

NAME: _____ **SOLUTION** _____

ECE 6340

Fall 2025

Exam 1

Nov. 10, 2025

INSTRUCTIONS:

This exam is open-book and open-notes. You may use any printed material or calculator that you wish, as long as it does not have any communication capability. Laptops, iphones, ipads, or other devices that may be used to communicate or connect to the Internet are not allowed.

- Put all of your answers in terms of the parameters given in the problems, unless otherwise noted.
- Include units with all numerical answers.
- Please circle your final answers.
- Please write all of your work on the sheets attached (if you need more room, you may write on the backs of the pages).

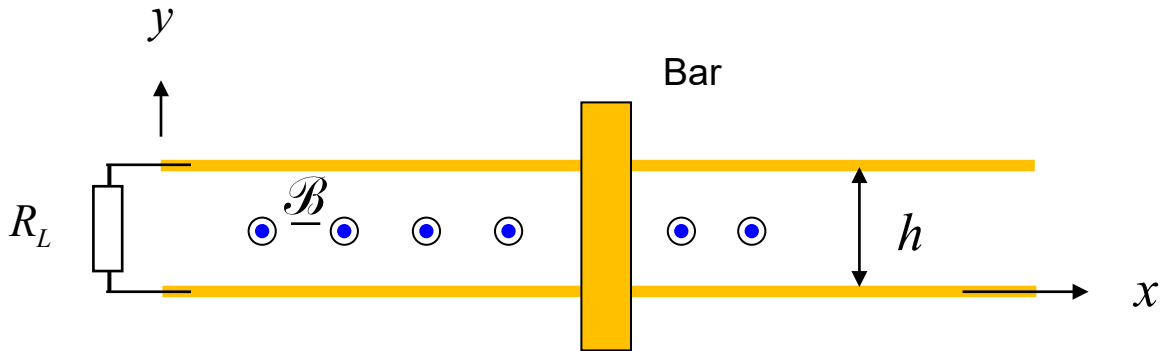
Please show all of your work and write neatly in order to receive proper credit.

Problem 1 (35 pts)

A sliding bar is located at $x = x_0$ and is moving to the right at a velocity of v_0 . The sliding bar has a resistance of R_b [Ω]. Each of the two rails has a resistance per unit length of R_l [Ω/m]. At the left end is a fixed resistance R_L [Ω]. A magnetic field exists between the rails, given by

$$\underline{\mathcal{B}} = \hat{z}(B_0 + B_1 \cos(\omega t)) \text{ [T]}.$$

- Find the current $i(t)$ flowing downward through the load resistance.
- From your answer to Part (a), find the Thevenin equivalent circuit of the sliding-bar system, i.e., the system that is to the right of the load. Find both the Thevenin voltage $v_{\text{Th}}(t)$ and the Thevenin resistance $R_{\text{Th}}(t)$. Do this by writing down an expression for the current $i(t)$ through the load resistance R_L that would be produced by the Thevenin equivalent circuit, and then match this to your answer from Part (a).



Solution

Part (a)

The EMF equation is

$$i(t)(R_b + R_L + 2R_l x_0) = -\frac{d\psi}{dt}.$$

We have

$$\psi = hx(t)\mathcal{B}_z(t).$$

Therefore,

$$\frac{d\psi}{dt} = hx(t)\mathcal{B}'_z(t) + hx'(t)\mathcal{B}_z(t)$$

or

$$\frac{d\psi}{dt} = hx_0\mathcal{B}'_z(t) + hv_0\mathcal{B}_z(t).$$

Hence, we have

$$i(t) = -\frac{hx_0\mathcal{B}'_z(t) + hv_0\mathcal{B}_z(t)}{R_L + R_b + 2R_l x_0},$$

where

$$\mathcal{B}_z = B_0 + B_1 \cos(\omega t)$$

$$\mathcal{B}'_z = -\omega B_1 \sin(\omega t).$$

Part (b)

From the Thévenin equivalent circuit we have

$$i(t) = \frac{v_{Th}(t)}{R_L + R_{Th}}.$$

Comparing, we have

$$v_{\text{Th}}(t) = -\left(hx_0\mathcal{B}'_z(t) + hv_0\mathcal{B}_z(t)\right)$$

$$R_{\text{Th}} = R_b + 2R_l x_0$$

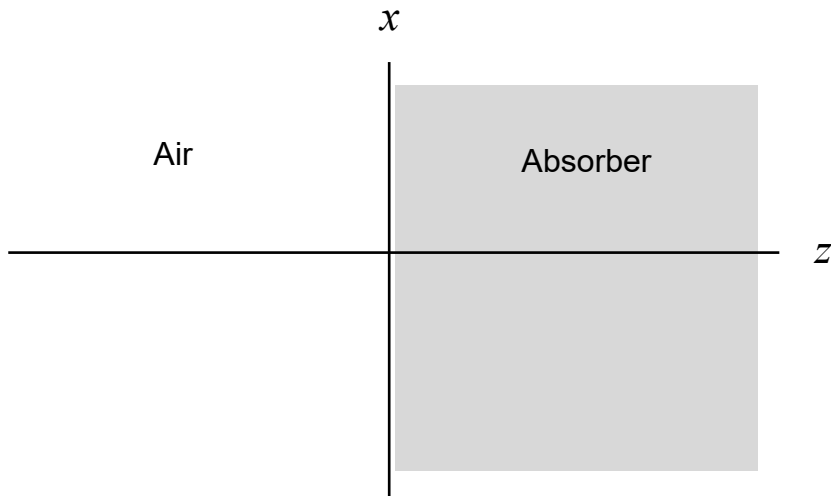
Problem 2 (35 pts)

A plane wave traveling in the z direction in the air region ($z \leq 0$) has the following fields in the phasor domain:

$$\underline{E} = \hat{x} e^{-jk_0 z}, \quad \underline{H} = \hat{y} \left(\frac{1}{\eta_0} \right) e^{-jk_0 z}$$

The plane wave hits a semi-infinite region of ideal absorber that lies in the region $z > 0$. The ideal absorber does not cause any reflected wave. In the region $z \leq 0$ (including at $z = 0$), the ideal absorber does not disturb the incident field.

- (a) Find the time-average force per unit area on the absorber in the z direction using the Maxwell stress tensor.
- (b) Find the time-average force per unit area on the absorber in the z direction using the concept of photons (note that all photons get absorbed and do not reflect).



Solution

Part (a)

The force in the z direction is given by

$$\mathcal{F}_z = -T_{zz}.$$

For the TEM _{z} plane wave we have

$$T_{zz} = \varepsilon_0 \mathcal{E}_z \mathcal{E}_z + \mu_0 \mathcal{H}_z \mathcal{H}_z - \left(\frac{1}{2} \varepsilon_0 |\underline{\mathcal{E}}|^2 + \frac{1}{2} \mu_0 |\underline{\mathcal{H}}|^2 \right) = - \left(\frac{1}{2} \varepsilon_0 |\underline{\mathcal{E}}|^2 + \frac{1}{2} \mu_0 |\underline{\mathcal{H}}|^2 \right).$$

For the time-average we have

$$\langle T_{zz} \rangle = - \left(\frac{1}{4} \varepsilon_0 |\underline{E}|^2 + \frac{1}{4} \mu_0 |\underline{H}|^2 \right)$$

or

$$\langle T_{zz} \rangle = - \left(\frac{1}{4} \varepsilon_0 |E_x|^2 + \frac{1}{4} \mu_0 |H_y|^2 \right)$$

or

$$\langle T_{zz} \rangle = - \left(\frac{1}{4} \varepsilon_0 + \frac{1}{4} \mu_0 \left| \frac{1}{\eta_0} \right|^2 \right)$$

or

$$\langle T_{zz} \rangle = - \left(\frac{1}{4} \varepsilon_0 + \frac{1}{4} \varepsilon_0 \right)$$

or

$$\langle T_{zz} \rangle = - \frac{\varepsilon_0}{2}.$$

Hence, we have

$$\langle \mathcal{F}_z \rangle = \frac{\varepsilon_0}{2} \left[\text{N/m}^2 \right].$$

Part (b)

From the photon point of view we have:

$$\langle \mathcal{F}_z \rangle = \frac{(\langle \mathcal{H}_z^{\text{inc}} \rangle)(Ac \Delta t)}{\Delta t} = (\langle \mathcal{H}_z^{\text{inc}} \rangle)(Ac) = \left[\frac{1}{c^2} \langle \mathcal{J}_z \rangle \right] [(1)c] = \frac{\langle \mathcal{J}_z \rangle}{c} = \frac{\text{Re}(S_z)}{c}.$$

We also have

$$S_z = \frac{1}{2\eta_0}.$$

Hence, we have

$$\langle \mathcal{F}_z \rangle = \frac{1}{2c\eta_0}$$

or

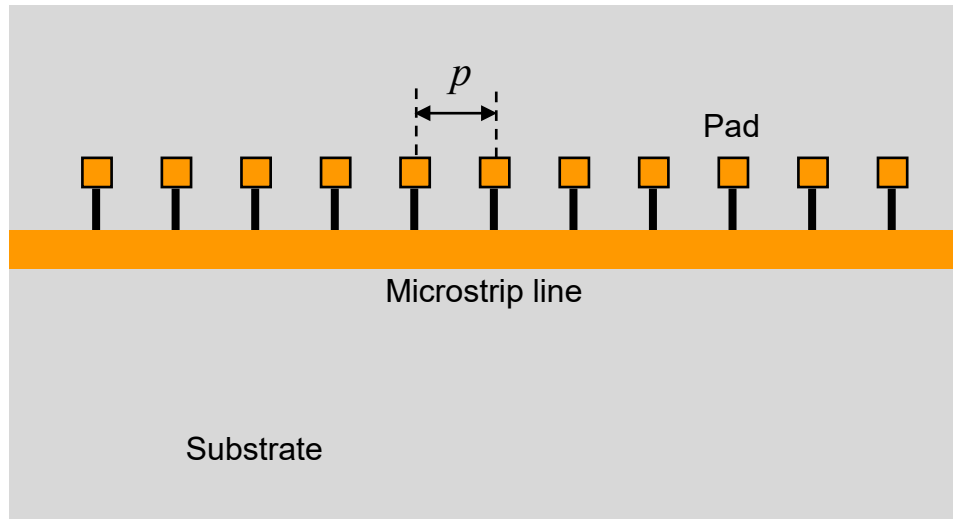
$$\langle \mathcal{F}_z \rangle = \frac{\epsilon_0}{2} \left[\text{N/m}^2 \right].$$

Problem 3 (30 pts)

A lossless microstrip line (shown below) has a characteristic impedance of Z_0 and an effective relative permittivity ϵ_r^{eff} at a radian frequency ω . The microstrip line is periodically loaded with short stubs (distance p apart in the z direction) that are connected to capacitive pads (modeled as a capacitance C_0 between the pad and ground), so that a capacitance C_0 is periodically added in parallel to the microstrip line. A top view is shown below. We can assume that $p \ll \lambda_0$.

For modeling purposes, we can assume that the microstrip line carries a TEM_z mode and has a homogeneous filling material everywhere above the ground plane, with $\epsilon_r = \epsilon_r^{\text{eff}}$ and $\mu = \mu_0$.

Find a formula for the phase constant β for the periodically-loaded line. Your answer should be in terms of Z_0 , ϵ_r^{eff} , p , C_0 , and ω , as well as fundamental constants such as μ_0, ϵ_0, c .



Solution

We have

$$\gamma = \sqrt{ZY}.$$

We also have

$$Z = j\omega L$$

$$Y = j\omega C + j\omega \left(\frac{C_0}{p} \right)$$

Therefore,

$$\gamma = \alpha + j\beta = \sqrt{(j\omega L) \left(j\omega C + j\omega \left(\frac{C_0}{p} \right) \right)}.$$

We thus have

$$\beta = \omega \sqrt{L \left(C + \left(\frac{C_0}{p} \right) \right)}.$$

We also have

$$\sqrt{LC} = \sqrt{\mu_0 \epsilon_0 \epsilon_r^{\text{eff}}}$$

$$\sqrt{\frac{L}{C}} = Z_0.$$

Hence, we have

$$L = Z_0 \sqrt{\mu_0 \epsilon_0 \epsilon_r^{\text{eff}}}$$

$$C = \frac{1}{Z_0} \sqrt{\mu_0 \epsilon_0 \epsilon_r^{\text{eff}}}.$$

We then have

$$\beta = \omega \sqrt{Z_0 \sqrt{\mu_0 \varepsilon_0 \varepsilon_r^{\text{eff}}} \left(\frac{1}{Z_0} \sqrt{\mu_0 \varepsilon_0 \varepsilon_r^{\text{eff}}} + \left(\frac{C_0}{p} \right) \right)}$$

or

$$\beta = \omega \sqrt{\mu_0 \varepsilon_0 \varepsilon_r^{\text{eff}}} \sqrt{\left(1 + \left(\frac{Z_0 C_0}{p \sqrt{\mu_0 \varepsilon_0 \varepsilon_r^{\text{eff}}}} \right) \right)}$$

or

$$\beta = k_0 \sqrt{\varepsilon_r^{\text{eff}}} \sqrt{\left(1 + \left(\frac{c Z_0 C_0}{p \sqrt{\varepsilon_r^{\text{eff}}}} \right) \right)}.$$

ROOM FOR WORK