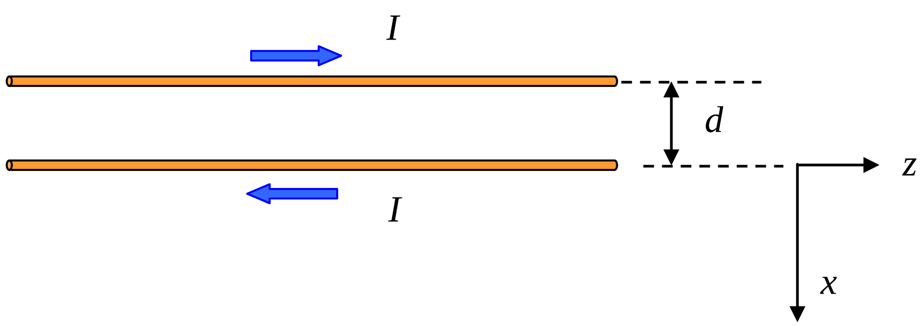
ECE 6340
Fall 2016

Homework 2

Please do the following problems (you may do the others for practice if you wish):
Probs. 1, 2, 3, 4, 5, 6, 7, 10, 12

1) Consider two parallel infinite wires in free space each carrying a DC current I in the opposite direction, as shown below. Use Ampere's law along with the Lorentz wire force law to show that the force per unit length on the bottom wire is

$$F_x = \frac{\mu_0 I^2}{2\pi d}.$$



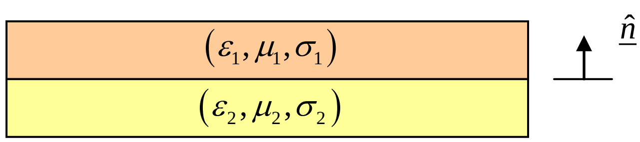
The Lorentz wire force law (which comes from the Lorentz force law for a charged particle) is

$$\underline{F} = \int_C I \, \underline{d\ell} \times \underline{B}.$$

In this equation the contour C runs in the direction of the reference-direction for the current (the direction of the arrow) for the current I on the wire that we are calculating the force on.

2) Consider a boundary between two conducting regions as shown below. Assume that there is no surface current on the boundary. Assuming that the frequency is not zero, show that in the time-harmonic steady state that

$$\underline{\hat{n}} \cdot \left(\varepsilon_{c1} \underline{E}_1\right) = \underline{\hat{n}} \cdot \left(\varepsilon_{c2} \underline{E}_2\right)$$



Why is it not true that the normal component of the flux density vector is continuous? That is, why is it not true that $\underline{\hat{n}} \cdot (\varepsilon_1 \underline{E}_1) = \underline{\hat{n}} \cdot (\varepsilon_2 \underline{E}_2)$?

Hint: Start with Ampere's law (in point form) and take the normal component of both sides. You may assume that the normal direction is the *z* direction for convenience, if you wish. Or, you can start with the boundary form of the continuity equation.

3) Consider a region with a uniform volume charge density of ρ_v = 1 [C/m^3] that is moving in the *x* direction with a speed of 1 [m/s] with respect to you, sitting in a laboratory.

a) What is the current density vector $\underline{J}^A$ that you see?

b) Next, consider a region with a volume charge density of ρ_v = 2 [C/m^3] that is moving in the *x* direction with a speed of 0.5 [m/s] with respect to the laboratory. What is the current density vector $\underline{J}^B$ that you see?

c) Next, imagine that you are an observer that is moving at a speed of 0.5 [m/s] in the *x* direction with respect to the laboratory. What would be the current densities $\underline{J}^A$ and $\underline{J}^B$ that you would observe from each of the two charge densities?

d) Given your answers to the above parts of this problem, answer the following question: Given a certain current density vector in one coordinate system (and no other information), can we uniquely determine what the current density vector is in another coordinate system that is moving with a known velocity vector relative to the first coordinate system? If so, how would you do it?

4) A sample of saltwater has a conductivity of $\sigma = 4$ [S/m] and a permittivity $\varepsilon = \varepsilon_0 (81 - j10)$ at a frequency of 10 GHz.

Inside the material there is an electric field $\underline{E} = \underline{\hat{z}}(1000)$ [V/m]. Determine the following inside the saltwater:

a) The electric flux density vector $\underline{D}$.
b) The effective complex relative permittivity ε_{rc} of the material.
c) The loss tangent of the water.
d) The conduction current density vector $\underline{J}$.
e) The polarization current density vector $\underline{J}^p$.
f) The equivalent current density vector $\underline{J}^{eq}$ (which may be used to replace the saltwater).

5) A parallel-plate capacitor has a plate area of *A* [m^2] and a separation of *h* [m] between the plates. The capacitor is filled with a material having conductivity σ and permittivity ε (which is complex). Derive a formula for the input impedance of the capacitor at any given frequency *f*. Hint: Use the principle of effective permittivity, starting with a formula for the input impedance of an air-filled capacitor.

6) A parallel-plate capacitor has a DC capacitance of 300 [pF] when it is air-filled. It is then filled with a lossy material, and the impedance of the capacitor is measured at a frequency of 159.15 [kHz]. The input impedance is Z_{in} = 27.45 - j (263.84) [Ω] Find ε'_{rc} and ε''_{rc}

   Note that $\varepsilon_c = \varepsilon'_c - j\varepsilon''_c = \varepsilon_0 \varepsilon_{rc} = \varepsilon_0 \left(\varepsilon'_{rc} - j\varepsilon''_{rc}\right)$.

   Is there any way to tell from this measurement what $\varepsilon_r = \varepsilon / \varepsilon_0$ is? If so, find this as well.

7) A parallel-plate capacitor has a plate area of A [m^2] and a separation of h [m] between the plates. The capacitor is filled with a material having a conductivity σ and a permittivity ε (which is complex). The effective complex permittivity (which accounts for conductivity) is

$$\varepsilon_c = \varepsilon - j\left(\sigma / \omega\right) = \varepsilon'_c - j\varepsilon_c''.$$

   Using the concept of effective permittivity, show that the input admittance of the capacitor can be written as

$$Y_{in} = G + j\omega C$$

   where

$$G = \left(\omega \varepsilon_c''\right) \frac{A}{h}$$

   and

$$C = \varepsilon_c' \frac{A}{h}.$$

   What is the equivalent circuit for this lossy capacitor?

   (Note that the capacitance is a real number, even though the capacitor is lossy.)

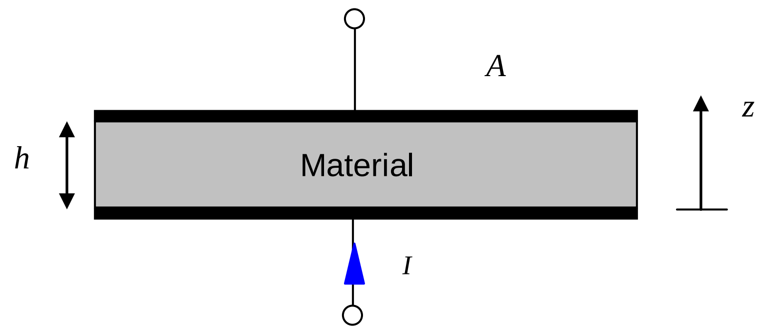
8) For the same lossy capacitor as above, define a complex capacitance $C^{complex}$ as the ratio of charge Q (in the phasor domain) on the positive plate of the capacitor to the voltage drop (in the phasor domain) between the two plates. Calculate this complex capacitance for this capacitance.

9) Assume that a parallel-plate capacitor is connected to a circuit, so that a current I (in the phasor domain) is flowing through the capacitor as shown below. The capacitor is filled with a material having a complex permittivity ε and conductivity σ. The plate area is A

[m^2]. Show that the current I that enters the bottom plate of the capacitor is equal to the "total effective current" that flows upwards from the bottom plate through the material. The "total effective current" is defined as the sum of the conduction current, the polarization current, and the free-space displacement current.

That is, prove that

$$I = A\left[\sigma E_z + j\omega\left(\varepsilon - \varepsilon_0\right)E_z + j\omega\varepsilon_0 E_z\right] = A\left(j\omega\varepsilon_c E_z\right).$$



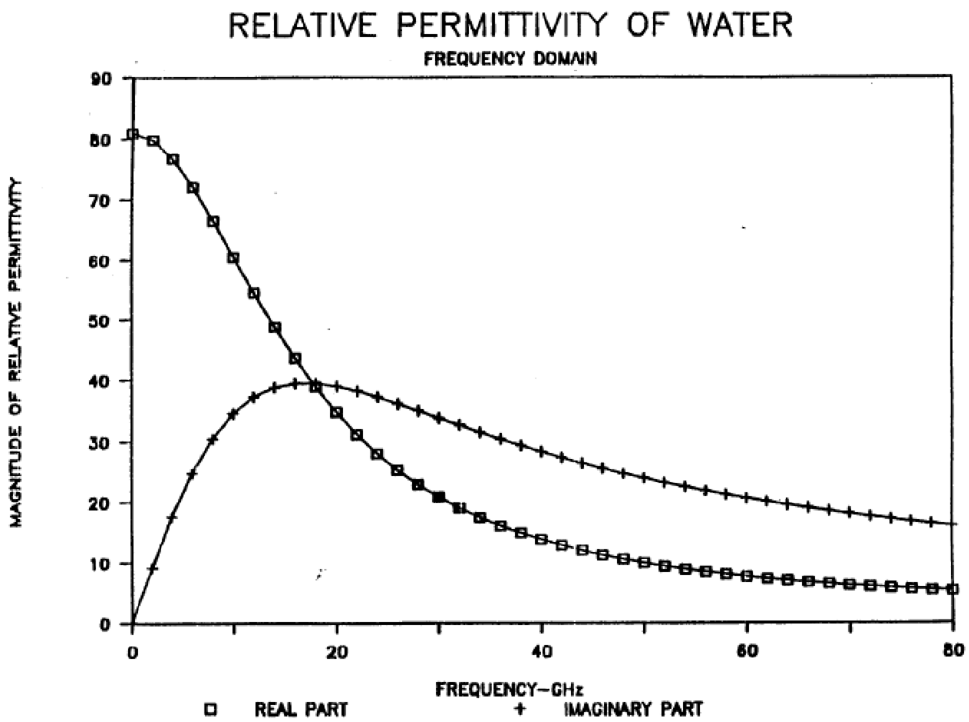
You may assume that the field inside the material is z-directed, and uniform. The field outside the capacitor may be assumed to be zero.

Start with the continuity equation in the phasor domain, applied to the bottom plate of the capacitor. Note that a current I flows into this plate, and also a current leaves the bottom plate due to ohmic current flow in the material (which is described by the conductivity of the medium). The difference of these two currents must be the rate of change of total charge Q_b on the bottom plate.

10) Assume that water obeys a Debye model for the permittivity. Make a plot of the real and imaginary parts of the complex permittivity versus frequency up to 80 [GHz]. Ignore the conductivity of the water (assume pure, or distilled, water).

To determine the unknown constants in the model, use the fact that the imaginary part of the permittivity is greatest at a frequency of 18 [GHz] (this determines the time constant τ). Then match with the measured value of the real part of the permittivity at zero frequency (this determines the value of $\varepsilon_r(0)$). Finally, match with the measured value of the real part of the permittivity at 80 [GHz]. (Do not assume that 80 GHz is high enough to be taken as infinite frequency!)

Plot the calculated real and imaginary parts of relative permittivity from the Debye model versus frequency from zero to 80 GHz, and on your calculated plot add enough data points from the measured results so that a comparison can be easily made. Note: A plot of the measured results may be found in the class notes, and it is also shown below. Feel free to enlarge and print out the plot to help in reading data from it.


RELATIVE PERMITTIVITY OF WATER
FREQUENCY DOMAIN
MAGNITUDE OF RELATIVE PERMITTIVITY
90
80
70
60
50
40
30
20
10
0
0
20
40
60
80
FREQUENCY–GHz
REAL PART
IMAGINARY PART

11) Assume that a material obeys the Lorentz model for the permittivity. Assume that the resonance peak is fairly sharp, so that $\omega \approx \omega_0$ for the frequencies of interest near the peak. Show that the imaginary part of the relative permittivity can be put in the following approximate form for frequencies close to resonance:

$$\varepsilon_r'' = \varepsilon_{r\max}'' \left[\frac{\bar{\omega}}{\bar{\omega}^2 + p^2\left(1-\bar{\omega}^2\right)^2} \right],$$

where

$$\bar{\omega} \equiv \frac{\omega}{\omega_0}$$

$\varepsilon_{r\max}'' \equiv$ maximum value of ε_r'' at the peak

$p \equiv \dfrac{\omega_0}{c_1} = \dfrac{m\,\omega_0}{c}$ (c is the coefficient of friction in the model).

12) Consider atmospheric absorption near 60 GHz. (This frequency range is often used for short-range communications when it is desired that the radiated signal does not propagate to large distances.) Assume that the atmospheric absorption peak near 60 GHz due to the O_2 molecule obeys a Lorentz model. Assume also that the atmospheric absorption A_{dB} in dB/km is proportional to ε_r''. Hence (from the result of the previous problem),

$$A_{\mathrm{dB}} = A_{dB}^{\max} \left[\frac{\bar{\omega}}{\bar{\omega}^2 + p^2\left(1-\bar{\omega}^2\right)^2} \right],$$

where

$$\bar{\omega} \equiv \frac{\omega}{\omega_0}.$$

a) Calculate the parameters ω_0, $A_{\mathrm{dB}}^{\max}$, and p for this resonance by using the experimental data shown on the plot below. (Note: choose a reasonable frequency within the bandwidth of the resonance to determine the value of p, by matching with the measured data.)

b) Plot the calculated attenuation A_{dB} from the Lorentz model versus frequency from 40 to 80 GHz. Add enough measured data points (from the plot below, which you need to convert to attenuation in dB/km) so that you can see how well the measured data is predicted by the Lorentz model.

Note 1: You can expand and print out this figure to more easily read values from it.

Note 2: The plot below gives the percent of the power absorbed after the wave travels through one km in air, called here P_a. This is related to the attenuation in dB/km by the equations

$$A_{\mathrm{dB}} = -10\log_{10}\left(e^{-2\alpha(1000)}\right)$$

$$P_a = 100\left(1 - e^{-2\alpha(1000)}\right).$$

Hence,

$$A_{\mathrm{dB}} = -10\log_{10}\left(1 - P_a/100\right).$$