NAME: ________________________________

ECE 6340
Fall 2010

EXAM I

INSTRUCTIONS:

This exam is open-book and open-notes. You may use any material or calculator that you wish. Laptops or other devices that may be used to communicate are not allowed.

- Put all of your answers in terms of the parameters given in the problems, unless otherwise noted.
- Include units with all answers in order to receive full credit.
- Please write all of your work on the sheets attached (if you need more room, you may write on the backs of the pages)

Please show *all of your work* and *write neatly* in order to receive credit.

Problem 1 (40 pts)

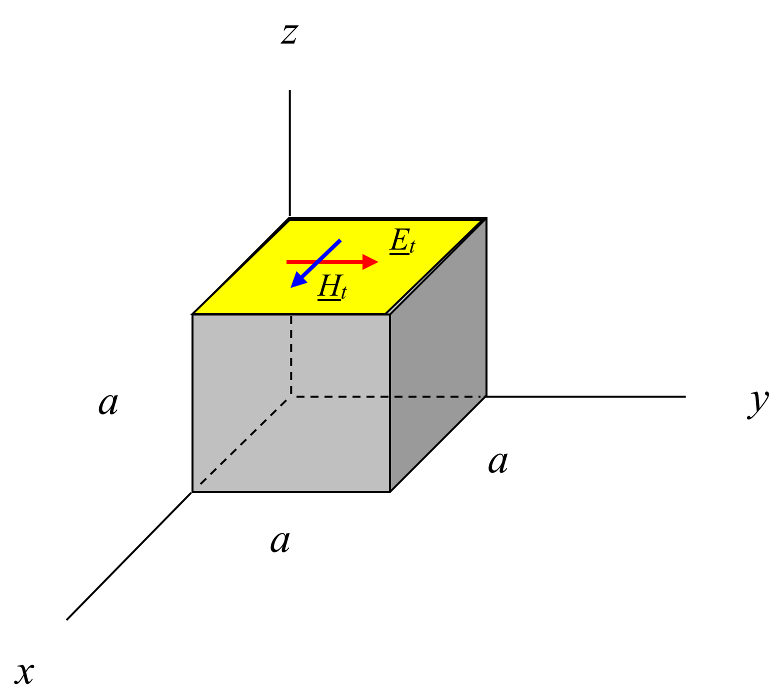
A cube consists of perfect conductor on all faces except the top face ($z = a$). On the top face the tangential electric and magnetic fields (the x and y components) at a frequency ω are given by

$$E_y = \sin\left(\frac{\pi x}{a}\right)$$
$$H_x = (A - jB)\sin\left(\frac{\pi x}{a}\right),$$

where A and B are positive real numbers. The cube is filled with a homogeneous lossy material having $\varepsilon_c = \varepsilon' - j\varepsilon''$.

a) Determine the time-average power being dissipated inside the cube due to material loses, in terms of the parameters given above.

b) Assume that the electric field inside the cube has only a y component and this component is a linear function of z, varying from zero at the bottom of the cube ($z = 0$) to the value that is given above at the top face ($z = a$). Determine the value of ε'' for the material, in terms of the parameters given above.



ROOM FOR WORK

Part (a)

$$\langle \mathscr{P}_d \rangle = \text{Re}\int_{top} S_z dx\, dy = \frac{1}{2}\text{Re}\int_0^a\int_0^a E_y H_x dx\, dy = \frac{1}{2}\text{Re}\int_0^a\int_0^a (A + jB)\sin^2\left(\frac{\pi x}{a}\right) dx\, dy$$
$$= \frac{1}{2}A\left(\frac{a^2}{2}\right)$$

Hence,

$$\langle \mathscr{P}_d \rangle = \left(\frac{a^2}{4}\right)A.$$

Part (b)

$$E_y = \left(\frac{z}{a}\right)\sin\left(\frac{\pi x}{a}\right).$$

Hence,

$$\langle \mathscr{P}_d \rangle = \frac{1}{2}(\omega\varepsilon'')\int_V |\underline{E}|^2\, dV$$
$$= \frac{1}{2}(\omega\varepsilon'')\int_0^a\int_0^a\int_0^a \left[\left(\frac{z}{a}\right)\sin\left(\frac{\pi x}{a}\right)\right]^2 dx\, dy\, dz$$
$$= \frac{1}{2}(\omega\varepsilon'')\left(\frac{a^2}{2}\right)\int_0^a \left(\frac{z}{a}\right)^2 dz$$
$$= \frac{1}{2}(\omega\varepsilon'')\left(\frac{a^2}{2}\right)\left(\frac{a}{3}\right)$$
$$= (\omega\varepsilon'')\left(\frac{a^3}{12}\right).$$

Equating the two formulas for the time-average power dissipation gives

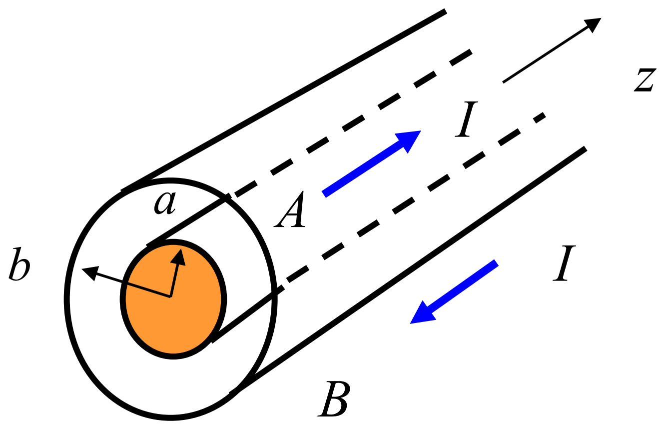
$$\left(\frac{a^2}{4}\right)A = (\omega\varepsilon'')\left(\frac{a^3}{12}\right)$$

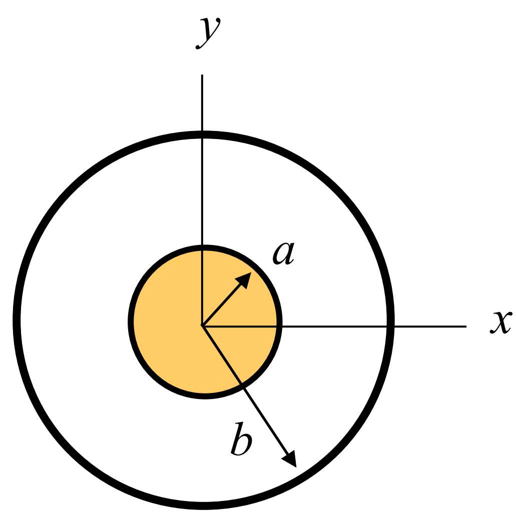
So that

$$\varepsilon'' = \frac{3A}{\omega a}.$$

Problem 2 (30 pts)

An air-filled coaxial cable is shown below.





Assume that the electric field of a TEM mode propagating in the z direction is described in the cross-sectional $z = 0$ plane by the function

$$\underline{E}(\rho) = \underline{\hat{\rho}}\, E_0 \left(\frac{1}{\rho} \right),$$

for some constant E_0.

Determine the time-average pressure (force per unit area) on the outer shield of the coax for this mode. For convenience, you may determine the pressure at $\phi = 0$ (i.e., $x = b$, $y = 0$), since the pressure is independent of the angle ϕ.

ROOM FOR WORK

The magnetic field is given by

$$\underline{H}(\rho) = \frac{1}{\eta}(\underline{\hat{z}} \times \underline{E}) = \underline{\hat{\phi}}\frac{1}{\eta}\left(\frac{E_0}{\rho}\right).$$

The Maxwell stress tensor is

$$T_{ij} = \varepsilon_0 \,\mathscr{E}_i \mathscr{E}_j + \mu_0 \,\mathscr{H}_i \mathscr{H}_j - \delta_{ij}\left(\frac{1}{2}\varepsilon_0 |\underline{\mathscr{E}}|^2 + \frac{1}{2}\mu_0 |\underline{\mathscr{H}}|^2\right).$$

At $\phi = 0$ we have

$$\underline{E} = \underline{\hat{x}}\left(\frac{E_0}{b}\right)$$

$$\underline{H} = \underline{\hat{y}}\left(\frac{E_0}{\eta b}\right).$$

The outward pressure is given by - T_{xx}.

We have

$$T_{xx} = \varepsilon_0 \,\mathscr{E}_x^2 - \left(\frac{1}{2}\varepsilon_0 |\underline{\mathscr{E}}|^2 + \frac{1}{2}\mu_0 |\underline{\mathscr{H}}|^2\right) = \frac{1}{2}\varepsilon_0 |\underline{\mathscr{E}}|^2 - \frac{1}{2}\mu_0 |\underline{\mathscr{H}}|^2.$$

The time-average is

$$\langle T_{xx} \rangle = \frac{1}{4}\varepsilon_0 |E_x|^2 - \frac{1}{4}\mu_0 |H_y|^2.$$

We then have

$$\langle T_{xx} \rangle = \frac{1}{4}\varepsilon_0 \left(\frac{E_0}{b}\right)^2 - \frac{1}{4}\mu_0 \left(\frac{E_0}{\eta_0 b}\right)^2 = \left(\frac{E_0}{2b}\right)^2 \left(\varepsilon_0 - \frac{\mu_0}{\eta_0^2}\right) = \left(\frac{E_0}{2b}\right)^2 (\varepsilon_0 - \varepsilon_0) = 0$$

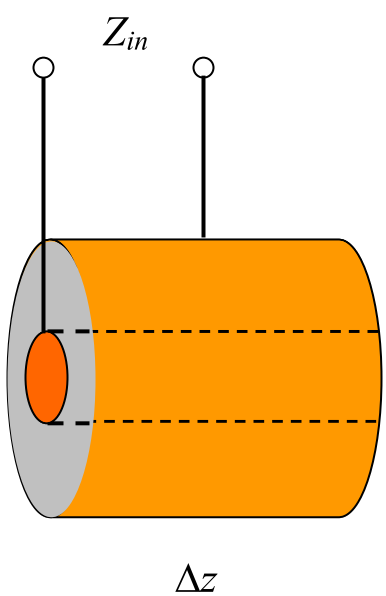
Hence the pressure is zero.

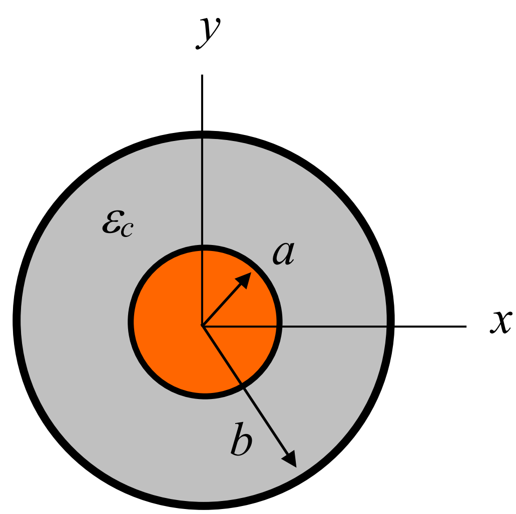
From a photon point of view, the photons travel straight down the cable, and do not reflect from the shield, and hence give no force to the shield.

Problem 3 (30 pts)

A coaxial cable is filled with a lossy dielectric material having $\varepsilon_c = \varepsilon' - j\varepsilon''$. An engineer takes a small length Δz of the cable and measures the input impedance Z_{in} seen between the inner and outer conductors at a frequency ω. For this small length of line it may be assumed that there is no z variation of the fields and that the structure is simply acting as a lossy capacitor.

Derive a formula for the propagation constant γ at the frequency ω in terms of Z_{in}, Δz, and the dimensions of a coax. It maybe assumed that the lossy dielectric material filing the coax is nonmagnetic. Assume that R (the resistance per unit length of the line) is small enough to be neglected.





ROOM FOR WORK

The propagation constant is

$$\gamma = jk_z = jk = j\omega\sqrt{\mu_0 \varepsilon_c}\ .$$

The input admittance when the line is air filled is

$$Y_{in}^0 = \frac{1}{Z_{in}^0} = j\omega C_0 \Delta z = j\omega\Delta z\left[\frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)}\right].$$

When it is filled with the lossy material it is

$$Y_{in}^0 = j\omega\Delta z\left[\frac{2\pi\varepsilon_c}{\ln\left(\frac{b}{a}\right)}\right].$$

Hence we have that

$$\varepsilon_c = \varepsilon' - j\varepsilon'' = \frac{1}{j\omega Z_{in}\Delta z 2\pi}\ln\left(\frac{b}{a}\right).$$

Hence,

$$\gamma = j\omega\sqrt{\mu_0 \frac{1}{j\omega Z_{in}\Delta z 2\pi}\ln\left(\frac{b}{a}\right)}.$$

Another approach:

$$\gamma = \sqrt{(j\omega L)(G + j\omega C)}\ .$$

Also, we have

$$Z_{in} = \frac{1}{(G + j\omega C)\Delta z}.$$

Therefore we have

$$\left(G + j\omega C\right) = \frac{1}{Z_{in}\Delta z}$$

Hence we have

$$\gamma = \sqrt{\left(j\omega L\right)\frac{1}{Z_{in}\Delta z}}\,.$$

Substituting for *L*, we have

$$\gamma = \sqrt{j\omega\left(\frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)\right)\left(\frac{1}{Z_{in}\Delta z}\right)}$$

or

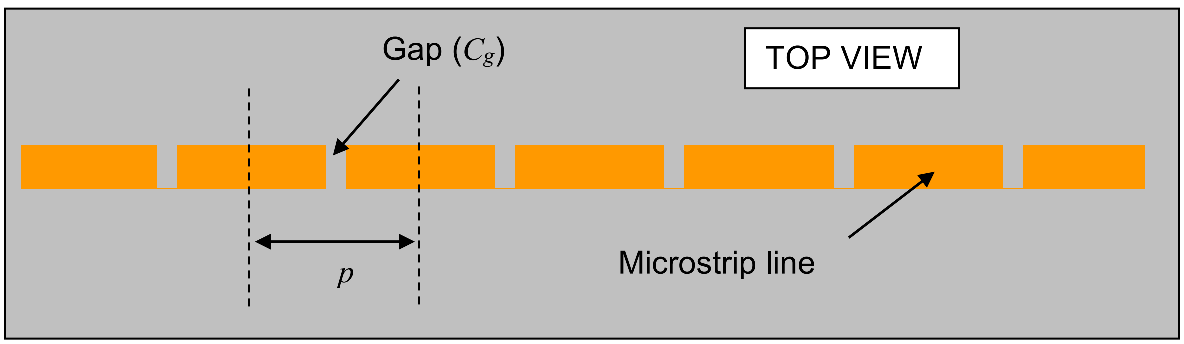
$$\gamma = j\omega\sqrt{\frac{1}{j\omega}\left(\frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)\right)\left(\frac{1}{Z_{in}\Delta z}\right)}\,.$$

Problem 4 (30 pts)

A lossless microstrip line has a characteristic impedance of Z_0^M and a wavenumber of k_z^M. A periodic set of gaps is placed along the line as shown in the figure below, where the period p is much smaller than a guided wavelength. Each gap has a capacitance of C_g.

(a) Derive a formula for the wavenumber k_z of the structure. The result should be in terms of Z_0^M, k_z^M, C_g, and p.

(b) Make a sketch of what α and β look like vs. frequency ω. The sketch should label the values of any important frequencies such as where α or β go to zero, and well as the value of α or β at zero frequency. The slope of α or β at high frequency should be indicated as well.



ROOM FOR WORK

Part (a)

The capacitance and inductance per unit length are given by

$$Z_0^M = \sqrt{\frac{L}{C}}$$
$$k_z^M = \omega\sqrt{LC}$$

so that

$$L = \frac{k_z^M Z_0^M}{\omega}$$
$$C = \frac{k_z^M}{\omega Z_0^M}.$$

We then have

$$Z = j\omega L + \frac{1}{p}\left(\frac{1}{j\omega C_g}\right)$$

and

$$Y = j\omega C .$$

Hence we have

$$\gamma = \alpha + j\beta = \sqrt{\left(j\omega L + \frac{1}{j\omega p C_g}\right)(j\omega C)}$$

or

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}\sqrt{1 - \frac{1}{\omega^2 L C_g p}} .$$

Define

$$\omega_c = \frac{1}{\sqrt{pLC_g}} .$$

Then we have

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}\sqrt{1-\frac{\omega_c^2}{\omega^2}}.$$

Part (b)

From the propagation constant we have the following results:

$$\omega < \omega_c: \quad \alpha = \omega\sqrt{LC}\sqrt{\frac{\omega_c^2}{\omega^2}-1}, \qquad \beta = 0$$

$$\omega > \omega_c: \quad \beta = \omega\sqrt{LC}\sqrt{1-\frac{\omega_c^2}{\omega^2}}, \qquad \alpha = 0$$