NAME: __________SOLUTION_________________________

ECE 6340

Fall 2016

Exam 1

Nov. 2, 2016

INSTRUCTIONS:

This exam is open-book and open-notes. You may use any material or calculator that you wish, as long as it does not have any communication capability. Laptops or other devices that may be used to communicate are not allowed.

- Put all of your answers in terms of the parameters given in the problems, unless otherwise noted.
- Include units with all numerical answers.
- Please circle your final answers.
- Please write all of your work on the sheets attached (if you need more room, you may write on the backs of the pages).

Please show *all of your work* and *write neatly* in order to receive credit.

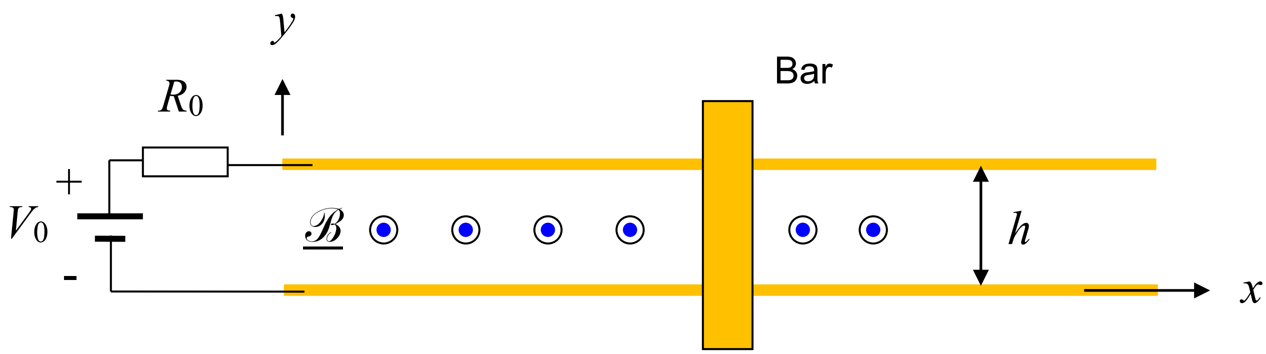
Problem 1 (25 pts)

A sliding bar is located at $x = L + v_0 t$, where L is fixed (the position at $t = 0$). The bar is moving to the right with a velocity of v_0. The sliding bar has a resistance of R_b. Each of the two rails has a resistance per unit length of R [Ω/m]. At the left end is a voltage source V_0 with a Thévenin resistance R_0. A magnetic field exists between the rails, given by

$$\underline{\mathscr{B}} = \underline{\hat{z}} B_0 \cos(\omega t) .$$

1) Find the current $i(t)$ flowing upward through the bar.

2) Find the voltage drop across the bar, with the + sign of the voltage drop being on the bottom of the bar.



ROOM FOR WORK

Part (1)

From the EMF form of the Faraday's law we have

$$EMF = -\frac{d\psi}{dt}.$$

This gives us (going counterclockwise around the path)

$$R_b i(t) + 2R(L + v_0 t) i(t) + R_0 i(t) + V_0 = -\frac{d\psi}{dt}.$$

The flux is

$$\psi = \left(B_0 \cos(\omega t)\right)\left(h(L + v_0 t)\right).$$

Hence, we have

$$\frac{d\psi}{dt} = -\omega h B_0 (L + v_0 t)\sin(\omega t) + (h v_0) B_0 \cos(\omega t).$$

Therefore, we have

$$i(t) = \frac{-V_0 + \omega h B_0 (L + v_0 t)\sin(\omega t) - (h v_0 B_0)\cos(\omega t)}{R_b + 2R(L + v_0 t) + R_0}.$$

Part (2)

Using the voltage form of Faraday's law, we have

$$\mathscr{V} = \oint_C \underline{\mathscr{E}} \cdot d\underline{r} = \int_S -\frac{\partial \underline{\mathscr{B}}}{\partial t} \cdot \underline{\hat{n}}\, dS = -h(L + v_0 t)\frac{\partial \mathscr{B}_z}{\partial t} = -h(L + v_0 t)\left(-\omega B_0 \sin(\omega t)\right).$$

For the voltage drop cross the bar $\mathscr{V}_b$, we then have

$$\mathscr{V}_b + 2R(L + v_0 t) i(t) + R_0 i(t) + V_0 = -h(L + v_0 t)\left(-\omega B_0 \sin(\omega t)\right).$$

Hence we have

$$\mathcal{V}_b(t) = -h(L + v_0 t)(-\omega B_0 \sin(\omega t)) - 2R(L + v_0 t)i(t) - R_0 i(t) - V_0 .$$

Problem 2 (25 pts)

Assume that water obeys the Debye model with the following parameters:

$$\varepsilon_r'(0) = 81$$
$$\varepsilon_r'(\infty) = 2$$
$$\tau = 1/\left(2\pi \cdot 18 \times 10^9\right) \text{ [s/rad]}.$$

Salt has been dissolved into the water so that is also has a conductivity of $\sigma = 4$ [S/m].

Determine the power in watts/m^3 being absorbed by the water at 2.54 GHz if the electric field inside the water has an amplitude of 1.0 [V/m] in the phasor domain.

Solution

The time-average power dissipated per cubic meter as heat is

$$\langle \mathscr{P}_d \rangle = \frac{1}{2}\left(\omega \varepsilon_c''\right)\left|\underline{E}\right|^2 .$$

Hence, we have

$$\langle \mathscr{P}_d \rangle = \pi f \varepsilon_c'' .$$

We have

$$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}.$$

We then have from the Debye model and from the concept of effective permittivity that

$$\varepsilon_c'' = \varepsilon_0 \varepsilon_r'' + \frac{\sigma}{\omega} = \varepsilon_0 \omega \tau \left[\frac{\varepsilon_r(0) - \varepsilon_r(\infty)}{1 + (\omega\tau)^2}\right] + \frac{\sigma}{\omega}.$$

This gives us

$$\varepsilon_c'' = \left(8.854\times10^{-12}\right)\left(2\pi\cdot2.54\times10^{9}\right)\left(2\pi\cdot18\times10^{9}\right)^{-1}\left[\frac{81-2}{1+\left(\frac{2\pi\cdot2.54\times10^{9}}{2\pi\cdot18\times10^{9}}\right)^{2}}\right]+\frac{4}{2\pi\cdot2.54\times10^{9}}.$$

Performing the calculation, we have

$$\varepsilon_c'' = \left(8.854\times10^{-12}\right)\left(2\pi\cdot2.54\times10^{9}\right)\left(2\pi\cdot18\times10^{9}\right)^{-1}\left[\frac{81-2}{1+\left(\frac{2\pi\cdot2.54\times10^{9}}{2\pi\cdot18\times10^{9}}\right)^{2}}\right]+\frac{4}{2\pi\cdot2.54\times10^{9}}$$

or

$$\varepsilon_c'' = \left(8.854\times10^{-12}\right)\left(0.14111\right)\left[\frac{79}{1+\left(0.14111\right)^{2}}\right]+2.506\times10^{-10}$$

or

$$\varepsilon_c'' = 0.96775\times10^{-10}+2.506\times10^{-10}$$

or

$$\varepsilon_c'' = 3.474\times10^{-10}\ \left[\text{F/m}\right].$$

We then have

$$\left\langle \mathscr{P}_d \right\rangle = 2.772\left[\text{W/m}^3\right].$$

Problem 3 (25 pts)

A coaxial cable has inner radius a and outer radius b. Inside the coax is a dielectric having a relative permittivity of ε_r and a loss tangent $\tan\delta$. A series inductor L_e [H] is periodically placed every p meters along the line. Assume that p is small compared to a wavelength. The resistance per unit length due to conductor loss at the operating frequency ω_0 is $R_0[\Omega/\text{m}]$ (assumed to be known).

a) Find the value of L_e that will make the line distortionless at the operating frequency ω_0.

b) Find the characteristic impedance of the loaded line at the operating frequency ω_0. You may neglect all losses for this calculation.

c) Assume that the model for the series inductor now also has a series resistance R_e in addition to the series inductance (to model a practical lossy inductor). Find a formula for the attenuation constant α on the loaded line at the operating frequency ω_0.

Leave your answers in terms of the dimensions of the coax, the relative permittivity, the loss tangent, the operating frequency ω_0, and the period p.

ROOM FOR WORK

Part (a)

$$\frac{R_0}{L+L_e/p}=\frac{G}{C}=\omega\frac{G}{\omega C}=\omega\tan\delta\,.$$

Hence, we have

$$L_e=p\left(\frac{R_0}{\omega\tan\delta}-L\right).$$

We have that

$$L=\frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$C=\frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)}.$$

Part (b)

For the characteristic impedance we have

$$Z_0=\sqrt{\frac{L+L_e/p}{C}}.$$

Part (c)

For the attenuation constant we have

$$\alpha=\mathrm{Re}(\gamma)$$

where

$$\gamma=\sqrt{\left(R_0+\frac{R_e}{p}+j\omega\left(L+\frac{L_e}{p}\right)\right)(G+j\omega C)}$$

and

$$\frac{G}{\omega C} = \tan\delta .$$

Hence, we have

$$\alpha = \operatorname{Re}\sqrt{\left(R_0 + \frac{R_e}{p} + j\omega\left(L + \frac{L_e}{p}\right)\right)(\omega C)(\tan\delta + j)} .$$

Problem 4 (25 pts)

A hollow (air-filled) coaxial cable has an inner radius a and an outer radius b. At $z = 0$ there is a short-circuiting metal plate across the coax. For $z < 0$ the voltage are current are given by:

$$V(z) = V^+\left(e^{-jk_0 z} + \Gamma e^{+jk_0 z}\right)$$

$$I(z) = \frac{V^+}{Z_0}\left(e^{-jk_0 z} - \Gamma e^{+jk_0 z}\right),$$

where $\Gamma = -1$.

a) Find the electric and magnetic fields inside the coax.

b) Find the total force on the metal plate using the Maxwell stress tensor.

c) Find the total force on the metal plate using the photon point of view.

You may leave your answers in terms of V^+ and Z_0, the coax dimensions, and the operating frequency ω.

ROOM FOR WORK

Part (a)

The electric and magnetic fields inside the coax are

$$\underline{E} = \underline{\hat{\rho}} V^{+} \left(e^{-jk_0 z} + \Gamma e^{+jk_0 z} \right) \left(\frac{1}{\rho} \right) \left(\frac{1}{\ln(b/a)} \right)$$

$$\underline{H} = \underline{\hat{\phi}} \frac{1}{\eta_0} V^{+} \left(e^{-jk_0 z} - \Gamma e^{+jk_0 z} \right) \left(\frac{1}{\rho} \right) \left(\frac{1}{\ln(b/a)} \right).$$

Part (b)

We have the force per unit area in the z direction on the metal plate is

$$\mathscr{F}_z = -T_{zz} = -\left(-\frac{1}{2} \mu_0 \mathscr{H}_\phi^2 \right).$$

Hence, we have

$$\langle \mathscr{F}_z \rangle = \frac{1}{4} \mu_0 \left| H_\phi \right|^2 .$$

This gives us

$$\langle \mathscr{F}_z \rangle = \frac{1}{4} \mu_0 \left| \frac{1}{\eta_0} V^{+} (2) \left(\frac{1}{\rho} \right) \left(\frac{1}{\ln(b/a)} \right) \right|^2$$

or

$$\langle \mathscr{F}_z \rangle = \mu_0 \frac{1}{\eta_0^2} \left| V^{+} \right|^2 \left(\frac{1}{\ln(b/a)} \right)^2 \frac{1}{\rho^2} .$$

Integrating over the plate, we have for the total force

$$\left\langle \mathscr{F}_z^T \right\rangle = \mu_0 \frac{1}{\eta_0^2} \left|V^+\right|^2 \left(\frac{1}{\ln(b/a)} \right)^2 \int_0^{2\pi} \int_a^b \frac{1}{\rho^2} \rho d\rho d\phi$$

or

$$\left\langle \mathscr{F}_z^T \right\rangle = \mu_0 \frac{1}{\eta_0^2} \left|V^+\right|^2 \left(\frac{1}{\ln(b/a)} \right)^2 2\pi \ln\left(\frac{b}{a}\right).$$

Simplifying, we have

$$\left\langle \mathscr{F}_z^T \right\rangle = 2\pi\varepsilon_0 \left|V^+\right|^2 \left(\frac{1}{\ln(b/a)} \right) \ [\mathrm{N}].$$

Part (c)

From the photon point of view have the momentum density as

$$\left\langle p_{inc} \right\rangle = \frac{1}{c^2} \left\langle \mathscr{P}_{inc} \right\rangle = \frac{1}{c^2} \frac{1}{2} \left(\underline{E} \times \underline{H}^* \right) \cdot \hat{\underline{z}}$$

so that

$$\left\langle p_{inc} \right\rangle = \frac{1}{2c^2} \frac{\left|V^+\right|^2}{\eta_0} \left(\frac{1}{\ln(b/a)} \right)^2 \frac{1}{\rho^2}.$$

Consider the momentum is a length L of the coax. We have

$$\left\langle p_{inc}^T \right\rangle = \frac{1}{2c^2} \frac{\left|V^+\right|^2}{\eta_0} \left(\frac{1}{\ln(b/a)} \right)^2 L \int_0^{2\pi} \int_a^b \frac{1}{\rho^2} \rho d\rho d\phi$$

or

$$\left\langle p_{inc}^T \right\rangle = \frac{1}{2c^2} \frac{\left|V^+\right|^2}{\eta_0} \left(\frac{1}{\ln(b/a)} \right)^2 L 2\pi \ln\left(\frac{b}{a}\right)$$

or

$$\left\langle \wp_{inc}^{T} \right\rangle = \frac{\pi}{c^2} \frac{\left|V^+\right|^2}{\eta_0} \left(\frac{1}{\ln(b/a)} \right) L \, .$$

The change in momentum from the this group of photons hitting the plate and bouncing off is

$$\left\langle \Delta\wp \right\rangle = \frac{2\pi}{c^2} \frac{\left|V^+\right|^2}{\eta_0} \left(\frac{1}{\ln(b/a)} \right) L \, .$$

The force is the change in momentum divided by the time, so that

$$\left\langle \mathscr{F}_z \right\rangle = \frac{2\pi}{c^2} \frac{\left|V^+\right|^2}{\eta_0} \left(\frac{1}{\ln(b/a)} \right) \frac{L}{\Delta t} \, .$$

Since the photons travel at the speed of light, we have

$$\left\langle \mathscr{F}_z \right\rangle = \frac{2\pi}{c^2} \frac{\left|V^+\right|^2}{\eta_0} \left(\frac{1}{\ln(b/a)} \right) c \, .$$

Simplifying, we have

$$\left\langle \mathscr{F}_z \right\rangle = 2\pi\varepsilon_0 \left|V^+\right|^2 \left(\frac{1}{\ln(b/a)} \right) \quad [\mathrm{N}] .$$

This agrees with the previous result obtained using the Maxwell stress tensor.