ECE 6345

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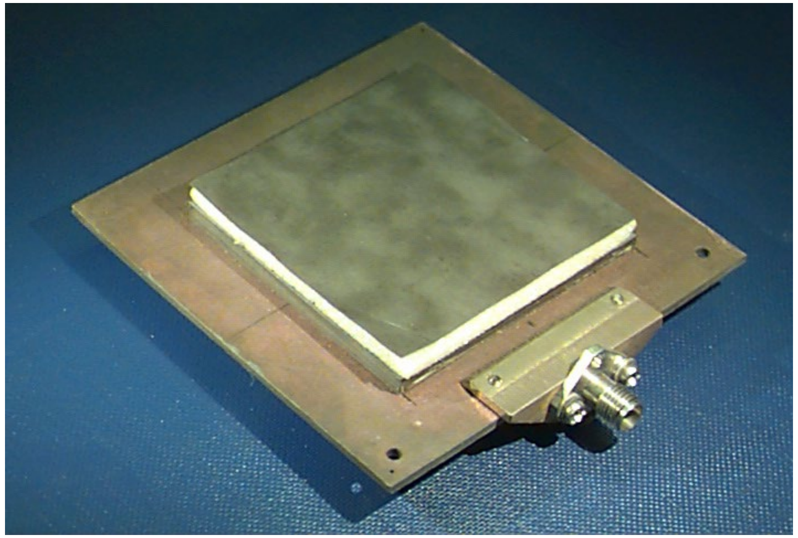


Notes 13

Overview

❖ In this set of notes we derive a general expression for the "p factor" that is used to determine the space-wave power radiated by the rectangular patch.

❖ In the next set of notes we will evaluate the integrals that appear and actually develop a final closed-form CAD expression for the p factor.

The p Factor

Definition of the p factor:

$$\boxed{p \equiv \frac{P_{\text{sp}}}{P_{\text{sp}}^{\text{dip}}}}$$

P_{sp} = power radiated by the actual rectangular patch

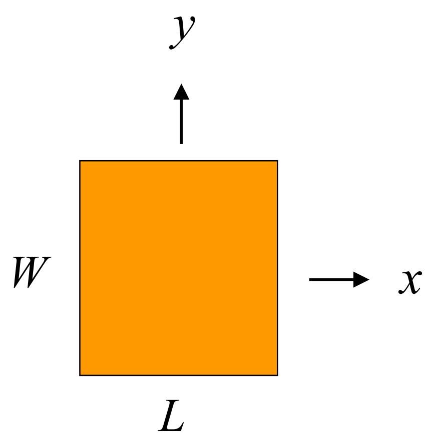
$P_{\text{sp}}^{\text{dip}}$ = power radiated by a dipole that has the equivalent dipole moment: $(Il)_{\text{patch}} = \frac{2}{\pi}(WL)$

This assumes: $J_{sx}^{1,0} = \cos\left(\frac{\pi x}{L}\right)$

The p Factor (cont.)

We can then write:

$$P_{\mathrm{sp}} = P_{\mathrm{sp}}^{\mathrm{dip}}\, p$$



From Notes 12, we have:

$$P_{\mathrm{sp}}^{\mathrm{dip}} = (Il)^2\, k_0^2 \left(\frac{\eta_0}{32\pi}\right) \int_0^{\pi/2} \left[|F(\theta)|^2 + |G(\theta)|^2 \right] \sin\theta \, d\theta$$

$$P_{\mathrm{sp}}^{\mathrm{dip}} \approx \left(\frac{2}{\pi} WL\right)^2 (k_0 h)^2\, k_0^2 \left(\frac{\eta_0}{6\pi}\right) \mu_r^2 c_1$$

$$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$

This assumes

$$J_{sx}^{1,0} = \cos\left(\frac{\pi x}{L}\right)$$

$$Il = (Il)_{\mathrm{patch}} = \frac{2}{\pi}(WL)$$

The p Factor (cont.)

Calculation of the patch space-wave radiated power:

$$P_{\mathrm{sp}} = \int_0^{2\pi}\int_0^{\pi/2} S_r(r,\theta,\phi)\, r^2 \sin\theta \, d\theta \, d\phi$$

where

$$S_r = \frac{1}{2\eta_0}\left[\left|E_\theta\right|^2 + \left|E_\phi\right|^2\right]$$

and

$$E_p\left(r,\theta,\phi\right) = E_p^{\mathrm{hex}}\left(r,\theta,\phi\right)\ \tilde{J}_{sx}^{1,0}\left(k_x,k_y\right) \qquad \left(p=\theta \text{ or } \phi\right)$$

and

E_p^{hex} = far field of unit-amplitude horizontal electric dipole in the x direction.

The p Factor (cont.)

Denote the patch space (array) factor as:

$$A(\theta,\phi)=\tilde{J}_{sx}^{1,0}\left(k_x,k_y\right)$$

Recall:

$$\tilde{J}_{sx}^{(1,0)}\left(k_x,k_y\right)=\int_S J_{sx}^{(1,0)}(x,y)\,e^{j(k_x x+k_y y)}dS$$
$$=\int_{-L/2}^{L/2}\cos\left(\frac{\pi x}{L}\right)e^{jk_x x}dx\int_{-W/2}^{W/2}e^{jk_y y}\,dy$$
$$=\left[\frac{\pi}{2}L\frac{\cos\left(k_x\dfrac{L}{2}\right)}{\left(\dfrac{\pi}{2}\right)^2-\left(k_x\dfrac{L}{2}\right)^2}\right]\left[W\mathrm{sinc}\left[k_y\frac{W}{2}\right]\right]$$

$$k_x=k_0\sin\theta\cos\phi$$
$$k_y=k_0\sin\theta\sin\phi$$

Then we have:

$$E_p\left(r,\theta,\phi\right)=E_p^{\mathrm{hex}}\left(r,\theta,\phi\right)A\left(\theta,\phi\right)$$

and

$$S_r\left(r,\theta,\phi\right)=S_r^{\mathrm{hex}}\left|A\left(\theta,\phi\right)\right|^2$$

The *p* Factor (cont.)

Hence, we have:

$$P_{\mathrm{sp}} = \int_0^{2\pi}\int_0^{\pi/2} S_r^{\mathrm{hex}} \left|A(\theta,\phi)\right|^2 r^2 \sin\theta \, d\theta \, d\phi$$

We also have:

$$P_{\mathrm{sp}}^{\mathrm{dip}} = \left|(Il)_{\mathrm{patch}}\right|^2 \int_0^{2\pi}\int_0^{\pi/2} S_r^{\mathrm{hex}}\left(r,\theta,\phi\right) r^2 \sin\theta \, d\theta \, d\phi$$

We can write the moment of the equivalent dipole as:

$$\left(Il\right)_{\mathrm{patch}} = \int_{-L/2}^{+L/2}\int_{-W/2}^{+W/2} J_{sx}^{1,0}(x,y)\,dy\,dx = \tilde{J}_{sx}^{1,0}\overset{\substack{(k_x,k_y)\\ \Uparrow}}{(0,0)} = A\overset{\substack{(\theta,\phi)\\ \Uparrow}}{\left(0,0\right)}$$

$$k_x = k_0 \sin\theta\cos\phi$$
$$k_y = k_0 \sin\theta\sin\phi$$

The p Factor (cont.)

Hence, we have:

$$p = \frac{\int_0^{2\pi}\int_0^{\pi/2} S_r^{\text{hex}}\left(r,\theta,\phi\right)\left|A(\theta,\phi)\right|^2 r^2 \sin\theta \, d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} S_r^{\text{hex}}\left(r,\theta,\phi\right)\left|A(0,0)\right|^2 r^2 \sin\theta \, d\theta d\phi}$$

The p Factor (cont.)

This may be written as (from Notes 12):

$$p = \frac{\int\limits_0^{2\pi}\int\limits_0^{\pi/2}\left[|F(\theta)|^2 \sin^2\phi + |G(\theta)|^2\cos^2\phi\right]|A(\theta,\phi)|^2 \sin\theta\, d\theta\, d\phi}{\int\limits_0^{2\pi}\int\limits_0^{\pi/2}\left[|F(\theta)|^2 \sin^2\phi + |G(\theta)|^2\cos^2\phi\right]|A(0,0)|^2 \sin\theta\, d\theta\, d\phi}$$

Recall:

$$E_\theta = (Il)\,E_0 \cos\phi\, G(\theta)$$
$$E_\phi = (Il)\,E_0 (-\sin\phi)\, F(\theta)$$

where

$$A(\theta,\phi) = \left(\frac{\pi}{2}WL\right)\operatorname{sinc}\left[k_y \frac{W}{2}\right]\left[\frac{\cos\left(k_x \frac{L}{2}\right)}{\left(\frac{\pi}{2}\right)^2 - \left(k_x \frac{L}{2}\right)^2}\right]$$

$$k_x = k_0 \sin\theta \cos\phi$$
$$k_y = k_0 \sin\theta \sin\phi$$

Note: $A(\theta,\phi)$ depends on W, L but not the substrate parameters.

The p Factor (cont.)

The patch space (array) factor:

$$A(\theta,\phi)=\left(\frac{\pi}{2}WL\right)\text{sinc}\left[k_y\frac{W}{2}\right]\left[\frac{\cos\left(k_x\dfrac{L}{2}\right)}{\left(\dfrac{\pi}{2}\right)^2-\left(k_x\dfrac{L}{2}\right)^2}\right]$$

$$k_x = k_0 \sin\theta\cos\phi$$
$$k_y = k_0 \sin\theta\sin\phi$$

Observation:

As $W, L \to 0$ $\quad A(\theta,\phi)\to\dfrac{2}{\pi}(WL)=A(0,0)$

Hence $p \to 1$