ECE 6340
Fall 2016

Homework 8

This assignment is not to be turned in. These are practice problems to help you understand the topics that were covered in Notes 23-29 ("EM Theorems").

1) Use duality and your knowledge of electrostatics to determine the magnetic field $\underline{B}$ that would be produced by a single static magnetic point charge q_m at the origin (if such a point charge really existed).

2) Determine formulas for surface electric and magnetic currents that flow on the surface of a sphere of radius R in free space, which produce the same field in the region $r > R$ as does a unit-amplitude z-directed infinitesimal dipole at the origin. Inside the sphere, $r < R$, these surface currents produce no fields. How would your answer change if the region $r < R$ was a dielectric sphere with a relative permittivity ε_r?

3) Determine formulas for surface electric and magnetic currents that flow on the $z = 0$ plane in free space, which produce a plane wave field in the region $z > 0$, having an electric field that is given by

$$\underline{E} = \underline{\hat{y}}\, e^{-j(k_x x + k_z z)}.$$

In the region $z < 0$ the surface currents produce no fields.

4) A current sheet

$$\underline{J}_s(x, y) = \underline{\hat{y}} \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

is placed at the plane $z = 0$ inside a hollow rectangular waveguide of dimensions $a \times b$ (x and y dimensions, respectively). (The lower left corner of the waveguide cross section is at the origin.) The x component of the magnetic field inside the waveguide is "guessed" to be

$$H_x(x, y) = \pm\frac{1}{2}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{\mp jk_{zmn}z}$$

where

$$k_{zmn} = \left(k_0^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2\right)^{1/2}.$$

The top (bottom) sign is for $z > 0$ ($z < 0$). The magnetic field inside the waveguide also has a z component (not shown above), but no y component. (The field is TM to the y direction, a fact that can be justified by using image theory for the waveguide.) Note that H_x satisfies the boundary conditions on the four walls of the waveguide.

a) Determine the field H_z inside the waveguide, using the fact that the divergence of the magnetic field must be zero. Verify that it satisfies the boundary conditions on the four walls of the waveguide.

b) Determine the electric field inside the waveguide from the magnetic field. Verify that it satisfies the boundary conditions on the four walls of the waveguide.

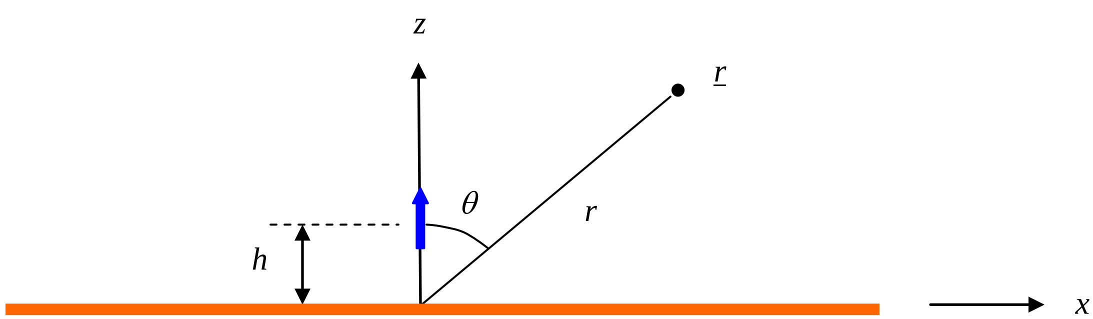
c) Explain how the uniqueness principle is satisfied, and why the fields calculated in the previous steps must be the correct fields launched inside the waveguide by the current sheet. (Take the surface S in the uniqueness principle to be the entire inner surface of the waveguide, together with ending surfaces (caps) at $z = \pm\infty$. You may assume a small amount of loss inside the waveguide, so that the uniqueness principle holds.)

5) Consider a y-directed unit-strength dipole ($Il = 1$) inside the same rectangular waveguide as in the previous problem, at the location $x = x_0$, $y = y_0$, $z = z_0 = 0$. By using a double Fourier series, the dipole current is thought of as a superposition of current sheets, of the form

$$J_{sy}(x,y) = \delta(x - x_0)\,\delta(y - y_0) = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} A_{mn} \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right).$$

Solve for the coefficients A_{mn} using Fourier series theory, and then use superposition along with the results from the previous problem to determine the magnetic field inside the waveguide for $z > 0$.

6) Use image theory to find the far field of a unit-strength z-directed infinitesimal electric dipole at a height h above an infinite ground plane. Simplify your answer so that it involves a term of the form $\cos(k_0 h \cos\theta)$.



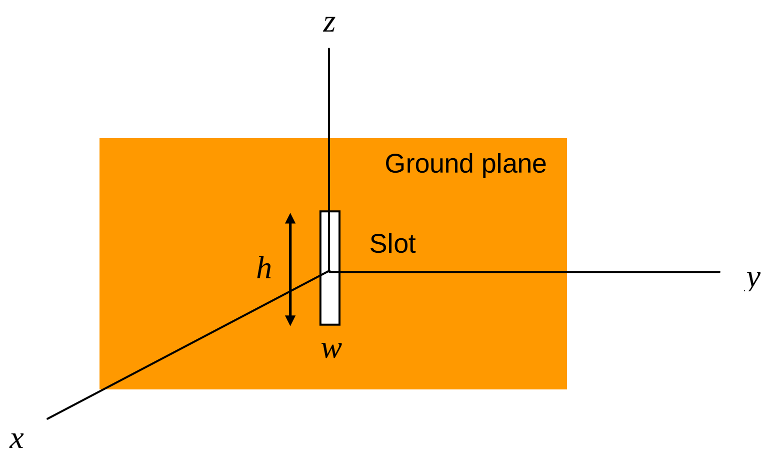
7) Consider the same unit-strength dipole inside a rectangular waveguide as in Prob. 5. Derive an expression for $A_y(x, y, z)$ inside the waveguide by using image theory. Your answer should be in the form of four infinite series (each series involving a double summation). Explain how you would calculate the magnetic and electric fields from a magnetic vector potential A_y. That is, derive general expressions for $\underline{E}$ and $\underline{H}$ in terms of A_y. You do not have to actually substitute your series expressions into these equations.

8) A slot antenna is in an infinite ground plane as shown below. The ground plane is the $x = 0$ plane. Assume that the width of the slot w (in the y direction) is very small compared to a wavelength. The slot is centered at the origin, and the length of the slot is $2h$ in the z direction. Derive an expression for the far-field electric and magnetic fields E_θ, E_ϕ, H_θ, H_ϕ in the region $x > 0$. Do this by converting the aperture electric field into an equivalent magnetic surface current on a ground plane (with no slot) by using boundary conditions, and then use image theory and duality. Assume that the electric field in the slot is given by

$$\underline{E} = \underline{\hat{y}}\, E_0 \sin\left(k\left(h - |z'|\right)\right).$$

Note: You may wish to use the result from the class notes on radiation (Notes 22) to help evaluate the integral that arises in this problem (you do not have to derive the integral result that appears in the class notes).

How do the fields in the region $x < 0$ compare to those in the region $x > 0$? Justify your answer.



9) Consider the same slot antenna as in the previous problem. Use the equivalence principle to show how the antenna can be modeled as a magnetic current on an infinite ground plane (with no slot) for the purposes of calculating the radiation in the region $x > 0$. Explain carefully how you are using the equivalence principle. Finally, apply image theory to obtain a magnetic current in free space that models the original slot antenna. (The magnetic current that you arrive at should be the same one that you ended up with in the previous problem, using boundary conditions.)

10) An infinitesimal unit-amplitude vertical electric dipole is at a height h above the ocean as shown below. Use reciprocity to determine the far-field components $E_\theta(r,\theta,\phi)$ and $E_\phi(r,\theta,\phi)$ (if both exist). Note that the field E_z in the plane-wave problem (where a plane wave from the testing dipole illuminates the ocean) can be found from the horizontal components of the magnetic field of the plane-wave field, by using Ampere's law. Also, note that a horizontal component of the magnetic field (either H_x or H_y) in the plane-wave problem can be modeled as current on the TEN.