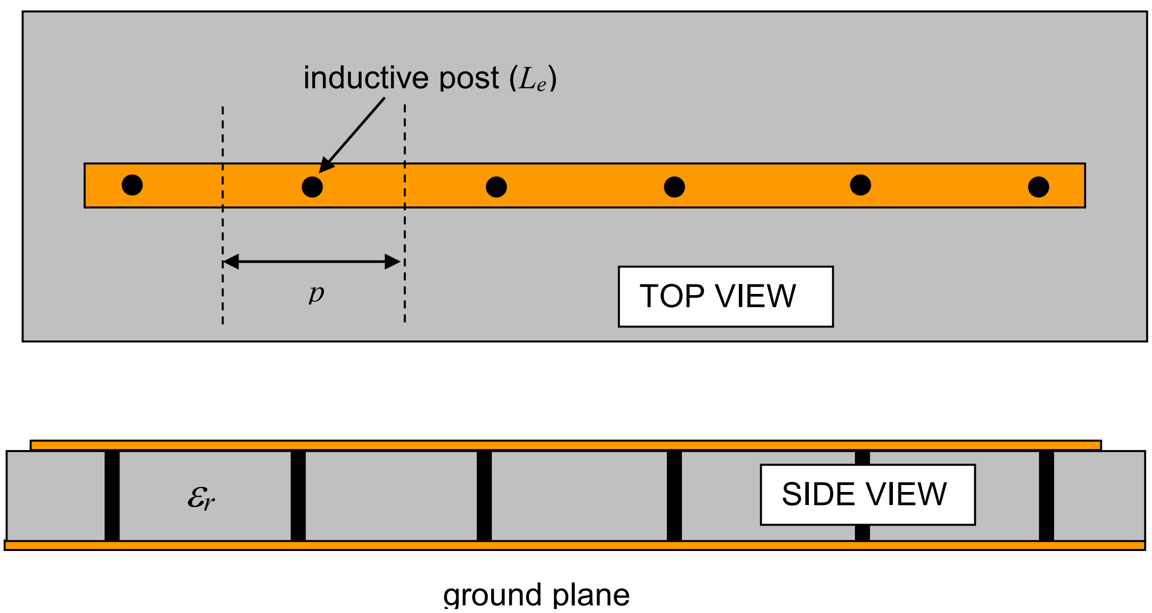
$$\langle \mathscr{P} \rangle = \frac{1}{4}\mu_0 \left|\underline{H}\right|^2 - \frac{1}{4}\varepsilon_0 \left|\underline{E}\right|^2.$$

Problem 4 (30pts)

A microstrip line is periodically loaded with shunt metal posts (vias) as shown below. Each via has an inductance L_e. The microstrip line is assumed to be lossless, with a known value of Z_0 and phase velocity v_p. Assume that the spacing p between the vias is small compared with a wavelength.

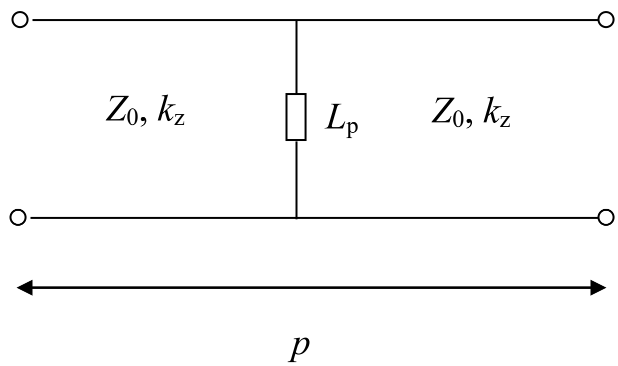
(a) Derive a formula for the propagation constant γ in terms of the known values.(Z_0, v_p, L_e, p).

(b) Derive a sketch of what the attenuation constant and the phase constant look like as a function of frequency ω. Clearly label any important points such as where the attenuation constant or the phase constant goes to zero, and include formulas for the frequency of such points.

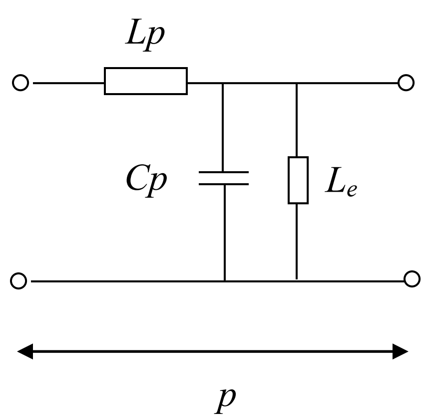


ROOM FOR WORK

The circuit model for the unit cell is shown below.



The equivalent model is then shown below.



For this model we have

$$Z = j\omega L$$

$$Y = j\omega C + \frac{1}{j\omega L_e p} = j\left(\omega C - \frac{1}{\omega L_e p}\right).$$

The propagation constant is then

$$\gamma = \sqrt{ZY} = j\omega\left(LC - \frac{L}{\omega^2 L_e p}\right)^{1/2}.$$

ROOM FOR WORK

Or, we can write

$$\gamma = j\omega\sqrt{LC}\left(1-\frac{1}{\omega^2 L_e pC}\right)^{1/2}$$

We also have that

$$\sqrt{LC} = \frac{1}{v_p}$$

$$\sqrt{\frac{L}{C}} = Z_0 .$$

Hence,

$$L = \frac{Z_0}{v_p}$$

$$C = \frac{1}{Z_0 v_p}.$$

We have pure propagation ($\alpha = 0$) above a critical frequency, and pure evanescence ($\beta = 0$) below the critical frequency. The critical frequency is given by

$$\omega_c = \frac{1}{\sqrt{L_e pC}}.$$

A sketch is shown below.

ROOM FOR WORK

$$\beta = \omega\sqrt{LC}\sqrt{1-\frac{1}{\omega^2 L_e pC}}$$

$$\alpha = \omega\sqrt{LC}\sqrt{\frac{1}{\omega^2 L_e pC}-1}$$