NAME: ________________________________

ECE 6340
Fall 2015

EXAM I

SOLUTION

INSTRUCTIONS:

This exam is open-book and open-notes. You may use any material or calculator that you wish, as long as it does not have any communication capability. Laptops or other devices that may be used to communicate are not allowed.

- Put all of your answers in terms of the parameters given in the problems, unless otherwise noted.
- Include units with all numerical answers.
- Please circle your final answers.
- Please write all of your work on the sheets attached (if you need more room, you may write on the backs of the pages).

Please show *all of your work* and *write neatly* in order to receive credit.

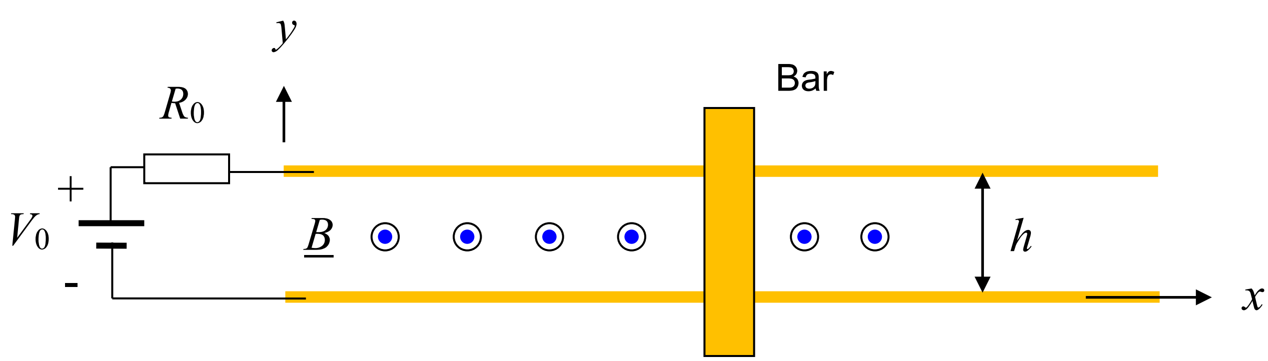
Problem 1 (25 pts)

A sliding bar is located at $x = f(t)$. (The function $f(t)$ is assumed to be known, but is arbitrary.) The sliding bar has a resistance of R_b. Each of the two rails has a resistance per unit length of R [Ω/m]. At the left end is a voltage source V_0 with a Thévenin resistance R_0. A magnetic field exists between the rails, given by

$$\underline{B} = \hat{\underline{z}} B_0 \, .$$

a) Derive a formula for the force $F_x\,(t)$ on the bar.

b) If the bar is sliding at a constant steady-state velocity in the x direction so that there is no force F_x on the bar, find the steady-state velocity of the bar in the x direction.



ROOM FOR WORK

Part (a)

The force comes from the current through the bar. Assuming that $i(t)$ is the current through the bar flowing in the y direction, The force is given by

$$\underline{F} = \int_0^h i(t)\,\underline{\hat{y}}dy \times \underline{B} = i(t)\,\underline{\hat{x}}B_0 \int_0^h dy = i(t)\,\underline{\hat{x}}B_0 h\,.$$

Hence,

$$\underline{F} = \underline{\hat{x}}\,i(t)\,B_0 h\,.$$

Therefore,

$$F_x = i(t)\,B_0 h\,.$$

To find the current, we use

$$EMF = R_b i(t) + 2Rx(t)i(t) + R_0 i(t) + V_0 = -\frac{d\psi}{dt}\,.$$

We also have

$$\psi = B_0 h x(t)\,.$$

Hence,

$$\frac{d\psi}{dt} = B_0 h x'(t) = B_0 h f'(t)\,.$$

Therefore, we have

$$R_b i(t) + 2Rf(t)i(t) + R_0 i(t) + V_0 = -B_0 h f'(t)\,.$$

This gives us

$$i(t) = -\frac{B_0 h f'(t) + V_0}{R_b + R_0 + 2Rf(t)}\,.$$

We then have

$$F_x = B_0 h\left(-\frac{B_0 h f'(t) + V_0}{R_b + R_0 + 2Rf(t)}\right).$$

Part (b)

If the force is zero, we have

$$B_0 h f'(t) + V_0 = 0.$$

This implies that

$$v_x = \frac{df}{dt} = -\frac{V_0}{B_0 h}.$$

Problem 2 (25 pts)

An incident plane wave at a frequency ω [rad/s] is propagating in the negative z direction, where it reflects off of a lossless semi-infinite dielectric region. The fields in the air region ($z > 0$) are given by

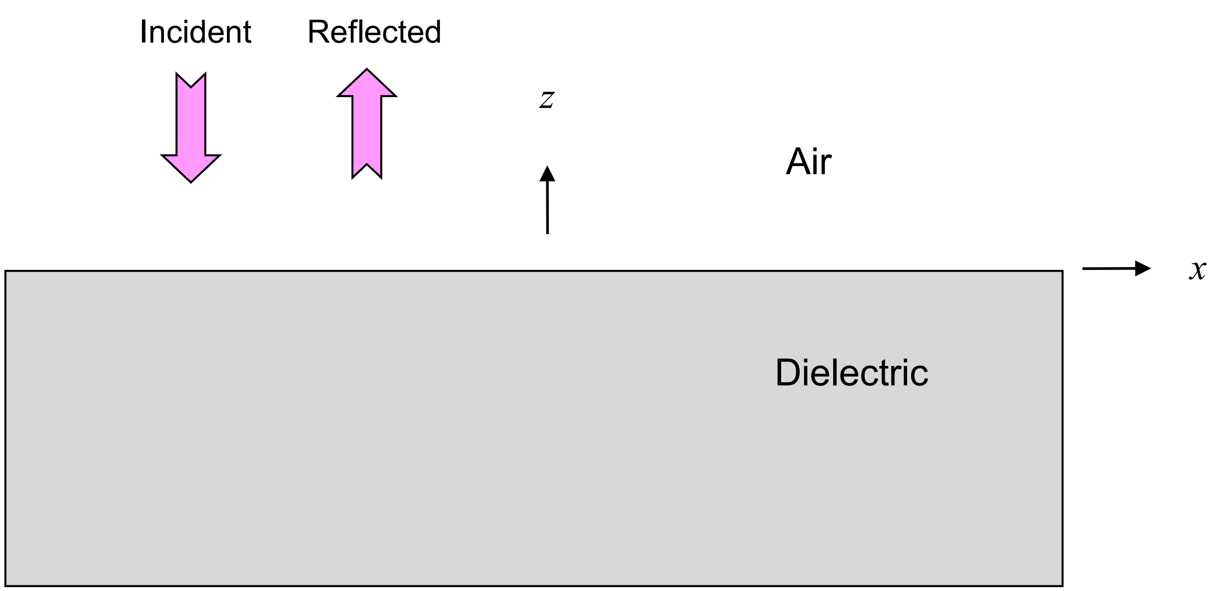
$$\underline{E} = \underline{\hat{x}}\, E_0 \left(e^{+jk_0 z} + \Gamma e^{-jk_0 z} \right)$$

$$\underline{H} = -\underline{\hat{y}} \frac{E_0}{\eta_0} \left(e^{+jk_0 z} - \Gamma e^{-jk_0 z} \right),$$

where Γ is a known real-valued reflection coefficient.

a) Determine the time-average force vector per unit area on the dielectric interface using the Maxwell stress tensor.

b) Determine the time-average force vector per unit area on the dielectric interface using the concept of photons bouncing from the interface. Note that the percentage of power being reflected from the interface is $100\,|\Gamma|^2$.



ROOM FOR WORK

Part (a)

We have

$$\mathscr{F}_z = \underline{\hat{z}} \cdot \left(\underline{\underline{T}} \cdot \underline{\hat{z}} \right) = T_{zz}$$

where

$$T_{zz} = -\frac{1}{2}\varepsilon_0 \left|\underline{\mathscr{E}}\right|^2 - \frac{1}{2}\mu_0 \left|\underline{\mathscr{H}}\right|^2 .$$

We then have

$$\left\langle \mathscr{F}_z \right\rangle = -\frac{1}{4}\varepsilon_0 \left|E_x\right|^2 - \frac{1}{4}\mu_0 \left|H_y\right|^2 .$$

We then have

$$\left\langle \mathscr{F}_z \right\rangle = -\frac{1}{4}\varepsilon_0 \left|E_0\right|^2 \left(1+\Gamma\right)^2 - \frac{1}{4}\mu_0 \left|E_0\right|^2 \frac{1}{\eta_0^2}\left(1-\Gamma\right)^2 .$$

This can be written as

$$\left\langle \mathscr{F}_z \right\rangle = -\frac{1}{4}\varepsilon_0 \left|E_0\right|^2 \left[\left(1+\Gamma\right)^2 + \left(1-\Gamma\right)^2 \right]$$

or

$$\left\langle \mathscr{F}_z \right\rangle = -\frac{1}{4}\varepsilon_0 \left|E_0\right|^2 \left[2\left(1+\Gamma^2\right) \right].$$

Hence we have

$$\left\langle \mathscr{F}_z \right\rangle = -\frac{1}{2}\varepsilon_0 \left|E_0\right|^2 \left(1+\Gamma^2\right).$$

Part (b)

From the photon point of view, we have

$$-\left\langle \mathscr{F}_z \right\rangle = \frac{1}{c^2}\left\langle -\mathscr{S}_z^{inc} \right\rangle c\left(1+\Gamma^2\right).$$

This gives us

$$\left\langle \mathscr{F}_z \right\rangle = \frac{1}{c^2}\left(-\frac{\left|E_0^2\right|}{2\eta_0} \right) c\left(1+\Gamma^2\right).$$

We then have

$$\left\langle \mathscr{F}_z \right\rangle = -\varepsilon_0 \left(\frac{\left|E_0\right|^2}{2} \right)\left(1+\Gamma^2\right)$$

or

$$\left\langle \mathscr{F}_z \right\rangle = -\frac{1}{2}\varepsilon_0 \left|E_0\right|^2 \left(1+\Gamma^2\right).$$

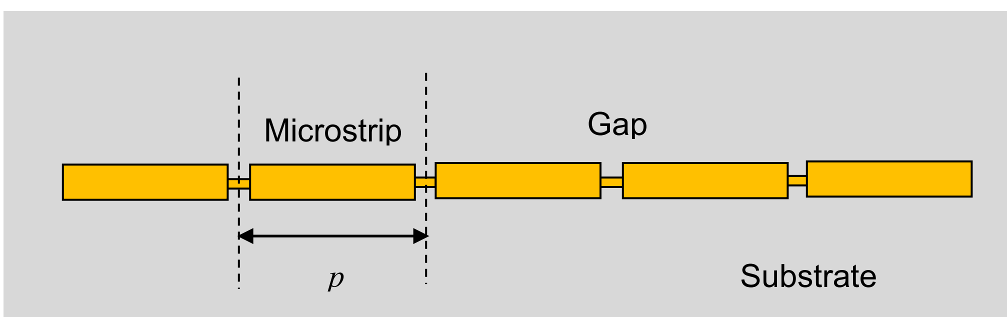
Problem 3 (25 pts)

A lossless microstrip line has a periodic set of small capacitive gaps. Inside each gap is a small narrow line that acts as an inductor. Each gap is thus modeled as a capacitor C_g in parallel with an inductor L_g. The length of each gap region is small enough to be ignored (but you cannot ignore C_g and L_g). The microstrip line has a characteristic impedance Z_0 and a phase constant β. Assume that the microstrip line acts as a TEM transmission line with an effective relative permittivity that is constant, where

$$\beta = k_0 \sqrt{\varepsilon_r^{eff}} \ .$$

a) Determine the propagation constant γ for this structure. Leave your answer in terms of ω, Z_0, ε_r^{eff}, p, L_g, C_g.

b) Determine the frequency range for ω over which the wave on the system will be attenuated. Show that this structure acts as a bandstop filter, meaning that it will attenuate waves in the frequency range $\omega_1 < \omega < \omega_2$, and solve for the frequency limits ω_1 and ω_2 in terms of the given parameters.



ROOM FOR WORK

Part (a)

For this artificial transmission line we have

$$\gamma = \sqrt{ZY} .$$

We also have

$$Z = \frac{1}{p}\left(j\omega Lp + \frac{1}{j\omega C_g + \frac{1}{j\omega L_g}} \right)$$

and

$$Y = j\omega C ,$$

where L and C are the per-unit-length parameters of the microstrip line, determined from

$$\omega\sqrt{LC} = k_0\sqrt{\varepsilon_r^{eff}}$$

and

$$\sqrt{\frac{L}{C}} = Z_0 .$$

Thus, we have

$$L = \frac{Z_0\sqrt{\varepsilon_r^{eff}}}{c}$$

and

$$C = \frac{\sqrt{\varepsilon_r^{eff}}}{cZ_0} ,$$

where c is the speed of light in vacuum.

We then have

$$\gamma = \sqrt{\left(j\omega L + \frac{j\omega L_g / p}{1 - \omega^2 L_g C_g} \right)\left(j\omega C \right)}$$

or

$$\gamma = j\omega\sqrt{LC}\sqrt{1 + \frac{L_g / \left(Lp\right)}{1 - \omega^2 L_g C_g}}.$$

Part (b)

The wave will be attenuated when

$$1 + \frac{L_g / \left(Lp\right)}{1 - \omega^2 L_g C_g} < 0.$$

This implies that

$$\frac{L_g / \left(Lp\right)}{\omega^2 L_g C_g - 1} > 1$$

or

$$L_g / \left(Lp\right) > \omega^2 L_g C_g - 1 > 0$$

or

$$1 < \omega^2 L_g C_g < 1 + L_g / \left(Lp\right)$$

or

$$\frac{1}{\sqrt{L_g C_g}} < \omega < \sqrt{\frac{1 + L_g / \left(Lp\right)}{L_g C_g}}.$$

Hence, the wave will be attenuated when

$$\omega_1 < \omega < \omega_2$$

where

$$\omega_1 = \frac{1}{\sqrt{L_g C_g}}, \quad \omega_2 = \sqrt{\frac{1 + L_g / \left(Lp\right)}{L_g C_g}}.$$

ROOM FOR WORK