

NAME: _____ **SOLUTION** _____

ECE 6340
Fall 2013
EXAM I

INSTRUCTIONS:

This exam is open-book and open-notes. You may use any material or calculator that you wish, as long as it does not have any communication capability. Laptops or other devices that may be used to communicate are not allowed.

- Put all of your answers in terms of the parameters given in the problems, unless otherwise noted.
- Include units with all numerical answers.
- Please circle your final answers.
- Please write all of your work on the sheets attached (if you need more room, you may write on the backs of the pages).

Please show *all of your work* and *write neatly* in order to receive credit.

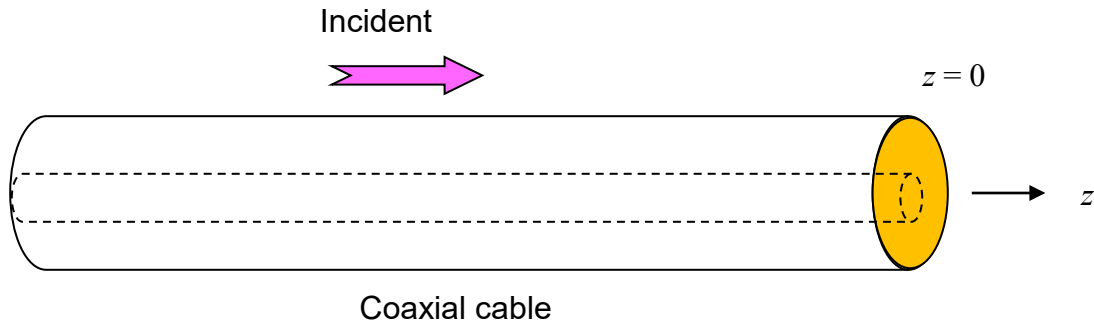
Problem 1 (25 pts)

A coaxial cable is air filled and has an inner radius of a and an outer radius of b . An incident wave at a frequency ω with a voltage of 1.0 [V] is incident on a short-circuit plate at the end of the line, at $z = 0$, as shown below. The incident voltage in the phasor domain is

$$V_{inc}(z) = e^{-jk_0 z}.$$

(Voltage is defined as the voltage on the inner conductor relative to the voltage on the outer conductor, and current is defined as the current in the z direction on the inner conductor.)

Determine the force in the z direction on the short-circuit plate using the Maxwell stress tensor.



ROOM FOR WORK

The voltage will be

$$V(z) = e^{-jk_0 z} - e^{+jk_0 z}.$$

The current will be

$$I(z) = \frac{1}{Z_0} (e^{-jk_0 z} + e^{+jk_0 z}).$$

The electric field and magnetic fields of the incident wave will be

$$\underline{E}^{inc}(\rho) = \underline{\hat{\rho}} \frac{1}{\rho} \left(\frac{1}{\ln(b/a)} \right) e^{-jk_0 z}$$

$$\underline{H}^{inc}(\rho) = \underline{\hat{\phi}} \frac{1}{\eta_0} \frac{1}{\rho} \left(\frac{1}{\ln(b/a)} \right) e^{-jk_0 z}.$$

The total electric and magnetic fields will be

$$\underline{E}(\rho) = \underline{\hat{\rho}} \frac{1}{\rho} \left(\frac{1}{\ln(b/a)} \right) (e^{-jk_0 z} - e^{+jk_0 z})$$

$$\underline{H}(\rho) = \underline{\hat{\phi}} \frac{1}{\eta_0} \frac{1}{\rho} \left(\frac{1}{\ln(b/a)} \right) (e^{-jk_0 z} + e^{+jk_0 z}).$$

At $z = 0$ there is no electric field, and the magnetic field is given by

$$\underline{H}(\rho) = \underline{\hat{\phi}} \frac{2}{\eta_0} \frac{1}{\rho} \left(\frac{1}{\ln(b/a)} \right).$$

The Maxwell stress tensor is

$$T_{ij} = \varepsilon_0 \mathcal{E}_i \mathcal{E}_j + \mu_0 \mathcal{H}_i \mathcal{H}_j - \delta_{ij} \left(\frac{1}{2} \varepsilon_0 |\underline{\mathcal{E}}|^2 + \frac{1}{2} \mu_0 |\underline{\mathcal{H}}|^2 \right).$$

For the TEM mode, this reduces to

$$T_{zz} = -\frac{1}{2}\mu_0 |\underline{\mathcal{H}}|^2.$$

The time-average of this is

$$\langle T_{zz} \rangle = -\frac{1}{4}\mu_0 |\underline{H}|^2.$$

The time-average force per unit area is then

$$\langle \mathcal{F}_z \rangle = \int_S \frac{1}{4}\mu_0 |\underline{H}|^2 dS.$$

Hence, we have

$$\langle \mathcal{F}_z \rangle = \int_S \frac{1}{4}\mu_0 |\underline{H}|^2 \rho d\rho d\phi = 2\pi \int_a^b \frac{1}{4}\mu_0 |\underline{H}|^2 \rho d\rho.$$

This gives us

$$\langle \mathcal{F}_z \rangle = 2\pi \int_a^b \frac{1}{4}\mu_0 \left| \frac{2}{\eta_0} \frac{1}{\rho} \left(\frac{1}{\ln(b/a)} \right) \right|^2 \rho d\rho.$$

Therefore, we have

$$\langle \mathcal{F}_z \rangle = 2\pi\mu_0 \left(\frac{1}{\eta_0 \ln(b/a)} \right)^2 \int_a^b \frac{1}{\rho} d\rho.$$

This gives us

$$\langle \mathcal{F}_z \rangle = \frac{2\pi\mu_0}{\eta_0^2} \left(\frac{1}{\ln(b/a)} \right).$$

This can then be written as

$$\langle \mathcal{F}_z \rangle = \left(\frac{2\pi\epsilon_0}{\ln(b/a)} \right) \text{ [N]}.$$

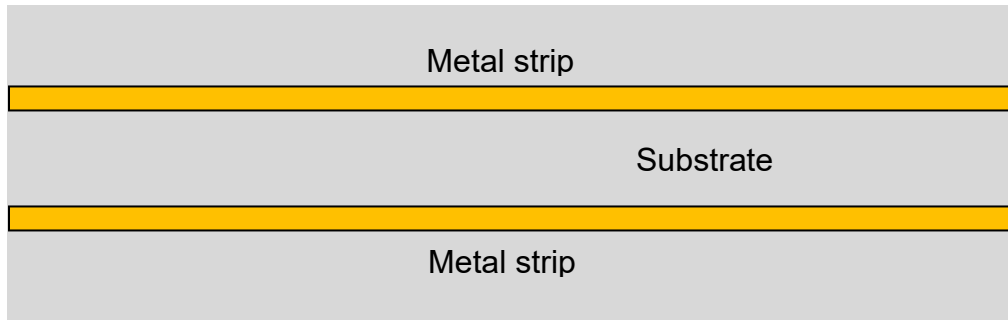
Problem 2 (25 pts)

A transmission line consists of two parallel strips of metal (microstrip lines) that are printed on top of a grounded substrate as shown in part (a) of the figure below (a top view is shown). This transmission line is called a “coplanar strips” transmission line. One of the lines is the “A” conductor and the other one is the “B” conductor. It is thus similar to twin-lead, except that the two conductors are flat strips instead of round wires, and they are printed on a grounded substrate instead of being in free space.

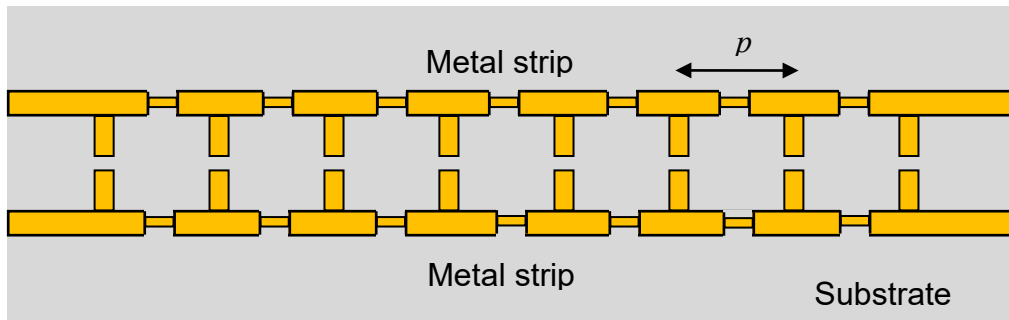
Assume that this transmission line has a known characteristic impedance of Z_0 and a known effective relative permittivity of ϵ_r^{eff} . (The phase velocity is the speed of light in vacuum (denoted as c) divided by the square root of the effective relative permittivity.) Assume that the mode is quasi-TEM, and thus may be taken as TEM for modeling purposes. Also assume that the structure is lossless.

The line is then modified by adding narrow series segments and shunt capacitive gaps as shown in part (b) of the figure below, with a period of p for the unit cell length. Assume that each narrow segment acts as a series inductance of L_e [H] and each shunt segment adds a shunt capacitance of C_e [F]. You can neglect the length of the series segments, since they are small compared to the unit cell length p ; a series segment is thus equivalent to an ideal series inductance of L_e that has no length.

Calculate R , defined as the ratio of the phase constant on the structure relative to that on the unmodified (coplanar strips) structure. You should carry out the algebra to the point where it is obvious that this ratio R is not a function of frequency, but only depends on c , Z_0 , L_e , C_e , p and ϵ_r^{eff} .



(a) Coplanar strips transmission line



(b) Modified transmission line

ROOM FOR WORK

The propagation constant is given by

$$\gamma = \sqrt{ZY}.$$

The equivalent circuit of the line has lumped circuit elements

$$L_c = Lp = \left(\frac{k_z Z_0}{\omega} \right) p = \left(\frac{k_0 \sqrt{\epsilon_r^{eff}} Z_0}{\omega} \right) p$$

$$C_c = Cp = \left(\frac{k_z}{\omega Z_0} \right) p = \left(\frac{k_0 \sqrt{\epsilon_r^{eff}}}{\omega Z_0} \right) p.$$

We then have

$$Z = j\omega(L_c + L_e) \frac{1}{p} = j\omega \left(\frac{pk_0 \sqrt{\epsilon_r^{eff}} Z_0}{\omega} + L_e \right) \frac{1}{p}$$

$$Y = j\omega \left(p \frac{k_0 \sqrt{\epsilon_r^{eff}}}{\omega Z_0} + C_e \right) \frac{1}{p}.$$

Hence

$$\gamma = j\omega \sqrt{\left(\frac{k_0 \sqrt{\epsilon_r^{eff}} Z_0}{\omega} + \frac{L_e}{p} \right) \left(\frac{k_0 \sqrt{\epsilon_r^{eff}}}{\omega Z_0} + \frac{C_e}{p} \right)}.$$

Hence we have

$$\beta = \omega \sqrt{\left(\frac{k_0 \sqrt{\epsilon_r^{eff}} Z_0}{\omega} + \frac{L_e}{p} \right) \left(\frac{k_0 \sqrt{\epsilon_r^{eff}}}{\omega Z_0} + \frac{C_e}{p} \right)}$$

$$\beta = k_0 \sqrt{\left(\sqrt{\epsilon_r^{eff}} Z_0 + \frac{\omega L_e}{k_0 p} \right) \left(\frac{\sqrt{\epsilon_r^{eff}}}{Z_0} + \frac{\omega C_e}{k_0 p} \right)}$$

or

$$\beta = k_0 \sqrt{\epsilon_r^{eff}} \sqrt{\left(Z_0 + \frac{\omega L_e}{k_0 p \sqrt{\epsilon_r^{eff}}} \right) \left(\frac{1}{Z_0} + \frac{\omega C_e}{k_0 p \sqrt{\epsilon_r^{eff}}} \right)}$$

or

$$\beta = k_0 \sqrt{\epsilon_r^{eff}} \sqrt{\left(Z_0 + \frac{c L_e}{p \sqrt{\epsilon_r^{eff}}} \right) \left(\frac{1}{Z_0} + \frac{c C_e}{p \sqrt{\epsilon_r^{eff}}} \right)}$$

where c is the speed of light in vacuum.

Hence, the ratio of the phase constant to the unmodified phase constant is

$$R = \sqrt{\left(Z_0 + \frac{c L_e}{p \sqrt{\epsilon_r^{eff}}} \right) \left(\frac{1}{Z_0} + \frac{c C_e}{p \sqrt{\epsilon_r^{eff}}} \right)}.$$

Problem 3 (25 pts)

A shallow dish of water is placed in a microwave oven as shown below. The radian frequency of the oven is $\omega = 2\pi f$. The volume of the water is V . Immediately above the water, in the air region, the electric field is perfectly vertical (in the z direction) and uniform, and is given (in the time domain) by

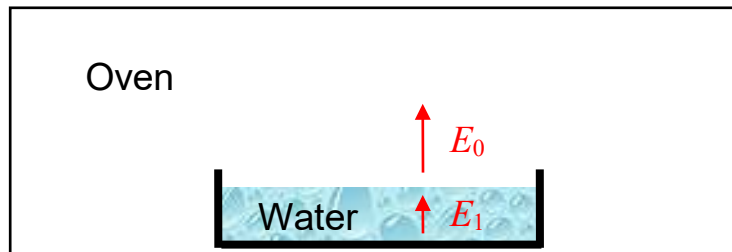
$$\underline{\mathcal{E}}_0 = \hat{z} A_0 \cos(\omega t).$$

Assume that the electric field $\underline{\mathcal{E}}_1$ inside the water is also uniform and vertical (but is not necessarily the same as the electric field $\underline{\mathcal{E}}_0$ above the water). The water has a complex relative permittivity given by

$$\epsilon_r = \epsilon'_r - j\epsilon''_r.$$

Also, the water has salt dissolved in it so that it has a conductivity σ . Hence, the water has a complex effective relative permittivity ϵ_{rc} .

- Derive a formula for the power being dissipated inside the water. Your final answer should be in terms of V , A_0 , ω , ϵ_0 , ϵ'_{rc} , and ϵ''_{rc} .
- Assume that the conductivity of the water is high so that the loss due to polarization is negligible compared to the loss from the conductivity. Solve for what the conductivity σ should be in order to maximize the power being dissipated within the water at a fixed frequency ω . Your answer should be in terms of ω , ϵ_0 and ϵ'_{rc} .



ROOM FOR WORK

In the phasor domain, the electric field above the water is

$$\underline{E}_0 = \hat{z} A_0.$$

The electric field inside the water is

$$\underline{E}_1 = \hat{z} A_0 \left(\frac{1}{\epsilon_{rc}} \right) = \hat{z} A_0 \left(\frac{1}{\epsilon'_{rc} - j\epsilon''_{rc}} \right).$$

The power dissipation inside the water is

$$P_d = \langle \mathcal{P}_d \rangle = \frac{1}{2} (\omega \epsilon''_c) |\underline{E}_1|^2 V.$$

Hence we have

$$P_d = \langle \mathcal{P}_d \rangle = \frac{1}{2} (\omega \epsilon''_c) |A_0|^2 \left(\frac{1}{|\epsilon_{rc}|^2} \right) V.$$

This may be written as

$$P_d = \langle \mathcal{P}_d \rangle = \frac{1}{2} |A_0|^2 \epsilon_0 \left(\frac{\omega \epsilon''_{rc}}{\epsilon'^2_{rc} + \epsilon''^2_{rc}} \right) V.$$

Under the assumption that the loss is mainly from conductivity, we have

$$P_d = \frac{1}{2} |A_0|^2 \epsilon_0 \left(\frac{\omega \left(\frac{\sigma}{\omega \epsilon_0} \right)}{\epsilon'^2_{rc} + \left(\frac{\sigma}{\omega \epsilon_0} \right)^2} \right) V$$

or

$$P_d = \frac{1}{2} |A_0|^2 V \omega \epsilon_0 \left(\frac{\left(\frac{\sigma}{\omega \epsilon_0} \right)}{\epsilon_{rc}'^2 + \left(\frac{\sigma}{\omega \epsilon_0} \right)^2} \right).$$

The term in parenthesis is maximized when

$$\frac{\sigma}{\omega \epsilon_0} = \epsilon_{rc}'.$$

Hence the result is that the optimum conductivity satisfies

$$\frac{\sigma}{\omega \epsilon_0 \epsilon_{rc}'} = 1.$$

This means that the loss tangent is one.

Problem 4 (25 pts)

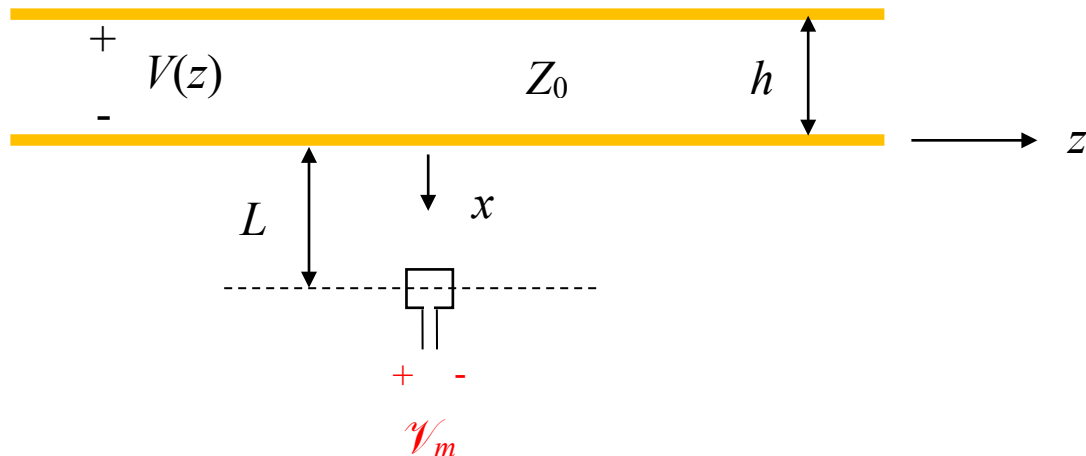
Two lossless narrow parallel wires form an infinite twin-lead transmission line in air. The separation between the centers of the wires is h . The lower wire has its axis aligned with the z axis. The characteristic impedance of the twin-lead transmission line is Z_0 . A small rectangular loop of wire is located next to the transmission line as shown below. The area of the loop is A , and the center of the loop is located at $z = 0$ and $x = L$. The size of the loop is small compared with the distance L . The loop is connected to an ideal voltmeter (not shown) so that the loop is open-circuited.

Assume that a sinusoidal steady-state voltage signal propagates down the line, with a phasor voltage given by

$$V(z) = V_0 e^{-jk_0 z}.$$

Assume that the frequency is low enough that the wavelength is very large compared with the distance to the loop and the distance between the wires. Therefore, the magnetic field produced by each wire in the time domain at any time t may be found by treating the current on each wire as a uniform DC current, with the value of the DC current being that of the actual sinusoidal current at that time t .

Determine the voltage reading on the voltmeter $\mathcal{V}_m(t)$ as a function of time.



ROOM FOR WORK

The magnetic field at the center of the loop in the phasor domain is

$$\underline{H} = \underline{\hat{y}} \left(\frac{V_0}{Z_0} \right) \left(\frac{1}{L+h} - \frac{1}{L} \right) \left(\frac{1}{2\pi} \right).$$

In the time domain we have

$$\mathcal{H} = \underline{\hat{y}} \left(\frac{V_0}{Z_0} \right) \left(\frac{1}{L+h} - \frac{1}{L} \right) \left(\frac{1}{2\pi} \right) \cos(\omega t).$$

The magnetic flux through the loop is (assuming a counterclockwise contour)

$$\psi = -\mu_0 H_y A = -\mu_0 A \left(\frac{V_0}{Z_0} \right) \left(\frac{1}{L+h} - \frac{1}{L} \right) \left(\frac{1}{2\pi} \right) \cos(\omega t).$$

The voltage output read by the voltmeter is

$$V_m = \oint_C \underline{E} \cdot d\underline{r} = -\frac{d\psi}{dt}.$$

Hence we have

$$V_m = -\omega \mu_0 A \left(\frac{V_0}{Z_0} \right) \left(\frac{1}{L+h} - \frac{1}{L} \right) \left(\frac{1}{2\pi} \right) \sin(\omega t).$$