ECE 6341

Spring 2016

# HW 2

Assigned problems: 1-6, 9-11, 13-15.

1. Assume that a TEN models a layered structure, where the *x* direction (the direction perpendicular to the layers) is the direction that the transmission line in the TEN runs. Normally, we would use waves that are TE*x* and TM*x*. Each polarization would then have a separate TEN model (no coupling of waves at the boundaries). Within each model (TE*x* or TM*x*), each layer would have a wave impedance that is unique for a given value of *kx*.

What would happen if you tried to use TE*z* and TM*z* waves in the TEN? Would you be able to uniquely define a wave impedance for the waves? For example, would the ratio *Ey* / *Hz* be the same as the ratio – *Ez* / *Hy* if you had a TE*z* or a TM*z* wave? Justify your answer by showing the calculation. Compare with what happens to these ratios when you use TE*x* and TM*x* waves.

1. Consider a hollow rectangular waveguide of dimensions *a* and *b* in the *x* and *y* directions, where the left side wall is at *x* = 0 and the bottom side wall is at *y* = 0. Assume a TE*z* (*m*,*n*) mode described by

.

Show that this mode can also be written as a sum of TM*x* and TE*x* modes by solving for *Ax* and *Fx* that will produce the same field inside the waveguide as does the original *Fz*. Hint: In writing down expressions for *Ax* and *Fx*, consider what boundary conditions they will have to satisfy at the walls.

Next, consider the TE10 mode as a special case. Solve for *Ax* and *Fx* for this mode.

1. A grounded dielectric slab of Teflon has a relative permittivity of 2.2 and a thickness of 60 mils (1.524 mm). Use a numerical search to find the normalized phase constant  of the TM0 surface-wave mode at the following frequencies: 1 GHz, 10 GHz, 100 GHz.
2. Assume that we have the same grounded slab as in the previous problem, but now we are interested in the TM1 mode. First, find the cutoff frequency of this mode. Then, by numerically searching for the improper surface-wave solutions, find the splitting point frequency *fs*. Then, for a frequency that is 10% lower than the splitting point frequency (i.e., *f* = 0.9 *fs*) do a numerical search to find *kz* / *k*0 for the complex TM1 leaky-mode solution (the one that has a positive attenuation constant *αz*). You may use whatever numerical search routine you want. Note that the secant method works in the complex plane, and is usually a good choice. This method is allows you to find the complex roots (zeros) of a complex function *f* (*z*). The method is represented by the following iterative formula (which requires two initial guesses *z*0 and *z*1):

.

Note: If you have trouble finding the root 10% below the splitting point frequency, you can try lowering the frequency gradually below the splitting-point frequency, and tracking the root continuously as a function of frequency.

1. Draw a ray picture for a leaky mode that has a negative value of *βz* (the phase velocity is in the negative *z* direction) but the wave is attenuating in the positive *z* direction. That leaky wave still radiates outward from the structure. That “rays” thus point in the backward direction (to the left of the positive *x* axis). Such waves, called “backward waves,” are very important on some types of guiding structures, such as periodic structures. Explain using the ray picture why a leaky wave that is backward is proper. Next, give a mathematical proof of why this wave must be proper, similar to the proof of why a leaky wave on a simple guiding structure such as a grounded slab (which is a “forward” leaky wave) must be improper.
2. A TE*x* leaky mode has a field on the interface (*x* = 0) due to a line source at *z* = 0 that is represented as



where  and *A* is an amplitude constant. The exact electric field above the interface is



where . The branch of the square root is chosen so that that  when *kx* is real, and  when *kx* is imaginary.

Perform the integration above in order to show that



and thus derive an exact expression for the field above the interface due to the leaky mode. (The expression will be in the form of an integral, as shown above). Explain how you are handling the limit at ±∞ in your integral evaluation.

1. Assume that we have a TE*x* leaky mode as in Prob. 5, with  and . Plot the magnitude of the field  versus *x*. Plot over the range  for the following fixed values of *z*: . Comment on the variation that you observe vertically. The field should be calculated numerically by using the result of Prob. 5. Assume that *A* = 1 (the amplitude of the leaky-wave field on the interface is unity).

*h*

*r*

*z*

*x*

Line source

1. According to the method of stationary phase (or the method of steepest-descent) which will be discussed later in the semester, we can asymptotically evaluate the field radiated by the leaky mode, by using the asymptotic result for *k*0*ρ* → ∞ that says



where  is an arbitrary function of , , , and in cylindrical coordinates , . Assuming this relation, derive the far-field pattern  of the TE*x* leaky mode in Problem 5.

1. The far-field pattern of a TE*x* bi-directional leaky mode on a grounded slab is given by

.

Evaluate this integral and show that the result is

.

1. Plot the far-field pattern of a TE*x* bi-directional leaky wave having the following properties:

a) 

b) .

Plot the magnitude of the far-field pattern (i.e., the array factor given in the problem above) vs. angle *θ* in degrees (a rectangular plot, not a polar plot), with the angle *θ* in the range-90o < *θ* < 90o. (Note: The pattern should be symmetric about *θ* = 0.) Normalize your patterns so that the magnitude of the far-field pattern is unity at the peaks of the beam.

1. Repeat the previous problem, assuming now that  and . This corresponds to a leaky mode that is in the non-physical (slow-wave) region.
2. Assume that we have a leaky-wave field described by



where

.

Introduce the following change of variables:

,

where is a complex number with  and  (which corresponds to , , , ). Also, introduce cylindrical coordinates as

,

where *θ* is the angle in cylindrical coordinates measured from the *x* axis, and *ρ* is the radial distance in cylindrical coordinates from the *y* axis (see the figure in Prob. 6). Show that the magnitude of the leaky-wave field may be written as

.

Next, show that

,

where *θ*0 is the angle from the *x* axis to the power flow vector (*β* vector), so that

.

(Hint: Use expressions for *βx* and *βx* in terms of *ζ*.)

Finally, conclude that

.

Use this result to explain why the magnitude of the leaky-wave field always decreases with distance *ρ* in cylindrical coordinates, as long as we are in the “leakage region” so that *θ* > *θ*0.

1. A sheet impedance *Zs* is a hypothetical sheet layer that has a continuous tangential electric field across it, but it supports a tangential (i.e., in the plane of the sheet, which we can assume is the *yz* plane) surface current *Js* in the direction of the tangential electric field *Et*. In general, we would have, from the definite of sheet impedance, that . Assume that a particular component of tangential electric field (e.g., *Ez*) is being modeled as voltage on a TEN model of a waveguiding structure that contains a sheet impedance, and that the corresponding perpendicular component of the tangential magnetic field (e.g., -*Hy*) is being modeled as current on the TEN. (The TEN runs in the *x* direction.) Show that the sheet impedance appears in the TEN as a lumped parallel impedance element, with a value of *Zs* Ohms.
2. Consider a “partially reflecting surface” (or “PRS”) type of leaky-wave antenna structure that consists of a lossless sheet impedance  (the PRS) above a grounded dielectric slab region as shown below. There is air above the structure. Assume a TM*x* leaky mode with a complex wavenumber *kz* is propagating on the structure in the *z* direction (there is no *y* variation). Draw a TEN for this structure and use the TRE method to derive a transcendental equation for the unknown complex wavenumber *kz* of the leaky wave. Choose a reference plane that is just below the sheet impedance.

In the transcendental equation, how do you choose the square root in order to calculate *kx*0(the vertical wavenumber in the air region)from *kz*?

Practical note: The sheet impedance could be modeling a partially-reflecting surface that is a periodic frequency selective surface (FSS) such as an array of metal patches, etc.



1. A rectangular waveguide is loaded with dielectric slabs on either side, as shown below. Derive transcendental equations for the wavenumber *kz* and the cutoff frequencies for all four possible modes that can exist inside the waveguide. The four mode types are: TM*x*e, TM*x*o, TE*x*e, and TE*x*o, where the (e,o) superscript denotes even or odd mode.

*a*

*b*

*t*

*t*

*εr*

*εr*

*ε*0

*x*

*y*