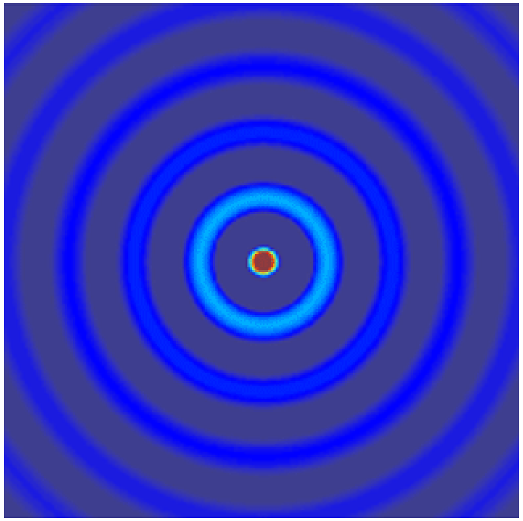
ECE 6341

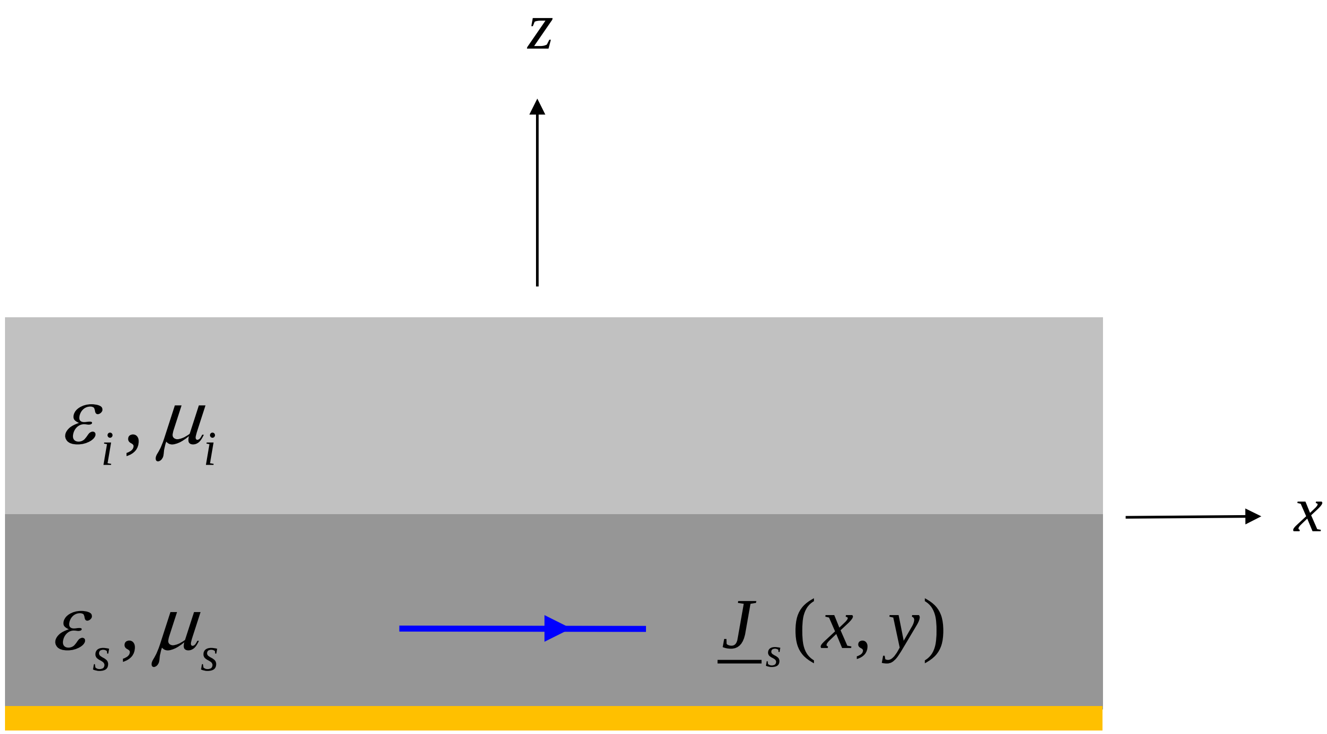
Spring 2016

Prof. David R. Jackson
ECE Dept.



Notes 38

Spectral Domain Immittance (SDI) Method



$$\underline{J}_s(x,y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \underline{\tilde{J}}_s(k_x, k_y) e^{-j(k_x x + k_y y)} dk_x dk_y$$

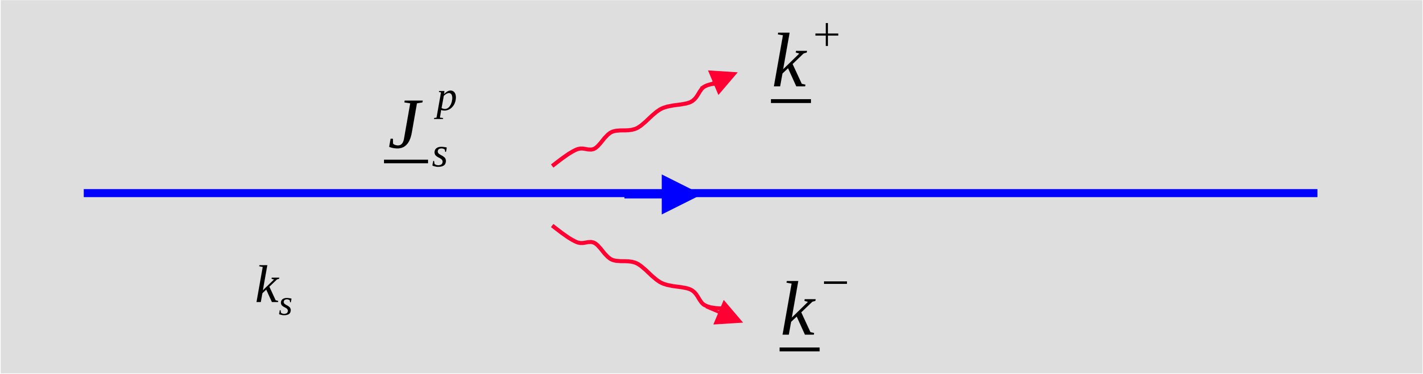
SDI Method (cont.)

The finite current sheet is thus represented as a set of infinite phased current sheets:

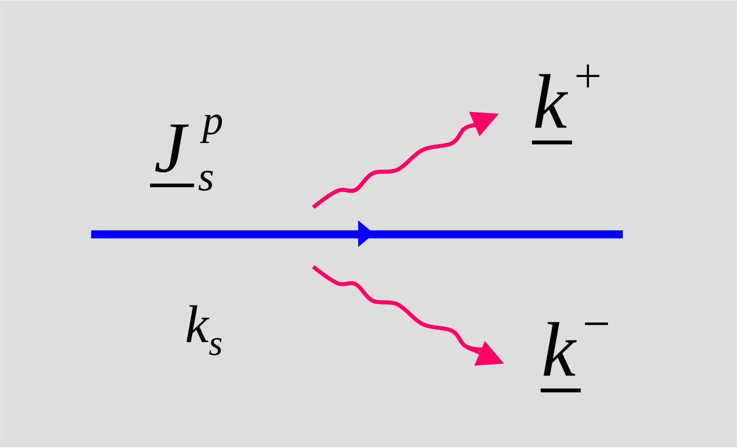
$$\underline{J}_s^p(x,y) = \underline{J}_{s0}^p \, e^{-j(k_x x + k_y y)}$$

where $$\underline{J}_{s0}^p = \frac{1}{(2\pi)^2} \underline{\tilde{J}}_s(k_x, k_y)\, dk_x dk_y$$

This phased current sheet launches a pair of plane waves as shown below:



SDI Method (cont.)



$$\underline{k}^+ = \underline{\hat{x}}\, k_x + \underline{\hat{y}}\, k_y + \underline{\hat{z}}\, k_{zs}$$

$$\underline{k}^- = \underline{\hat{x}}\, k_x + \underline{\hat{y}}\, k_y - \underline{\hat{z}}\, k_{zs}$$

$$k_{zs} = \left(k_s^2 - k_t^2\right)^{1/2}$$

$$k_t^2 = k_x^2 + k_y^2$$

Goal: Decompose the current into two parts: one that excites only a pair of TM_z plane waves, and one that excites only a pair of TE_z plane waves.

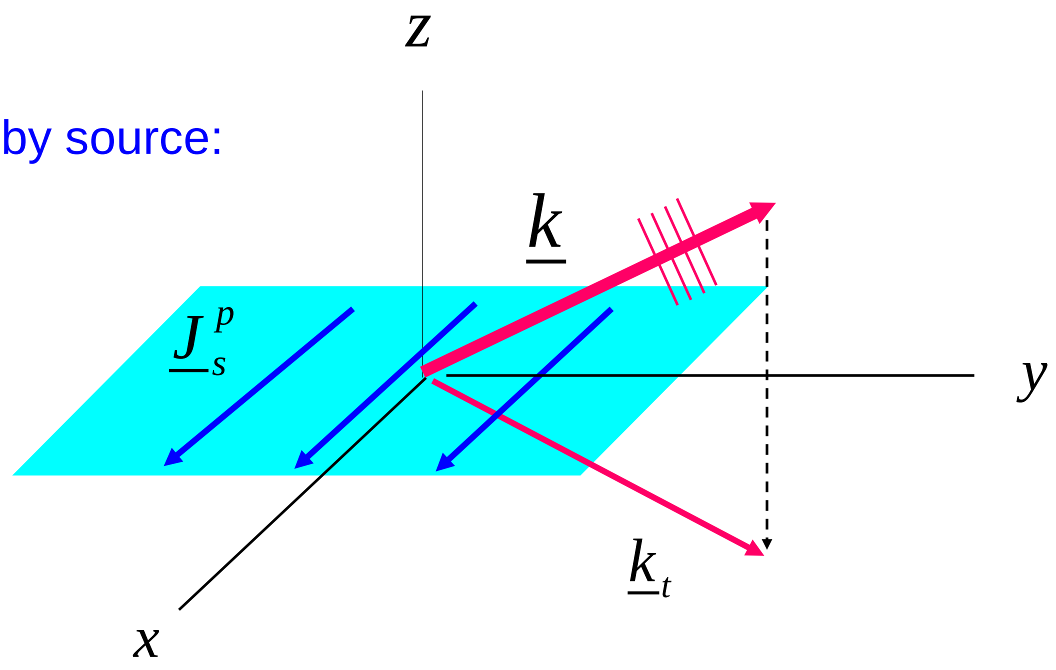
SDI Method (cont.)

Define:

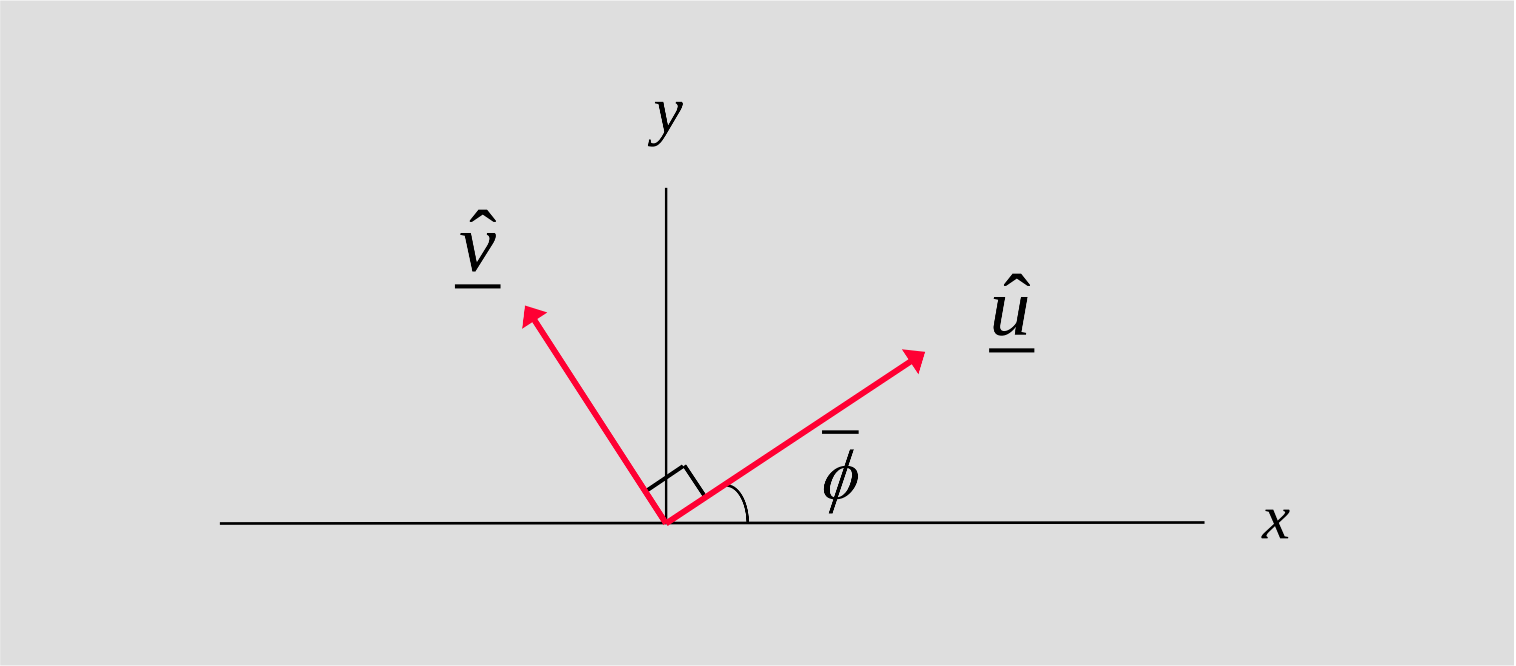
$$\underline{k}_t \equiv \underline{\hat{x}}\, k_x + \underline{\hat{y}} k_y$$

$$\underline{\hat{u}} \equiv \frac{\underline{k}_t}{k_t} \qquad \underline{\hat{v}} \equiv \underline{\hat{z}} \times \underline{\hat{u}}$$

Launching of plane wave by source:

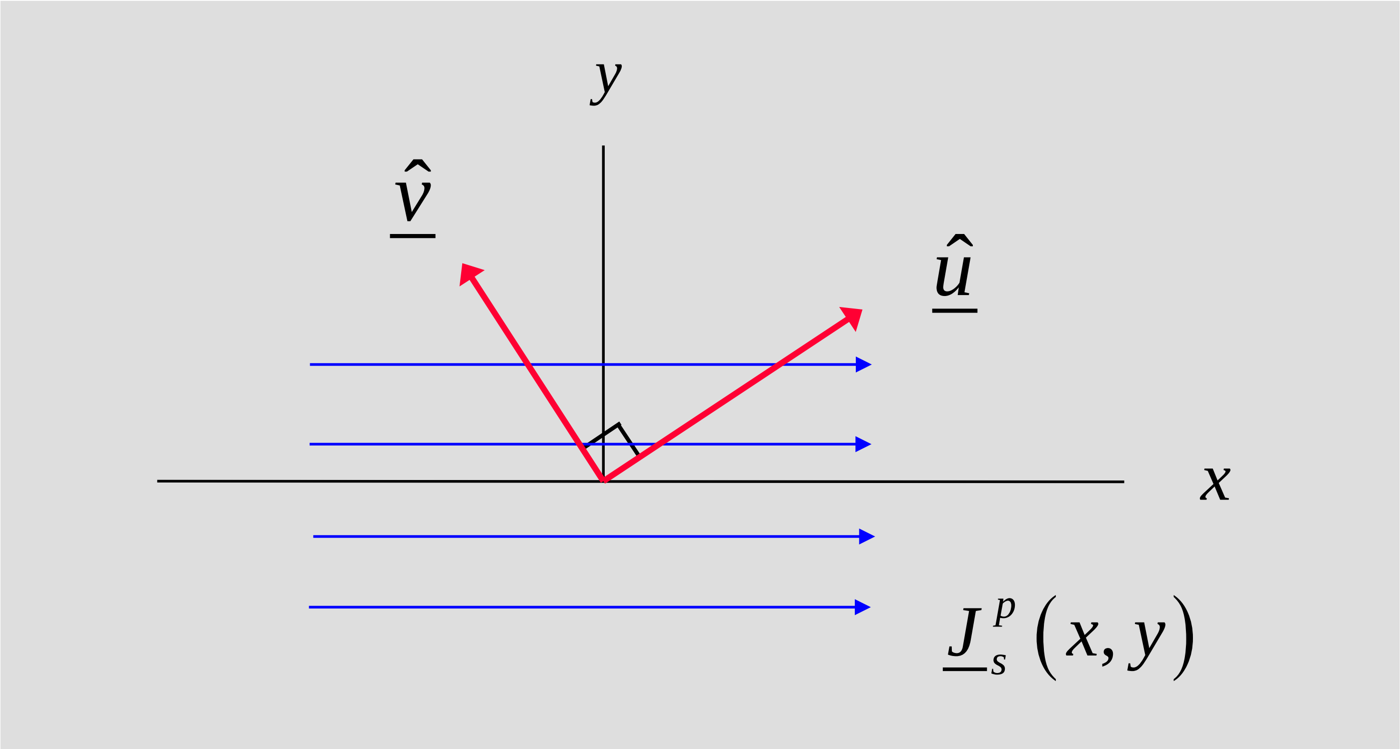


SDI Method (cont.)



$$\cos\bar{\phi} = \frac{k_x}{k_t} \qquad \sin\bar{\phi} = \frac{k_y}{k_t}$$

SDI Method (cont.)



$\underline{J}_{su}^p(x,y)$: Launches a TM_z wave

$\underline{J}_{sv}^p(x,y)$: Launches a TE_z wave

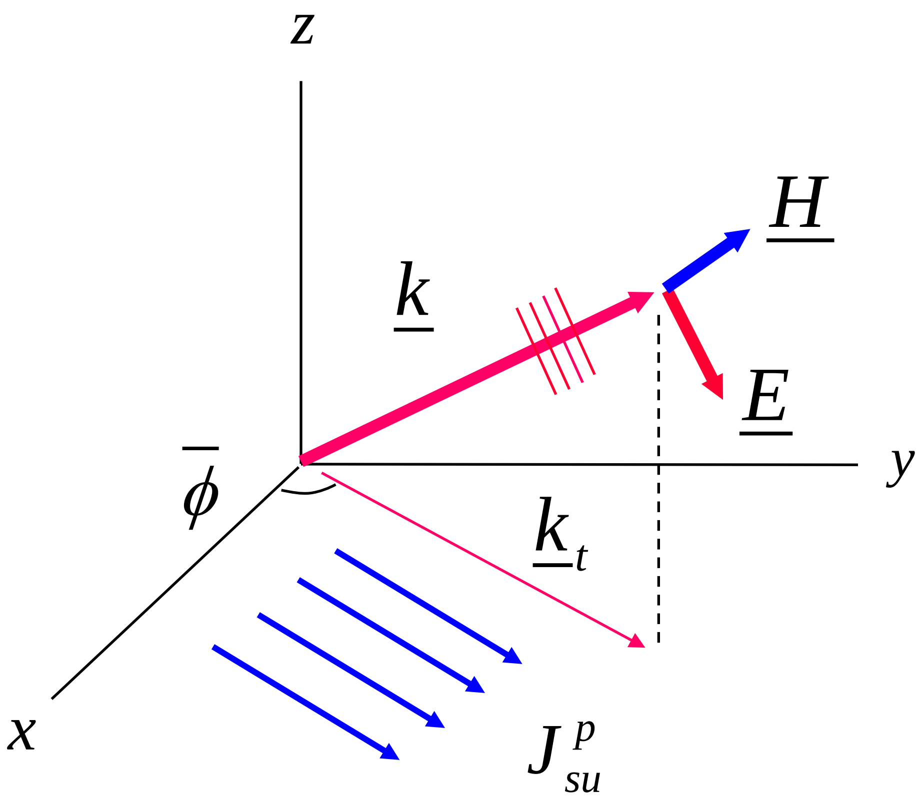
A proof of this is given next.

SDI Method (cont.)

TM$_z$

$$\underline{E}_t = \underline{\hat{u}}\, E_u$$

$$\underline{H}_t = \underline{\hat{v}}\, H_v$$



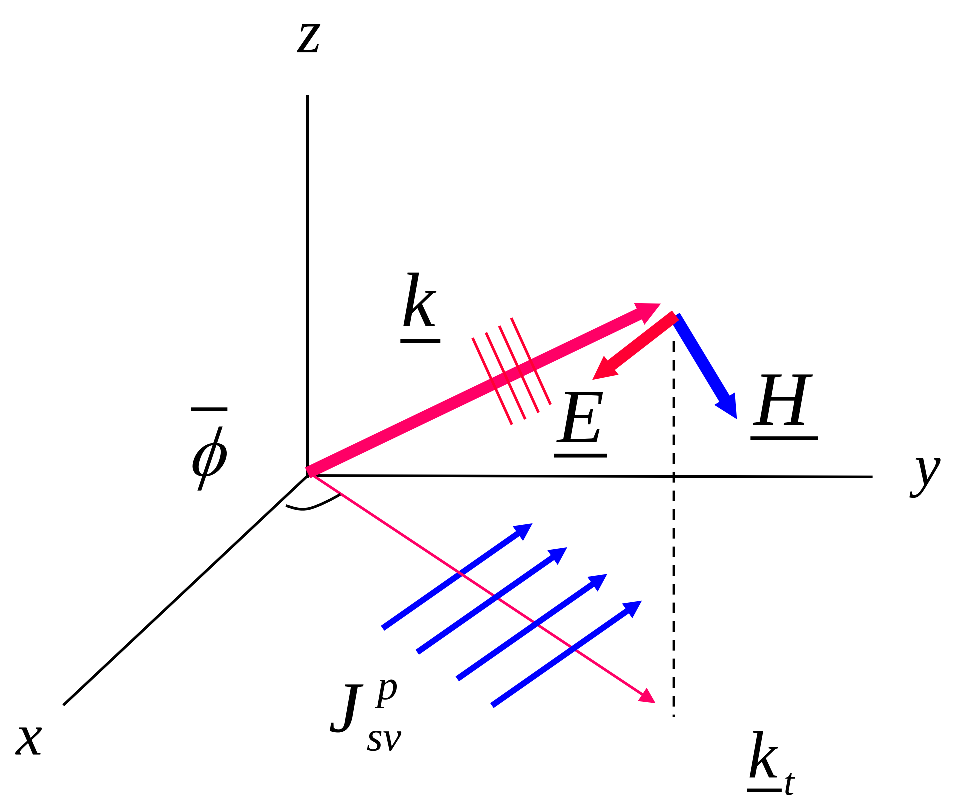
This figure shows the u component of the current launching a TM$_z$ plane wave.

SDI Method (cont.)

TE_z

$$\underline{E}_t = \underline{\hat{v}} E_v$$

$$\underline{H}_t = \underline{\hat{u}} H_u$$



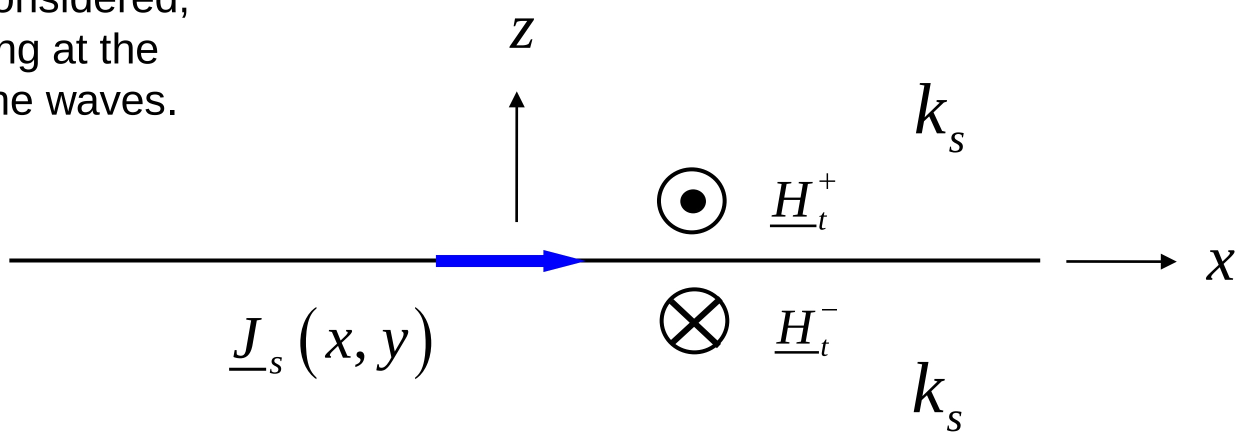
This figure shows the v component of the current launching a TE_z plane wave.

SDI Method (cont.)

Proof of launching property:

Consider an arbitrary surface current.

An infinite medium is considered, since we are only looking at the launching property of the waves.

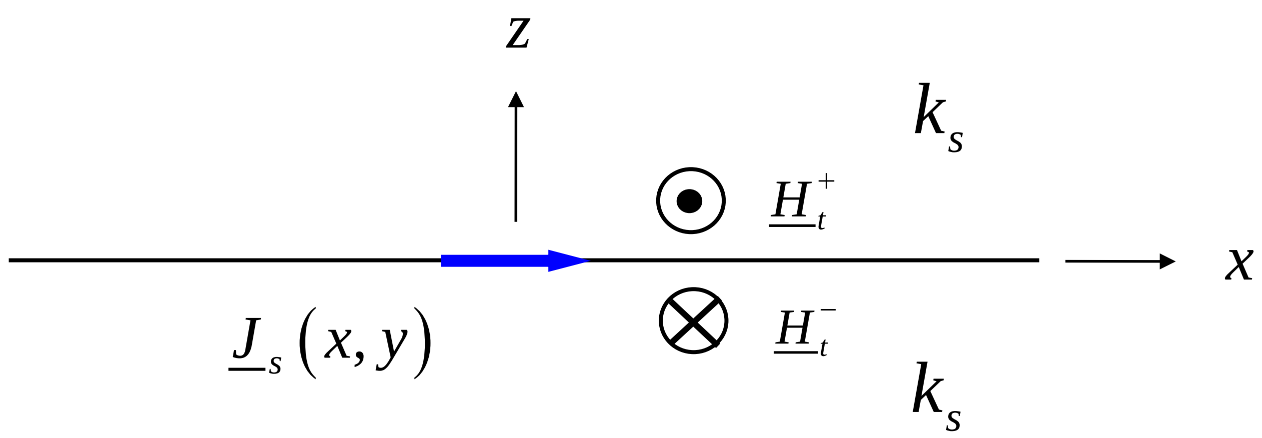


At $z = 0$ we have that $\underline{H}_t^+ = -\underline{H}_t^-$

Note: This follows from the fact that the magnetic vector potential component A_x is an even function of z (imagining the current to be in the x direction for simplicity here).

$$H_y = \frac{1}{\mu}\frac{\partial A_x}{\partial z}$$

SDI Method (cont.)



$$\underline{\hat{z}} \times \left(\underline{H}_t^+ - \underline{H}_t^- \right) = \underline{J}_s$$

$$2\,\underline{\hat{z}} \times \underline{H}_t^+ = \underline{J}_s$$

Hence

$$\underline{H}_t^+ = -\frac{1}{2}\underline{\hat{z}} \times \underline{J}_s$$

SDI Method (cont.)

$$\underline{H}_t^{+} = -\frac{1}{2}\,\underline{\hat{z}} \times \underline{J}_s$$

This is the tangential magnetic field just above the infinite current sheet, which gets launched by the current sheet.

Note: This result is general, and does not assume phased current sheets.

Two cases of interest for phased current sheets:

$$\underline{J}_s(x,y) = \underline{\hat{u}}\, J_{su}^{p}(x,y) \quad \Rightarrow \quad \underline{H}_t^{+} = \underline{\hat{v}}\, H_v^{+} \quad (TM_z)$$

$$\underline{J}_s(x,y) = \underline{\hat{v}}\, J_{sv}^{p}(x,y) \quad \Rightarrow \quad \underline{H}_t^{+} = \underline{\hat{u}}\, H_u^{+} \quad (TE_z)$$

SDI Method (cont.)

Split original current into two parts:

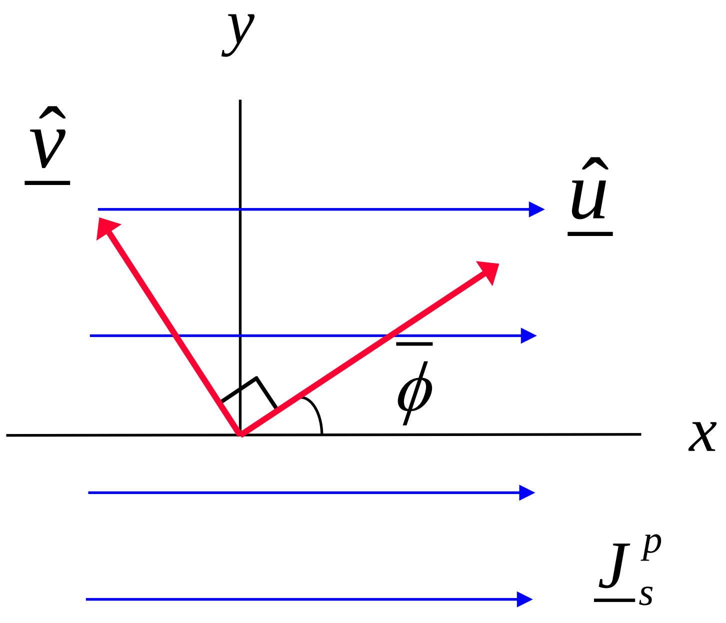
$$\underline{J}_s^p(x,y) = \underline{\hat{u}}\, J_{su}^p(x,y) + \underline{\hat{v}}\, J_{sv}^p(x,y)$$

Launches TM_z

Launches TE_z

$$J_{su}^p(x,y) = \underline{J}_s^p(x,y)\cdot\underline{\hat{u}}$$

$$J_{sv}^p(x,y) = \underline{J}_s^p(x,y)\cdot\underline{\hat{v}}$$

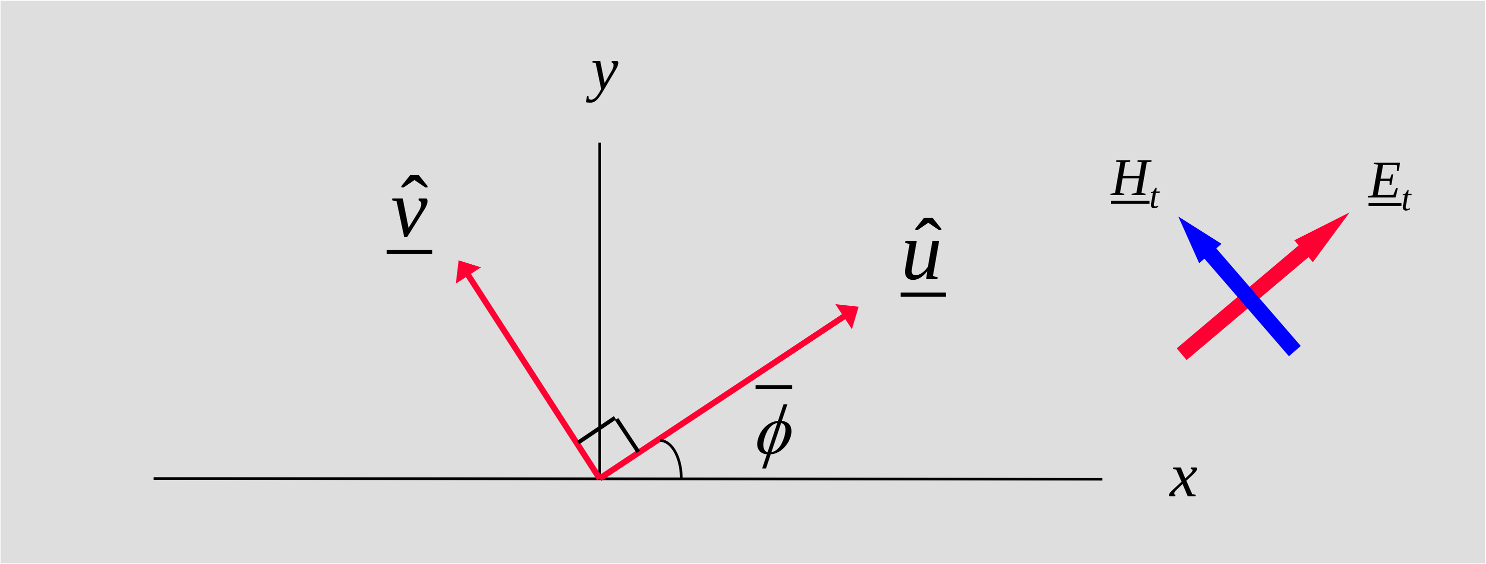


Transverse Fields

TM$_z$:

$$\underline{E}_t = \underline{\hat{u}}\, E_u(x, y, z) = \underline{\hat{u}}\, E_{u0}(z)\, e^{-j\left(k_x x + k_y y\right)}$$

$$\underline{H}_t = \underline{\hat{v}}\, H_v(x, y, z) = \underline{\hat{v}}\, H_{v0}(z)\, e^{-j\left(k_x x + k_y y\right)}$$

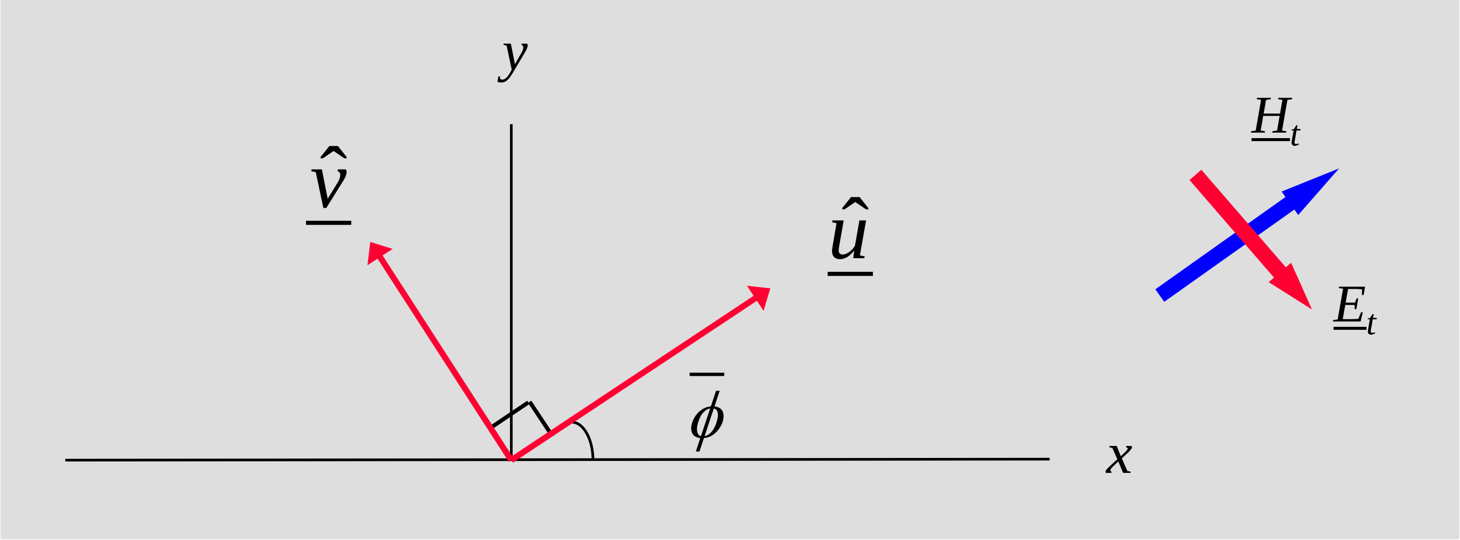


Transverse Fields (cont.)

TE$_z$:

$$\underline{E}_t = \underline{\hat{v}}\, E_v\left(x,y,z\right) = \underline{\hat{v}}\, E_{v0}\left(z\right) e^{-j\left(k_x x + k_y y\right)}$$

$$\underline{H}_t = \underline{\hat{u}}\, H_u\left(x,y,z\right) = \underline{\hat{u}}\, H_{u0}\left(z\right) e^{-j\left(k_x x + k_y y\right)}$$



Transverse Equivalent Network

TEN modeling equations:

TM$_z$:

$$V^{TM}(z) = E_{u0}(z)$$

$$I^{TM}(z) = H_{v0}(z)$$

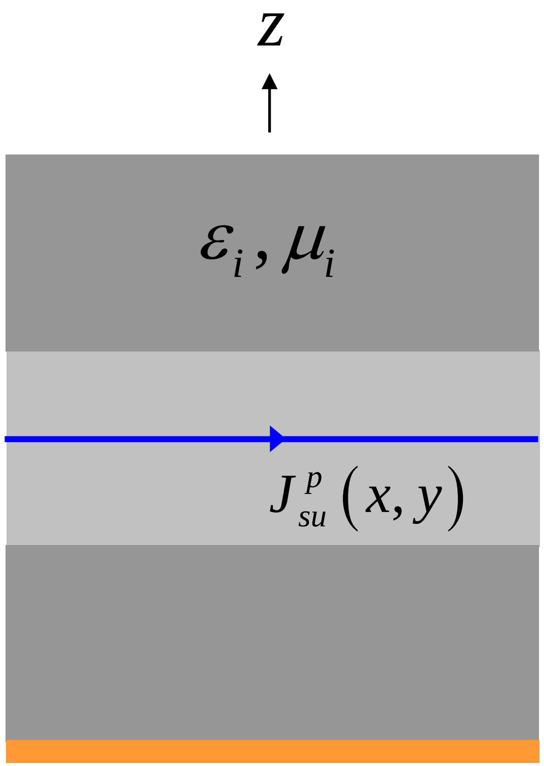
TE$_z$:

$$V^{TE}(z) = -E_{v0}(z)$$

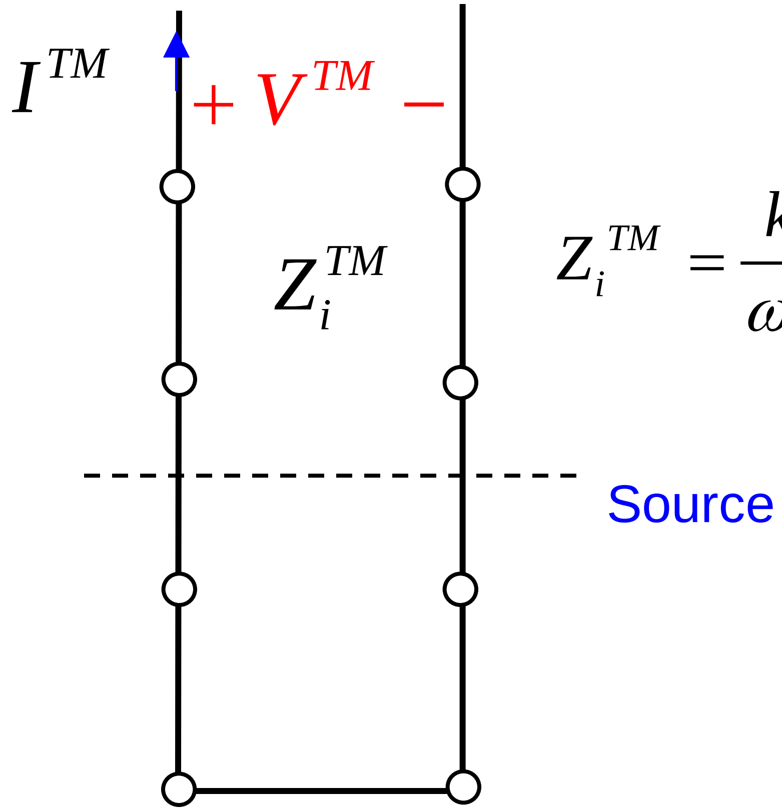
$$I^{TE}(z) = H_{u0}(z)$$

TEN (cont.)

TM_z



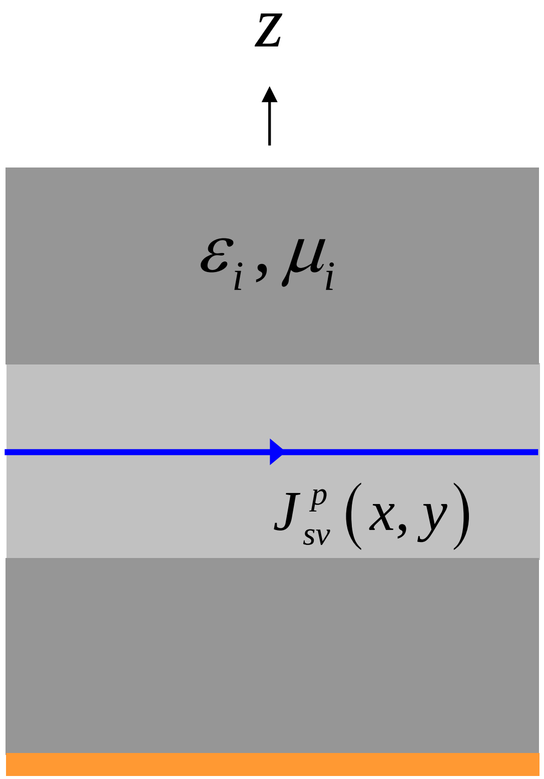
$$k_{zi} = \left(k_i^2 - k_x^2 - k_y^2\right)^{1/2}$$

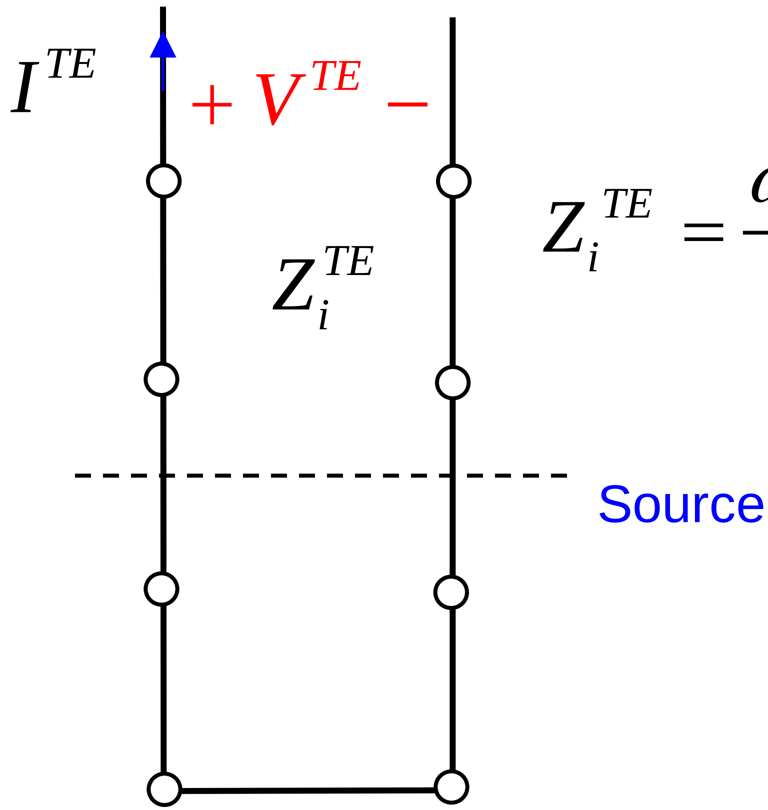


$$Z_i^{TM} = \frac{k_{zi}}{\omega \varepsilon_i}$$

TEN (cont.)

TE$_z$





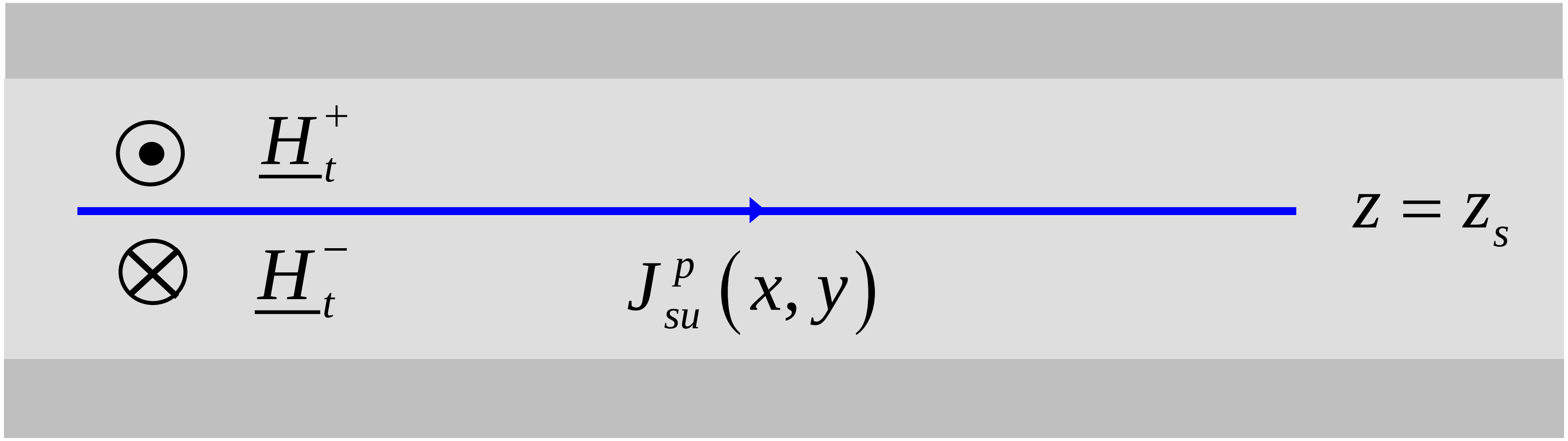
$$Z_i^{TE} = \frac{\omega \mu_i}{k_{zi}}$$

$$k_{zi} = \left(k_i^2 - k_x^2 - k_y^2\right)^{1/2}$$

Source Model

TM$_z$

Infinite phased current sheet inside layered medium



$$\underline{\hat{z}} \times \left(\underline{H}_t^{+} - \underline{H}_t^{-} \right) = \underline{J}_s(x,y) = \underline{\hat{u}}\, J_{su}^{p}(x,y)$$

$$\underline{\hat{z}} \times \left[\underline{\hat{v}} \left(H_v^{+} - H_v^{-} \right) \right] = \underline{\hat{u}}\, J_{su}^{p}(x,y)$$

$$-\underline{\hat{u}} \left(H_v^{+} - H_v^{-} \right) = \underline{\hat{u}}\, J_{su}^{p}(x,y)$$

Source Model (cont.)

$$H_v^+(x,y) - H_v^-(x,y) = -J_{su}^p(x,y)$$

so

$$H_{v0}^+ - H_{v0}^- = -J_{su0}^p$$

Hence

$$I^{TM}(z_s^+) - I^{TM}(z_s^-) = -J_{su0}^p$$

Also $\underline{\hat{z}} \times \left(\underline{E}_t^+ - \underline{E}_t^-\right) = \underline{0}$

$$\Rightarrow \left(\underline{\hat{z}} \times \underline{\hat{u}}\right)\left(E_u^+ - E_u^-\right) = \underline{0} \Rightarrow E_u^+ = E_u^- \Rightarrow E_{u0}^+ = E_{u0}^-$$

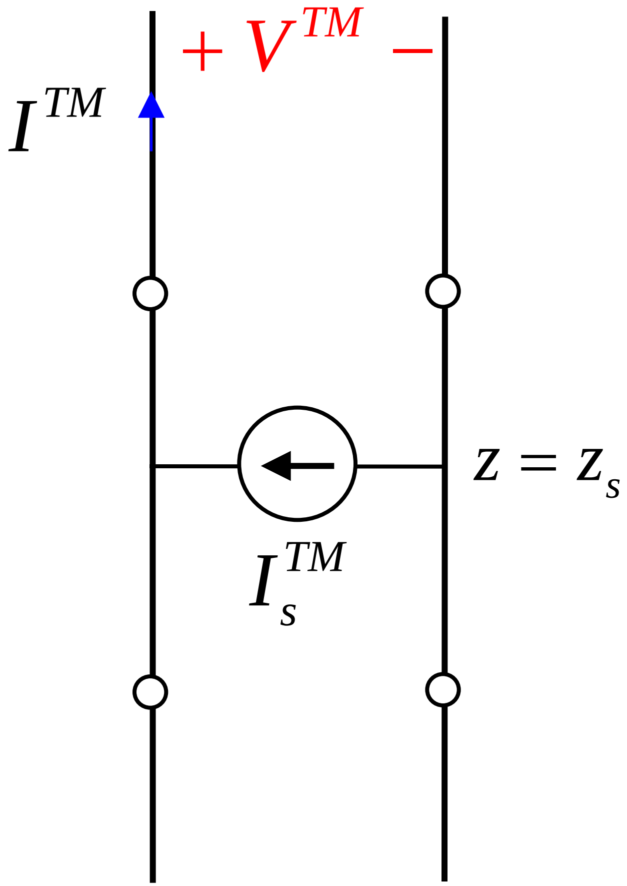
Hence

$$V^{TM}(z_s^+) = V^{TM}(z_s^-)$$

Source Model (cont.)

TM$_z$ Model:

$$I_s^{TM} = -\underline{J}_{s0}^{p} \cdot \underline{\hat{u}}$$

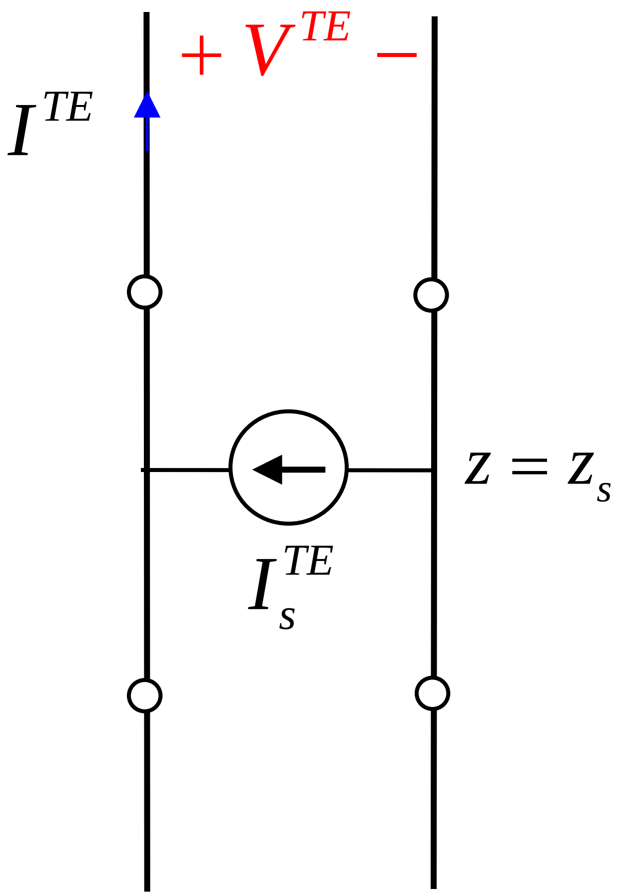


Source Model (cont.)

TE$_z$ Model:

$$I_s^{TE} = \underline{J}_{s0}^{p} \cdot \underline{\hat{v}}$$

(derivation omitted)



Source Model (cont.)

The transverse fields can then be found from

$$E_u(x,y,z) = V^{TM}(z)\, e^{-j(k_x x + k_y y)}$$

$$H_v(x,y,z) = I^{TM}(z)\, e^{-j(k_x x + k_y y)}$$

$$-E_v(x,y,z) = V^{TE}(z)\, e^{-j(k_x x + k_y y)}$$

$$H_u(x,y,z) = I^{TE}(z)\, e^{-j(k_x x + k_y y)}$$

Source Model (cont.)

Introduce normalized voltage and current functions:
(following the notation of K. A. Michalski)

$$V_i^{TM/TE}(z) = V^{TM/TE}(z) \quad \text{when} \quad I_s^{TM/TE} = 1\ [A]$$

$$I_i^{TM/TE}(z) = I^{TM/TE}(z) \quad \text{when} \quad I_s^{TM/TE} = 1\ [A]$$

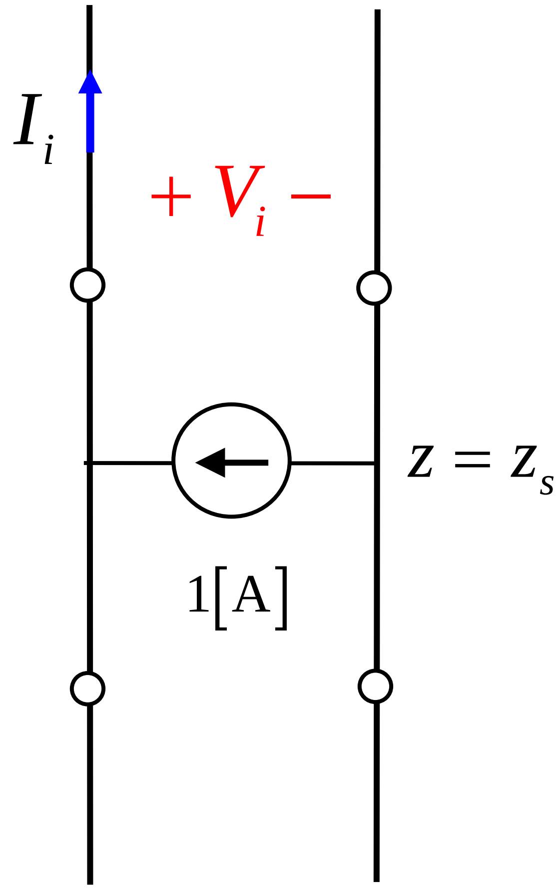
Then we can write

$$V^{TM}(z) = V_i^{TM}(z)\left(-\underline{J}_{s0}^{p} \cdot \underline{\hat{u}}\right)$$

$$I^{TM}(z) = I_i^{TM}(z)\left(-\underline{J}_{s0}^{p} \cdot \underline{\hat{u}}\right)$$

$$V^{TE}(z) = V_i^{TE}(z)\left(\underline{J}_{s0}^{p} \cdot \underline{\hat{v}}\right)$$

$$I^{TE}(z) = I_i^{TE}(z)\left(\underline{J}_{s0}^{p} \cdot \underline{\hat{v}}\right)$$



Source Model (cont.)

Hence

$$E_u(x,y,z) = V_i^{TM}(z)\left(-\underline{J}_{s0}^{p} \cdot \underline{\hat{u}}\right) e^{-j\left(k_x x + k_y y\right)}$$

$$H_v(x,y,z) = I_i^{TM}(z)\left(-\underline{J}_{s0}^{p} \cdot \underline{\hat{u}}\right) e^{-j\left(k_x x + k_y y\right)}$$

$$-E_v(x,y,z) = V_i^{TE}(z)\left(+\underline{J}_{s0}^{p} \cdot \underline{\hat{v}}\right) e^{-j\left(k_x x + k_y y\right)}$$

$$H_u(x,y,z) = I_i^{TE}(z)\left(+\underline{J}_{s0}^{p} \cdot \underline{\hat{v}}\right) e^{-j\left(k_x x + k_y y\right)}$$

Example

Find $E_x\,(x, y, x)$ for $z \geq 0$ due to a surface current at $z = 0$

$$\underline{J}_s^{\,p} = \underline{\hat{x}}\, e^{-j\left(k_x x + k_y y\right)}$$

The current is radiating in free space.

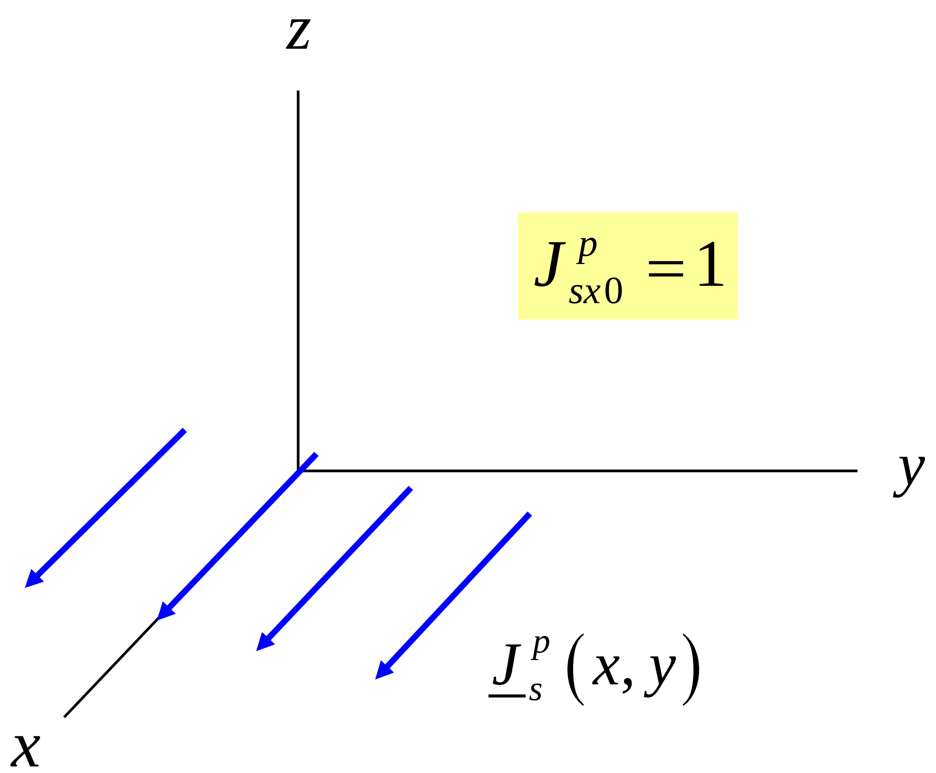
with

$$k_x = 2k_0$$

$$k_y = k_0$$

$$k_t = \sqrt{5}k_0$$

$$J_{sx0}^{\,p} = 1$$



$$\underline{J}_s^{\,p}\left(x, y\right)$$

Example

Decompose current into two parts:

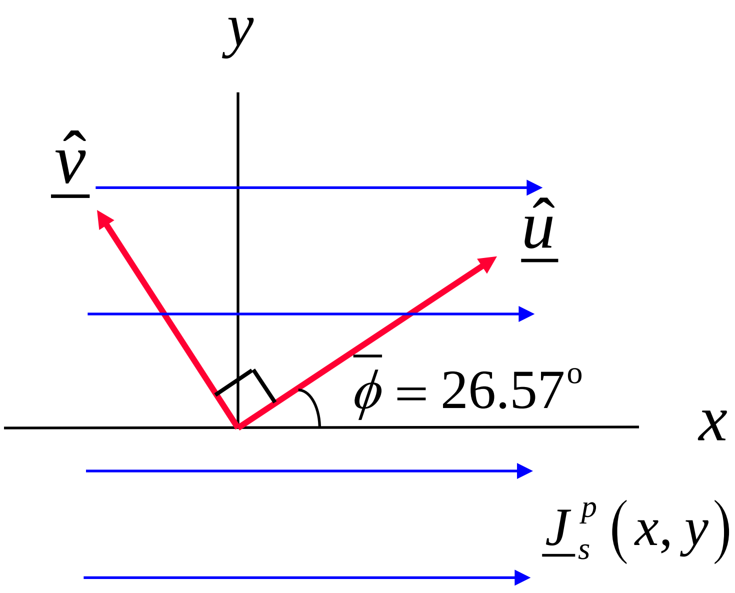
$$E_x = E_u\left(\underline{\hat{u}} \cdot \underline{\hat{x}}\right) + E_v\left(\underline{\hat{v}} \cdot \underline{\hat{x}}\right)$$

$$\tan\overline{\phi} = \frac{k_y}{k_x} = 0.5$$

$$\overline{\phi} = 26.57^{\circ}$$

$$\underline{\hat{u}} \cdot \underline{\hat{x}} = \cos\overline{\phi} = \frac{k_x}{k_t} = \frac{2}{\sqrt{5}}$$

$$\underline{\hat{v}} \cdot \underline{\hat{x}} = -\sin\overline{\phi} = -\frac{k_y}{k_t} = -\frac{1}{\sqrt{5}}$$



Example (cont.)

$$
\begin{aligned}
E_x &= E_u\left(\cos\overline{\phi}\right) + E_v\left(-\sin\overline{\phi}\right) \\
&= E_u\left(\frac{k_x}{k_t}\right) + E_v\left(-\frac{k_y}{k_t}\right) \\
&= \frac{1}{k_t}\left[k_x E_{u0} - k_y E_{v0}\right] e^{-j\left(k_x x + k_y y\right)} \\
&= \frac{1}{k_t}\left[k_x V^{TM}(z) + k_y V^{TE}(z)\right] e^{-j\left(k_x x + k_y y\right)} \\
&= \frac{1}{k_t}\left[k_x V_i^{TM}(z)\left(I_s^{TM}\right) + k_y V_i^{TE}(z)\left(I_s^{TE}\right)\right] e^{-j\left(k_x x + k_y y\right)} \\
&= \frac{1}{k_t}\left[k_x V_i^{TM}(z)\left(-\underline{J}_{s0}^{p}\cdot\underline{\hat{u}}\right) + k_y V_i^{TE}(z)\left(+\underline{J}_{s0}^{p}\cdot\underline{\hat{v}}\right)\right] e^{-j\left(k_x x + k_y y\right)}
\end{aligned}
$$

Example (cont.)

$$E_x = \frac{1}{k_t}\left[k_x V_i^{TM}(z)\left(-\underline{J}_{s0}^{\,p}\cdot\underline{\hat{u}}\right)+k_y V_i^{TE}(z)\left(+\underline{J}_{s0}^{\,p}\cdot\underline{\hat{v}}\right)\right]e^{-j\left(k_x x+k_y y\right)}$$

$$= \frac{1}{k_t}\left[k_x V_i^{TM}(z)\left(-J_{sx0}^{\,p}\right)\left(\underline{\hat{x}}\cdot\underline{\hat{u}}\right)+k_y V_i^{TE}(z)\left(+J_{sx0}^{\,p}\right)\left(\underline{\hat{x}}\cdot\underline{\hat{v}}\right)\right]e^{-j\left(k_x x+k_y y\right)}$$

$$= \frac{1}{k_t}\left[k_x V_i^{TM}(z)\left(-J_{sx0}^{\,p}\right)\left(\cos\overline{\phi}\right)+k_y V_i^{TE}(z)\left(+J_{sx0}^{\,p}\right)\left(-\sin\overline{\phi}\right)\right]e^{-j\left(k_x x+k_y y\right)}$$

$$= \frac{1}{k_t}\left[k_x V_i^{TM}(z)\left(-J_{sx0}^{\,p}\right)\left(\frac{k_x}{k_t}\right)+k_y V_i^{TE}(z)\left(+J_{sx0}^{\,p}\right)\left(-\frac{k_y}{k_t}\right)\right]e^{-j\left(k_x x+k_y y\right)}$$

Example (cont.)

Hence

$$E_x(x,y,z) = \frac{-1}{k_t^{\,2}}\left[k_x^2 V_i^{TM}(z) + k_y^2 V_i^{TE}(z)\right] J_{sx0}^{\,p}\, e^{-j\left(k_x x + k_y y\right)}$$

$$J_{sx0}^{\,p} = 1 \qquad \begin{matrix} k_x = 2k_0 \\ k_y = k_0 \end{matrix} \qquad k_t = \sqrt{5}k_0$$

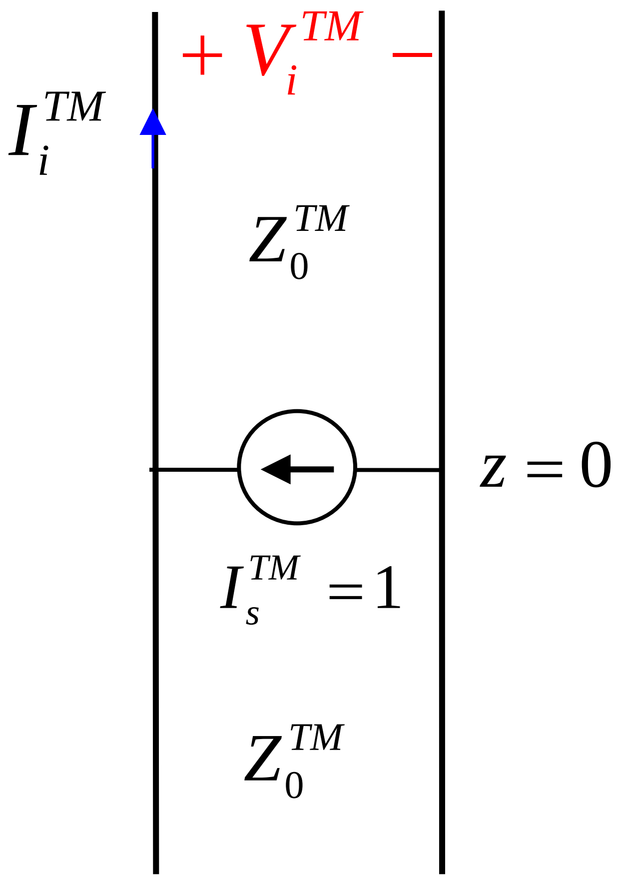
Example (cont.)

TM$_z$ Model

For $z > 0$:

$$V_i^{TM}(z) = \frac{Z_0^{TM}}{2} e^{-jk_{z0}z}$$

$$I_i^{TM}(z) = \frac{1}{2} e^{-jk_{z0}z}$$



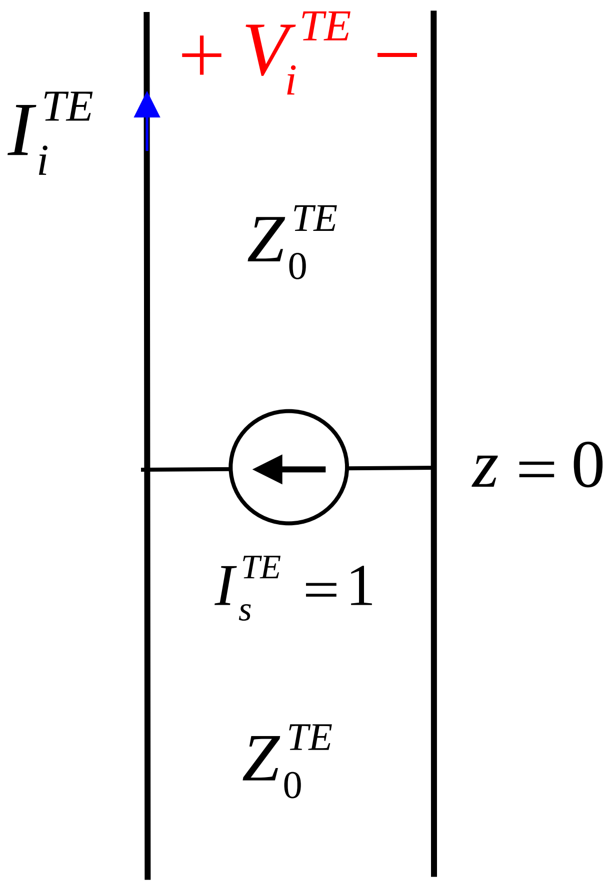
Example (cont.)

TE$_z$ Model

For $z > 0$:

$$V_i^{TE}(z) = \frac{Z_0^{TE}}{2} e^{-jk_{z0}z}$$

$$I_i^{TE}(z) = \frac{1}{2} e^{-jk_{z0}z}$$



Example (cont.)

Hence

$$E_x(x,y,z) = \frac{-1}{k_t^2}\left[k_x^2 V_i^{TM}(z) + k_y^2 V_i^{TE}(z)\right] J_{sx0}^{p}\, e^{-j(k_x x + k_y y)}$$

⇓

$$E_x(x,y,z) = \frac{-1}{k_t^2}\left[k_x^2\left(\frac{Z_0^{TM}}{2}\right) + k_y^2\left(\frac{Z_0^{TE}}{2}\right)\right] J_{sx0}^{p}\, e^{-j(k_x x + k_y y)}\, e^{-jk_{z0} z}$$

where $Z_0^{TM} = \dfrac{k_{z0}}{\omega\varepsilon_0}$ $\quad Z_0^{TE} = \dfrac{\omega\mu_0}{k_{z0}}$

$$k_{z0} = \left(k_0^2 - k_x^2 - k_y^2\right)^{1/2}$$

Example (cont.)

Substituting in values, we have: $k_x = 2k_0$ $k_y = k_0$ $k_t^2 = 5k_0^2$

$$k_{z0} = -j2k_0 \qquad Z_0^{TM} = -j2\eta_0 \qquad Z_0^{TE} = j\frac{\eta_0}{2} \qquad J_{sx0}^{p} = 1$$

Hence

$$E_x(x,y,z) = \frac{-1}{5k_0^{\,2}}\left[4k_0^2(-j\eta_0) + k_0^2\left(\frac{j\eta_0}{4}\right)\right] e^{-j(2k_0x+k_0y)}\, e^{-2k_0z}$$

or

$$E_x(x,y,z) = \frac{j\eta_0}{5}\left[\frac{15}{16}\right] e^{-jk_0(2x+y)}\, e^{-2k_0z}$$

or

$$E_x(x,y,z) = \frac{j3}{16}\,\eta_0\, e^{-jk_0(2x+y)}\, e^{-2k_0z} \qquad (z \ge 0)$$