ECE 6345

Fall 2024

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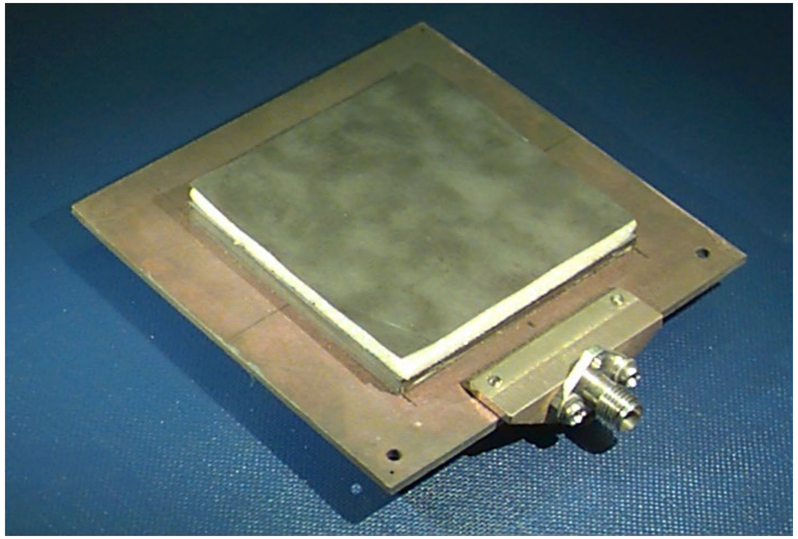
ECE Dept.



Notes 1

Overview

This set of notes discusses the Q (quality factor) of a patch and its different components, as well as the radiation efficiency of the patch based on the Q components.

- Define Q and its components: Q_d, Q_c, Q_{sp}, Q_{sw}
- Calculate Q_d, Q_c, Q_{sp}, Q_{sw}
- Calculate the radiation efficiency based on the Q components.

Q of a Resonator

$$Q \equiv 2\pi \left(\frac{U_S}{U_D^T} \right)$$

$U_s =$ energy stored inside resonator

$U_D^T =$ energy dissipated per cycle (period T)

$$Q \equiv \frac{2\pi}{T} \left(\frac{U_S}{U_D^T / T} \right)$$

$T = 1 / f_0 =$ period

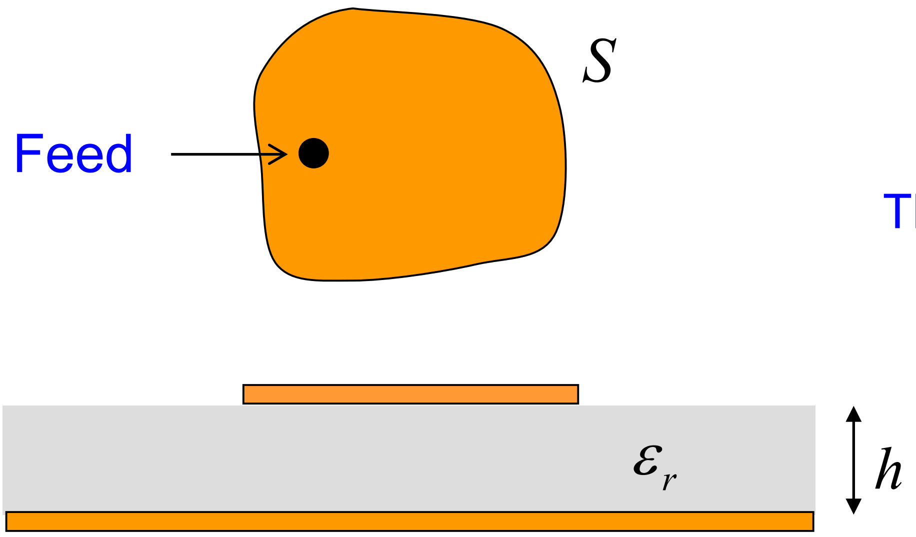
or

$$Q \equiv \omega_0 \left(\frac{U_S}{P_D^{\text{ave}}} \right)$$

$P_D^{\text{ave}} =$ average power "dissipated"

(This includes radiation loss)

Q of the Patch



The patch is allowed to have an arbitrary shape here.

$$Q = \omega_0 \frac{U_S}{P_D^{\text{ave}}} \qquad \text{so} \qquad \frac{1}{Q} = \frac{P_D^{\text{ave}}}{\omega_0 U_S}$$

$$P_D^{\text{ave}} = P_D^{\text{sp}} + P_D^{\text{sw}} + P_D^{c} + P_D^{d}$$

Note: ω_0 denotes the resonance frequency (radians/second).

Q of the Patch (cont.)

Hence, we have:

$$\frac{1}{Q} = \frac{P_D^{sp}}{\omega_0 U_S} + \frac{P_D^{sw}}{\omega_0 U_S} + \frac{P_D^{c}}{\omega_0 U_S} + \frac{P_D^{d}}{\omega_0 U_S}$$

or

$$\frac{1}{Q} = \frac{1}{Q_{sp}} + \frac{1}{Q_{sw}} + \frac{1}{Q_c} + \frac{1}{Q_d}$$

Note: Combining Q terms is like combining resistors in parallel.

Note: A smaller Q is a more dominant one!

Calculation of Q_d

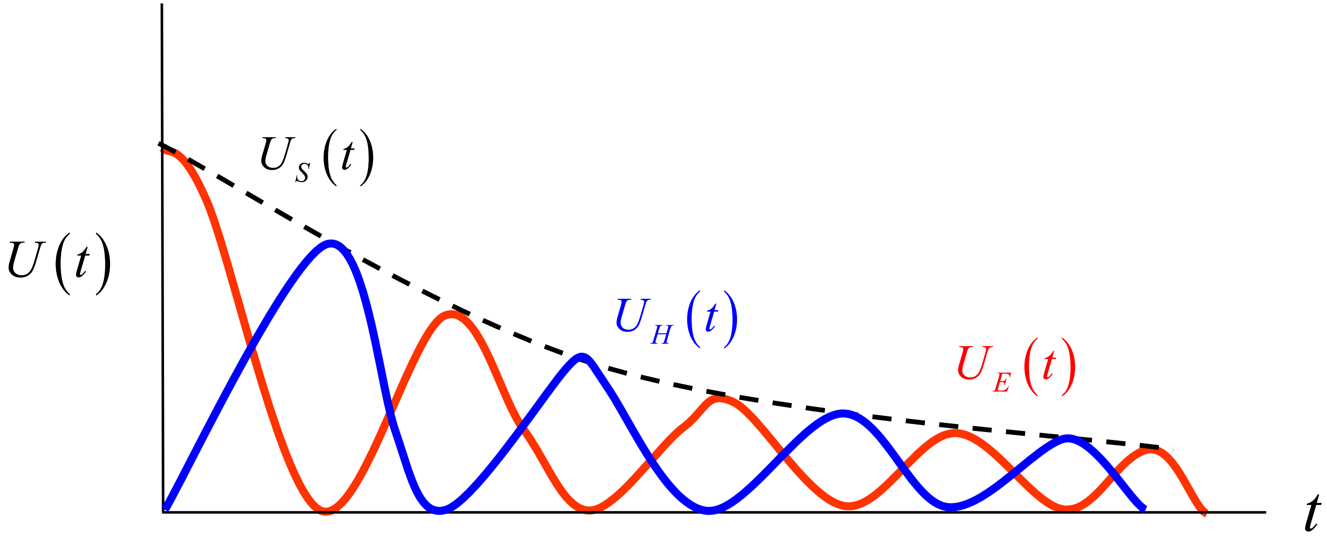
$$U_S = \langle U_S \rangle = \langle U_E \rangle + \langle U_H \rangle = 2\langle U_E \rangle$$

(We have equal time-average stored energies at resonance.)

$$= 2\int_V \frac{1}{4}\varepsilon' \left|\underline{E}\right|^2 dV$$

$$= \frac{1}{2}\varepsilon' h \int_S \left|\underline{E}\right|^2 dS$$

Note:
The total stored energy U_S is a slowly decaying function of time (approximately constant over a cycle).



From ECE 6340:

$$P_D^d = \int_V \frac{1}{2}\left(\omega_0 \varepsilon''\right)\left|\underline{E}\right|^2 dV$$

$$= \frac{1}{2}\omega_0 \varepsilon'' h \int_S \left|\underline{E}\right|^2 dS$$

Calculation of Q_d (cont.)

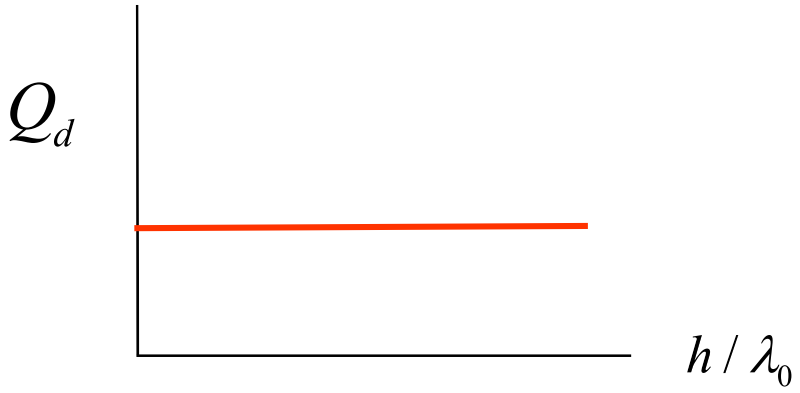
Hence

$$Q^d = \omega_0 \left[\frac{\frac{1}{2}\varepsilon' h \int_S |\underline{E}|^2 \, dS}{\frac{1}{2}\omega_0 \varepsilon'' h \int_S |\underline{E}|^2 \, dS} \right] = \frac{\varepsilon'}{\varepsilon''}$$

Therefore, we have:

$$Q^d = \frac{1}{\tan \delta}$$

The dielectric Q factor is independent of the substrate thickness.



Calculation of Q_c

$$\left\langle P_D^c \right\rangle = \int_{S_P} \frac{1}{2} R_s^{\text{patch}} \left|\underline{H}\right|^2 dS + \int_{S_G} \frac{1}{2} R_s^{\text{ground}} \left|\underline{H}\right|^2 dS$$

S_P denotes patch
S_G denotes ground plane

Hence, we have:

$$R_s^{\text{ave}} = \left(R_s^{\text{patch}} + R_s^{\text{ground}} \right) / 2$$

$$\left\langle P_D^c \right\rangle = \frac{1}{2} \left(R_s^{\text{patch}} + R_s^{\text{ground}} \right) \int_S \left|\underline{H}\right|^2 dS$$

$$= R_s^{\text{ave}} \int_S \left|\underline{H}\right|^2 dS$$

$$R_s = \frac{1}{\sigma\delta}$$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

Calculation of Q_c (cont.)

For the stored energy,

$$U_S = \langle U_S \rangle = \langle U_E \rangle + \langle U_H \rangle = 2\langle U_H \rangle$$

where

$$\langle U_H \rangle = \int_V \frac{1}{4} \mu \left| \underline{H} \right|^2 dV$$

$$= \frac{1}{4} \mu_0 \mu_r h \int_S \left| \underline{H} \right|^2 dS$$

Calculation of Q_c (cont.)

Hence,

$$Q_c = \omega_0 \frac{2\left(\frac{1}{4}\mu_0\mu_r h \int_S \left|\underline{H}\right|^2 dS\right)}{R_s^{\text{ave}} \int_S \left|\underline{H}\right|^2 dS}$$

Use

$$\omega_0 = \frac{k_0}{\sqrt{\mu_0 \varepsilon_0}}$$

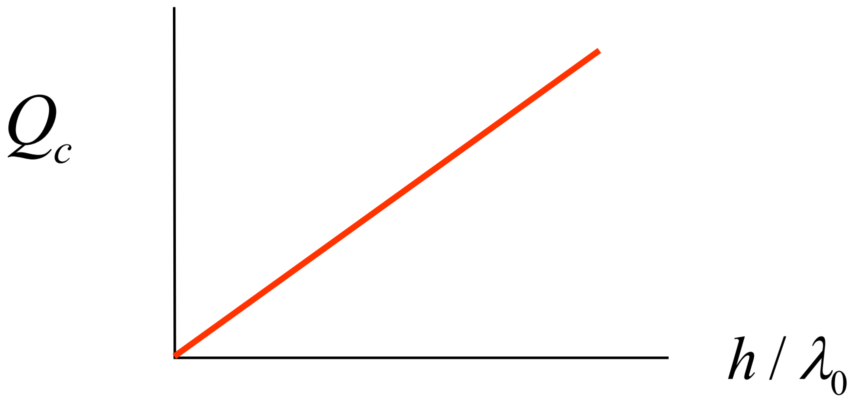
We then have:

$$Q_c = \frac{\frac{1}{2}\mu_0\mu_r(k_0 h)}{R_s^{\text{ave}}\sqrt{\mu_0\varepsilon_0}} = \frac{\frac{1}{2}\mu_r(k_0 h)}{R_s^{\text{ave}}}\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Calculation of Q_c (cont.)

Hence, we have:

$$Q_c = \left(\frac{\eta_0}{2}\right)\mu_r\left[\frac{(k_0 h)}{R_s^{\text{ave}}}\right]$$



The conducting Q factor becomes more important (smaller) as the substrate gets thinner.

Calculation of Q_{sp}

Rectangular Patch: (The derivation is given later.)

$$Q_{\mathrm{sp}} \approx \frac{3}{16}\left(\frac{\varepsilon_r}{pc_1}\right)\left(\frac{L_e}{W_e}\right)\left(\frac{1}{h/\lambda_0}\right)$$

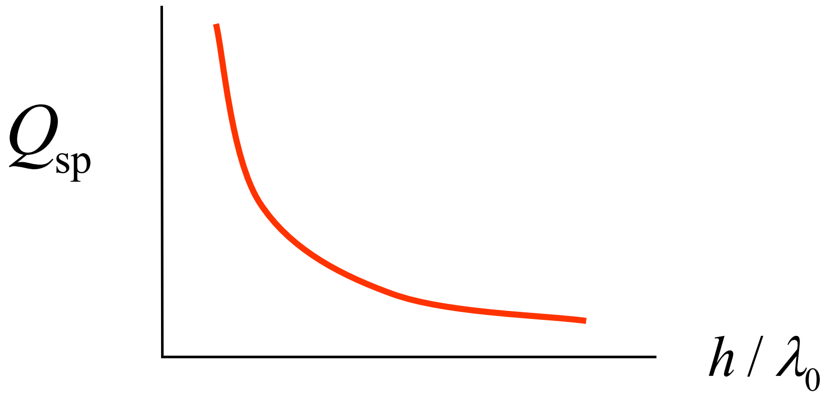
$$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4} \qquad \left(n_1 = \sqrt{\varepsilon_r \mu_r}\right)$$

where

$$p = 1 + \frac{a_2}{10}\left(k_0 W_e\right)^2 + \left(a_2^2 + 2a_4\right)\left(\frac{3}{560}\right)\left(k_0 W_e\right)^4$$
$$+ c_2\left(\frac{1}{5}\right)\left(k_0 L_e\right)^2 + a_2 c_2\left(\frac{1}{70}\right)\left(k_0 W_e\right)^2\left(k_0 L_e\right)^2$$

$$a_2 = -0.16605$$
$$a_4 = 0.00761$$
$$c_2 = -0.0914153$$

Calculation of Q_{sp} (cont.)



The radiation Q factor becomes more important (smaller) as the substrate gets thicker.

$$Q_{\mathrm{sp}} \approx \frac{3}{16}\left(\frac{\varepsilon_r}{p c_1}\right)\left(\frac{L_e}{W_e}\right)\left(\frac{1}{h / \lambda_0}\right)$$

Calculation of Q_{SW}

Surface-wave radiation efficiency:

$$e_r^{\mathrm{sw}} \equiv \frac{P_{\mathrm{sp}}}{P_{\mathrm{sp}} + P_{\mathrm{sw}}}$$

so

$$\frac{1}{e_r^{\mathrm{sw}}} = \frac{P_{\mathrm{sp}} + P_{\mathrm{sw}}}{P_{\mathrm{sp}}} = 1 + \frac{P_{\mathrm{sw}}}{P_{\mathrm{sp}}}$$

Hence

$$P_{\mathrm{sw}} = P_{\mathrm{sp}} \left(\frac{1}{e_r^{\mathrm{sw}}} - 1 \right)$$

Calculation of Q_{SW} (cont.)

Now look at the Q's :

$$Q_{\mathrm{sw}} = \omega_0 \frac{U_S}{P_{\mathrm{sw}}}$$

$$Q_{\mathrm{sp}} = \omega_0 \frac{U_S}{P_{\mathrm{sp}}}$$

Hence, we have

$$\frac{Q_{\mathrm{sw}}}{Q_{\mathrm{sp}}} = \frac{P_{\mathrm{sp}}}{P_{\mathrm{sw}}}$$

$$= \frac{1}{\frac{1}{e_r^{\mathrm{sw}}} - 1}$$

$$= \frac{e_r^{\mathrm{sw}}}{1 - e_r^{\mathrm{sw}}}$$

Calculation of Q_{sw} (cont.)

Hence, we have:

$$Q_{\text{sw}} = Q_{\text{sp}} \left(\frac{e_r^{\text{sw}}}{1 - e_r^{\text{sw}}} \right)$$

For the radiation efficiency, the patch can be approximated by a horizontal electric dipole (HED) at the center of the patch.

$$e_r^{\text{sw}} \approx e_r^{\text{hed}}$$

(This is approximately true for any shape of patch.)

Calculation of Q_{SW} (cont.)

$$e_r^{\text{hed}} = \frac{P_{\text{sp}}^{\text{hed}}}{P_{\text{sp}}^{\text{hed}} + P_{\text{sw}}^{\text{hed}}} = \frac{1}{1 + \left(P_{\text{sw}}^{\text{hed}} / P_{\text{sp}}^{\text{hed}}\right)}$$

$$P_{\text{sp}}^{\text{hed}} \approx \frac{1}{\lambda_0^2}\left(k_0 h\right)^2 \left(80\pi^2 \mu_r^2 c_1\right)$$

$$P_{\text{sw}}^{\text{hed}} \approx \frac{1}{\lambda_0^2}\left(k_0 h\right)^3 \left(60\pi^3 \mu_r^3 \left(1 - \frac{1}{n_1^2}\right)^3\right)$$

where

$$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4} \qquad n_1 = \sqrt{\varepsilon_r \mu_r}$$

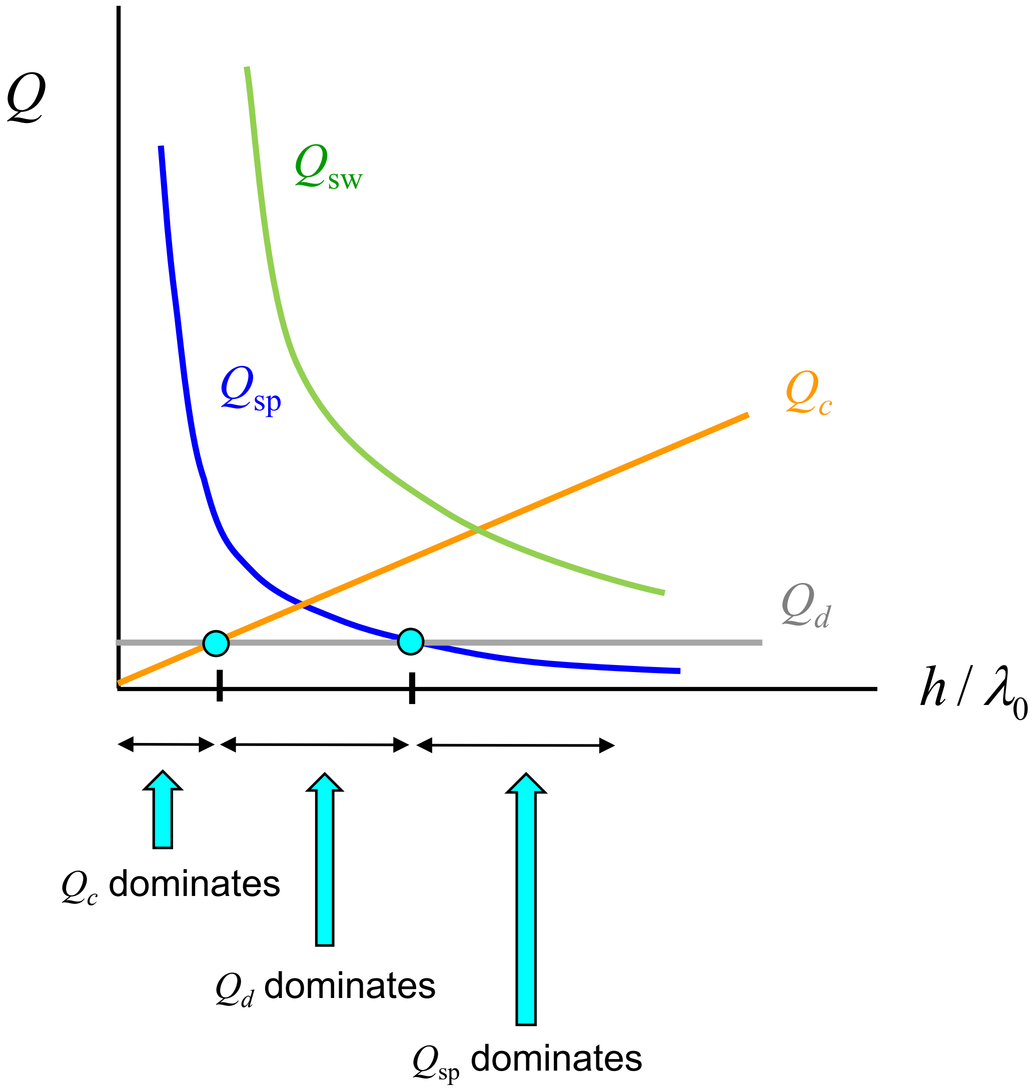
Calculation of Q_{SW} (cont.)

Hence, we have

$$e_r^{\mathrm{sw}} \approx e_r^{\mathrm{hed}} = \frac{1}{1+(k_0 h)\left(\frac{3\pi}{4}\right)\mu_r \frac{1}{c_1}\left(1-\frac{1}{n_1^2}\right)^3}$$

This holds for any shape patch
(assuming the HED approximation is accurate).

Comparison of Qs



Radiation Efficiency

$$e_r \equiv \frac{P_{\text{sp}}}{P_{\text{tot}}} = \frac{P_{\text{sp}}}{P_{\text{sp}} + P_{\text{sw}} + P_c + P_d}$$

Note that

$$Q_i = \omega_0 \frac{U_S}{P_i}$$

Hence

$$P_i \propto \frac{1}{Q_i}$$

Radiation Efficiency (cont.)

Hence

$$e_r = \frac{\dfrac{1}{Q_{\text{sp}}}}{\dfrac{1}{Q_{\text{sp}}} + \dfrac{1}{Q_{\text{sp}}} + \dfrac{1}{Q_c} + \dfrac{1}{Q_d}}$$

or $$e_r = \frac{\dfrac{1}{Q_{\text{sp}}}}{\dfrac{1}{Q}}$$ or $$e_r = \frac{Q}{Q_{\text{sp}}}$$

Radiation Efficiency (cont.)

Also, we can write

$$e_r = \frac{P_{\text{sp}}}{P_{\text{tot}}} = \left(\frac{P_{\text{sp}}}{P_{\text{spw}}}\right)\left(\frac{P_{\text{spw}}}{P_{\text{tot}}}\right)$$

where $$P_{\text{spw}} = P_{\text{sp}} + P_{\text{sw}}$$

Define:

$$e_r^{\text{sw}} = \frac{P_{\text{sp}}}{P_{\text{spw}}}$$ (efficiency due to surface wave loss only)

$$e_r^{\text{diss}} = \frac{P_{\text{spw}}}{P_{\text{tot}}}$$ (efficiency due to material (dissipation) loss only)

Radiation Efficiency (cont.)

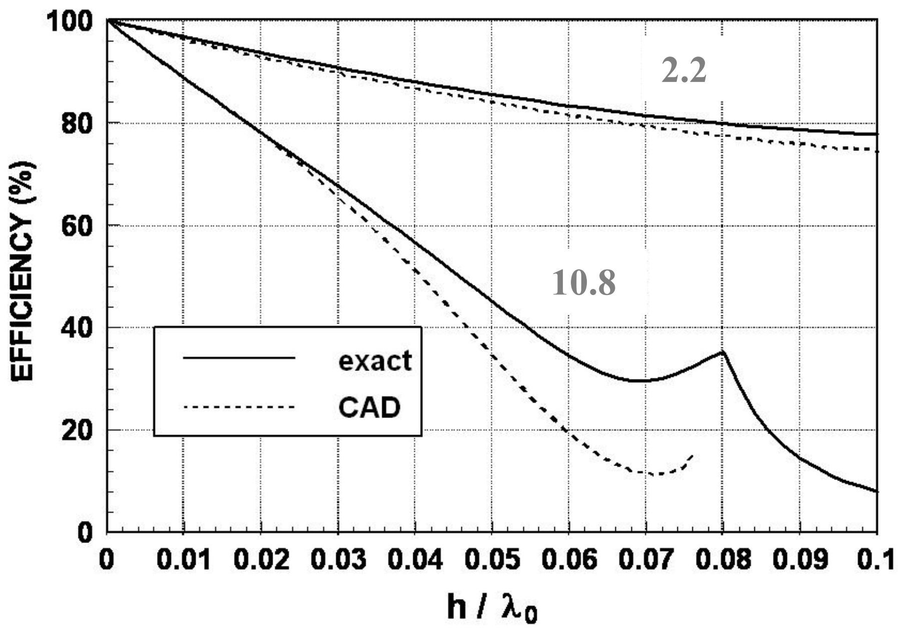
Then $$e_r = e_r^{\mathrm{sw}} e_r^{\mathrm{diss}}$$

where $$e_r^{\mathrm{sw}} = \frac{Q_{\mathrm{spw}}}{Q_{\mathrm{sp}}}$$ $$e_r^{\mathrm{diss}} = \frac{Q}{Q_{\mathrm{spw}}}$$

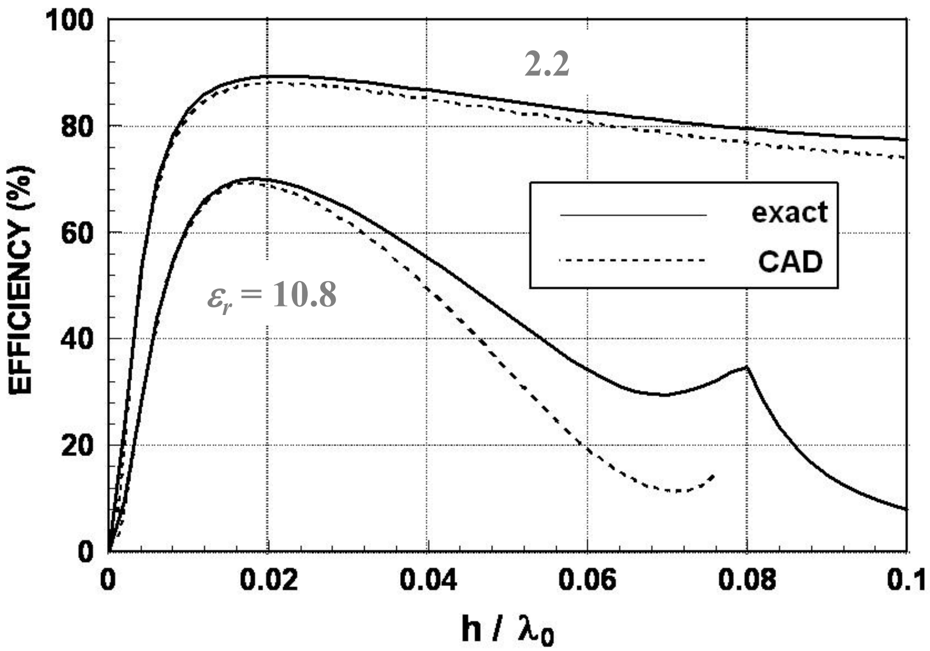
and $$\frac{1}{Q_{\mathrm{spw}}} = \frac{1}{Q_{\mathrm{sp}}} + \frac{1}{Q_{\mathrm{sw}}}$$

Radiation Efficiency (cont.)

e_r^{sw}



e_r



$\varepsilon_r = 2.2$ or 10.8

$W / L = 1.5$

$\tan\delta = 0.001$

$\sigma = 3.0\times10^{7}$ [S/m]